Exchange Rate Disconnect and the Trade Balance*

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Abstract

Unlike mechanisms of exchange rate determination that imply a stark dichotomy between the exchange rate and macroeconomic behavior, we offer a theory that naturally reunites macroeconomic fluctuations and the exchange rate through movements in the trade balance while being consistent with standard metrics of the exchange rate disconnect. We introduce time-varying preferences for domestic goods into an open economy model of exchange rate determination. These preferences induce trade rebalancing that shifts export demand towards domestic goods and reduces imports of foreign goods. We derive theoretical and quantitative implications of trade rebalancing as a candidate mechanism to explain several exchange rate puzzles observed in the data. The role of exogenous UIP deviations is substantially reduced in the presence of trade rebalancing.

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1 Introduction

Real exchange rate (RER) determination is central to international macroeconomics. Yet, existing mechanisms of exchange rate determination imply a stark dichotomy between the exchange rate and macroeconomic behavior to resolve long-standing puzzles observed in the data (Fama 1984; Backus and Smith 1993; Obstfeld and Rogoff 2000). Instead, we offer a theory that naturally reunites macroeconomic fluctuations and the exchange rate through movements in the trade balance while being consistent with standard metrics of the exchange rate disconnect. Thus, we contribute to the significant progress in explaining exchange rate puzzles using optimizing models of open economies (Gabaix and Maggiori 2015; Eichenbaum, Johannsen, and Rebelo 2021; Itskhoki and Mukhin 2021).

We show that trade and financial integration are crucial in explaining the relationship between macroeconomic variables and the exchange rate in our model. More specifically, the link between the real exchange rate and the macroeconomy emerges from the interaction between trade rebalancing shocks and the degree of financial integration that facilitates trade in financial assets. These two features are crucial to explaining all the puzzling moments about exchange rates we observe in the data.

It is commonplace in the exchange rate disconnect literature to ignore shocks directly impacting trade flows—see Itskhoki and Mukhin (2021), Eichenbaum, Johannsen, and Rebelo (2021). In such frameworks, UIP shocks are the main drivers of the real exchange rate. However, these models have counterfactual implications for the dynamics of international trade and the trade balance. In particular, they induce excess volatility in the trade balance and cannot capture the weak correlation between the real exchange rate and the trade balance at business cycle frequencies. In our model, rebalancing shocks induce terms of trade and real exchange rate fluctuations without much impact on aggregate variables such as output and consumption, thus offering an alternative mechanism for explaining the macroeconomic disconnect of exchange rates. In addition, our model can account for the lack of international risk sharing documented Backus and Smith (1993) and the forward premium puzzle (Fama 1984).

We offer three contributions relative to existing work. First, we develop a tractable model of flexible prices with incomplete markets in which we can obtain analytical solutions that characterize the exchange rate disconnect and the role of trade rebalancing in exchange rate determination. Second, our simple model goes a long way in matching many exchange rate moments. Crucial for this result is that we can use the trade rebalancing shocks to target moments that relate to the real exchange rate and the trade balance. Third, we extend our analytical insights to a New Keynesian open economy model of the U.S. and the Rest of the World that we confront with the data. In our estimated model, we quantify the main shocks that drive the exchange rate and find that trade rebalancing shocks account for close to 50 percent of the variance of the real exchange rate and the trade balance at business cycle frequencies (8-32 quarters). The presence of trade shocks also reduces the importance of exogenous UIP deviations in the medium term.

Relation to the literature. Our work relates to three strands in the literature. First, we allow financial-type shocks to contribute to exchange rate fluctuations, in line with a large body of work building on (Gabaix and Maggiori 2015). Relative to these papers, we demonstrate that data on trade flows are informative about the importance of different shocks in explaining the exchange rate and, through implications for international risk sharing, for UIP deviations. We attribute an important but not dominant role to UIP shocks while showing that shocks related to trade flows play an important role. This sets us apart from, for example, Itskhoki and Mukhin (2021), Itskhoki and Mukhin (2023), and Eichenbaum, Johannsen, and Rebelo (2021), who argue in favor of exogenous deviations from UIP as the main driver of the RER. In particular, the former two papers argue against rebalancing shocks as a key driver of the RER. Their conclusion rests on the behavior of the economy that approaches a trade autarky in the limit, i.e., complete home bias, which eliminates a role for rebalancing shocks by assumption. However, as can be seen from our results, rebalancing shocks can be consistent with the disconnect patterns once we step away from this limit. Trade data becomes informative for telling different shocks apart as drivers of the RER. In addition, it is also important to keep in mind that UIP shocks still play an important role in explaining the RER in our results, especially at business cycle frequency, and that our rebalancing shocks also generate moments consistent with the exchange rate disconnect when looking beyond trade flows itself. As such, there is no tension with the structural analysis of Eichenbaum, Johannsen, and Rebelo (2021) or the VAR-based analysis in Miyamoto, Nguyen, and Oh (2022).

Second, we build on insights from quantitative models using trade data to inform on the drivers of the RER in other contexts. Here, our contribution is to show that rebalancing shocks can be consistent with the disconnect puzzles and informative for the degree of asset market frictions, which allows us to draw sharper inferences on their role. Alessandria and Choi (2021) quantifies the sizable role of changes in trade costs for the path of the U.S. trade balance and RER after 1980. Gornemann, Guerrón-Quintana, and Saffie (2020), MacMullen and Woo (2023), Ayres, Hevia, and Nicolini (2021) point to the importance of trade data, trade frictions, and commodity prices in capturing the dynamics of the RER in quantitative international DSGE models. However, all these papers treat the degree of friction in international financial markets as given. As a result, most of the deviations from UIP result from shocks. We show both analytically and in a quantitative model that trade data and its correlation with other series, like the RER, are, in fact, informative for these frictions as well. Our paper is, therefore, closer to work by Fitzgerald (2012), who uses bilateral trade data to learn about the degree of trade and asset market frictions between countries. However, while Fitzgerald (2012) derives a set of test statistics to probe for these frictions between many countries using mainly trade data in line with the literature testing for deviations from complete markets, we utilize the co-movement of many macro-variables to draw inferences about the key frictions and shocks, more in line with work in quantitative DSGE models. Our strategy also allows us to speak directly to various puzzles in the international literature.

Third, we contribute to the literature by studying the drivers of the exchange rate more broadly, bringing new results to the discussion and complementing the analysis aimed at telling them apart. In this sense, here we are complementing Itskhoki and Mukhin (2021) and Itskhoki and Mukhin (2023) in putting less emphasis on TFP and TFP-news based stories as in Corsetti, Dedola, and Leduc (2008), Colacito and

Croce (2011), Heathcote and Perri (2014), and Chahrour, Cormun, Leo, Guerron-Quintana, and Valchev (2021). Our emphasis on different drivers of the RER in concert with the UIP shock also aligns well with Fukui, Nakamura, and Steinsson (2023).

Layout. The rest of the paper is organized as follows. Section 2 presents the analytical model and derives the main theoretical results. Section 3 revisits the exchange rate disconnect in the data and our simple model. Section 4 explores the role of trade and financial integration to account for the exchange rate disconnect observed in the data. Section 5 estimates a medium-scale model featuring real and nominal rigidities to quantify the main drivers of the real exchange rate in the data. We offer our conclusions in Section 6.

2 Analytical Model

For traceability, we develop our argument in a simple model before turning to a quantitative business cycle model that encompasses the features of the analytical model in Section 5. A continuum of agents of mass one lives in each of two equally-sized countries. Each country produces one good using labor as the only input into production. Both prices and wages are fully flexible. Home and foreign goods are imperfect substitutes and traded across borders. International financial markets are incomplete, as captured by restricting financial flows to a single non-state-contingent bond. In addition, we assume that there are limits to the amount of debt intermediated internationally. This feature gives rise to *endogenous* departures from the uncovered interest rate parity (UIP) condition similar to the financial intermediation model of Gabaix and Maggiori (2015) or the convenience yield model of Valchev (2020).

The model features three sources of uncertainty: technology shocks that alter total factor productivity, rebalancing shocks that alter the relative demand for the domestically-produced and the imported goods, and a financial shock that gives rise to *exogenous* departures from the UIP condition as in Itskhoki and Mukhin (2023).[1]

[1] Itskhoki and Mukhin (2023) discusses several micro-foundations for UIP deviation shocks, including shocks

2.1 Assumptions

The intertemporal preferences of the representative household in Country 1, the home country, are

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln \left(C_{1,t+j} \right) - L_{1,t+j} \right\}. \tag{1}$$

The felicity function in period $t+j$ depends on consumption, $C_{1,t+j}$, as well as hours worked, $L_{1,t+j}$. The household chooses consumption, labor supply, and asset holdings to maximize the intertemporal utility given the budget constraint

$$P^c_{1,t+j} C_{1,t+j} + P^b_{1,t+j} B_{1,t+j} + J_{1,t+j} = W_{1,t+j} L_{1,t+j} + B_{1,t-1+j}. \tag{2}$$

Expenditures for consumption are given by $P^c_{1,t+j} C_{1,t+j}$, and wage income is given by $W_{1,t+j} L_{1,t+j}$. The household purchases (or issues) the non-state-contingent bond in the amount of $B_{1,t+j}$ and receives (or pays out) the amount $B_{1,t-1+j}$. The term $J_{1,t+j}$ stands for mechanisms that limit the ability of households to issue/hold debt. Among the many mechanisms offered in the literature are

1. a financial intermediation cost, as in Turnovsky (1985) or Bodenstein, Erceg, and Guerrieri (2011), with

$$J_{1,t+j} = \left(\frac{1}{\phi^b_{1,t+j}} - 1 \right) P^b_{1,t+j} B_{1,t+j} \tag{3}$$

where, as shown in more detail below, $\phi^b_{1,t+j}$ is a function of aggregate aggregate amount of bonds issued;

2. a quadratic portfolio adjustment cost, as in Fukui, Nakamura, and Steinsson (2023) or Guo, Ottonello, and Perez (2023), with

$$J_{1,t+j} = \frac{1}{2} \tau \left(\frac{B_{1,t+j}}{P^d_{1,t+j} M^*_{2,t+j}} \right)^2 P^d_{1,t+j} M^*_{2,t+j} \tag{4}$$

to the utility from holding specific assets, noise traders, and time-varying risk premia. Valuation shocks that alter households' effective time preference as in Albuquerque, Eichenbaum, Luo, and Rebelo (2016) and Bodenstein, Corsetti, and Guerrieri (2020) can also be a source of UIP deviations in our setting.

where here the costs are measured by household bond holdings relative to the aggregate value of exports (not the individual household's choice), $P^d_{1,t+j}M^*_{2,t+j}$;

3. a borrowing constraint on debt, as in de Groot, Durdu, and Mendoza (2023), with

$$J_{1,t+j} = -\mu_{1,t+j}\left(B_{1,t+j} + \bar{B}\right) \tag{5}$$

where household assets cannot fall below the adhoc level $-\bar{B}$, i.e., $B_{1,t+j} > -\bar{B}$. Denoting the Lagrange multiplier on the time-t budget constraint by $\beta^t\lambda_{1,t+j}$, the borrowing constraint can be absorbed into the budget constraint since $\beta^t\lambda_{1,t+j}\mu_{1,t+j}\left(B_{1,t+j} + \bar{B}\right) = 0$.

Approaches 1 and 2 affect the household's ability to smooth consumption via international financial markets because the level of asset holdings directly influences the price of debt. When asset holdings are high, the higher cost of borrowing discourages further accumulation of debt and limits risk sharing (or consumption smoothing). The two approaches are first-order equivalent. Approach 3 introduces a hard limit on the quantity of bonds in circulation. Once the constraint binds, no more bonds can be issued. The closer the quantity of outstanding debt is to the limit $\bar{B}$, the higher the cost of borrowing.[2]

Each of these mechanisms limits the extent of international financial intermediation and thus consumption risk sharing across countries. In addition, they lead to endogenous departures from the UIP condition and give rise to excess returns on foreign assets. In this regard, all these approaches capture the essence of the micro-founded model of financial intermediation presented in Gabaix and Maggiori (2015), in which international financiers are constrained in their ability to bear risks from international imbalances.

In the following, we pursue the approach with costly financial intermediation with $\phi^b_{1,t} = \exp\left(-\frac{\chi}{2}\frac{B^*_{1,t}}{P^d_{1,t}M^*_{2,t}}\right)$, where $B^*_{1,t}$ denotes the aggregate amount of bonds

[2] Approaches 1 and 2 are widely used in the international macro literature to render the dynamics of the NFA position stationary under popular local perturbation methods. See Schmitt-Grohé and Uribe (2003), Bodenstein (2011) and references therein. As shown in de Groot, Durdu, and Mendoza (2023), under suitable parameter choices, these two approaches imply similar dynamics for the endogenous variables as Approach 3, for which obtaining the equilibrium dynamics requires global solution methods and numerical algorithms.

issued, not the individual household's holdings normalized by the aggregate value of exports, $M^*_{2,t}$. In other words, households do not internalize the effects of their asset choice on the intermediation cost. The cost parameter χ governs the extent of intermediation and, therefore, intertemporal consumption risk sharing through international financial markets: a given value of the net-foreign-asset (NFA) position is associated with higher bond prices from the perspective of the country if χ is larger, which in turn reduces the country's inclination to borrow. If χ approaches infinity, international risk sharing is shut down, and countries exist in financial autarchy. Setting $\chi = 0$ shuts down endogenous deviations from the UIP condition. Thus, the parameter governs the speed with which the NFA position unwinds over time.[3]

We assume a similar intermediation function for the foreign country with $\phi^b_{2,t} = \exp\left(-\frac{\chi}{2}\frac{\frac{1}{e_{1,t}}B^*_{2,t}}{P^d_{2,t}M_{1,t}} + \xi^{UIP}_{1,t}\right)$.[4] Notice that we assume in the foreign country's intermediation function the presence of the stochastic component $\xi^{UIP}_{1,t}$ which introduces exogenous departures from the uncovered interest rate parity (UIP) condition as discussed in Itskhoki and Mukhin (2021).[5]

The final consumption good, $C_{1,t}$, is an aggregate of the home good, $C^d_{1,t}$, and imports of the foreign good, $M_{1,t}$,

$$C_{1,t} = \left(\left(\omega^c_{1,t}\right)^{\frac{\rho^c}{1+\rho^c}}\left(C^d_{1,t}\right)^{\frac{1}{1+\rho^c}} + \left(1-\omega^c_{1,t}\right)^{\frac{\rho^c}{1+\rho^c}}\left(M_{1,t}\right)^{\frac{1}{1+\rho^c}}\right)^{1+\rho^c}. \tag{6}$$

The elasticity of substitution between the domestic and foreign goods is measured by $\frac{1+\rho^c}{\rho^c}$. The shared parameter $\omega^c_{1,t} = \omega^c_1 \exp\left(\xi^{trade}_{1,t}\right)$ is time-varying to allow for shifts in the relative demand for the home and foreign goods, that are not directly induced by a change in the relative price. Accordingly, we label shocks to the shared parameter "rebalancing shocks." As discussed in Appendix C, the rebalancing shock can be obtained as a shock to import tariffs, export subsidies, transportation costs, or a combination thereof.

[3] When applying local perturbation methods to solve for the equilibrium dynamics as we do here, the NFA position follows a unit-root process for $\chi = 0$. See Bodenstein (2011) for an extensive discussion.

[4] Aggregate bond holdings abroad, $B^*_{2,t}$ are normalized by foreign exports, $P^d_{2,t}M^*_{1,t}$.

[5] In the linearized model, there is no difference whether the UIP shock enters only in one of the functions. We keep the subscript of the home country because we refer to a positive UIP shock as an increase in the home country's assets.

We denote the price of the home good by $P^d_{1,t}$ and the price of the imported foreign good by $P^m_{1,t}$. The price of the imported foreign good satisfies $P^m_{1,t} = e_{1,t}P^d_{2,t}$, where $e_{1,t}$ is the nominal exchange rate and $P^d_{2,t}$ is the price of the foreign good in the foreign country. The terms of trade, $\delta_{1,t}$, are the ratio of import prices of the two countries expressed in common currency

$$\delta_{1,t} = \frac{e_{1,t}P^d_{2,t}}{P^d_{1,t}}. \tag{7}$$

Relatedly, their real consumption exchange rate is defined as

$$q_{1,t} = \frac{e_{1,t}P^c_{2,t}}{P^c_{1,t}}. \tag{8}$$

We assume that prices and wages are flexible. Production of each country's goods is linear in the country's labor, which is prized at the wage $W_{1,t}$. Total output, $Y_{1,t}$, in country 1 is

$$Y_{1,t} = exp(z_{1,t})L_{1,t}. \tag{9}$$

The assumptions for preferences and productions in country 2 mirror those of country 1 in equations 1-6 and 9. The rebalancing shock in country 2 follows $\omega^c_{2,t} = \omega^c_2 \exp\left(\xi^{trade}_{2,t}\right)$. As already discussed, the financial intermediation costs in the foreign country are $\phi^b_{2,t} = \exp\left(-\frac{\chi}{2}\frac{\frac{1}{e_{1,t}}B^*_{2,t}}{P^d_{2,t}M_{1,t}} + \xi^{UIP}_{1,t}\right)$ and include the exogenous UIP shock, $\xi^{UIP}_{1,t}$.

Market clearance in goods and financial markets requires

$$Y_{1,t} = C^d_{1,t} + M_{2,t} \tag{10}$$

$$Y_{2,t} = C^d_{2,t} + M_{1,t} \tag{11}$$

$$0 = B_{1,t} + B_{2,t}. \tag{12}$$

We define the trade balance normalized by the value of exports, $P^d_{1,t}M_{2,t} = e_tP^m_{2,t}M_{2,t}$, as

$$\tilde{T}_{1,t} = \frac{T_{1,t}}{e_tP^m_{2,t}M_{2,t}} \equiv \frac{e_tP^m_{2,t}M_{2,t} - P^m_{1,t}M_{1,t}}{e_tP^m_{2,t}M_{2,t}}. \tag{13}$$

We choose the price of domestic goods to be the numeraire.

The exogenous technology, rebalancing, and financial (UIP) shocks follow autoregressive processes of order 1 with

$$\xi_{1,t}^{trade} = \rho_1^{trade}\xi_{1,t-1}^{trade} + \sigma_1^{trade}\epsilon_{1,t}^{trade} \quad (14)$$

$$\xi_{2,t}^{trade} = \rho_2^{trade}\xi_{2,t-1}^{trade} + \sigma_2^{trade}\epsilon_{2,t}^{trade} \quad (15)$$

$$\xi_{1,t}^{UIP} = \rho_1^{UIP}\xi_{1,t-1}^{UIP} + \sigma_1^{UIP}\epsilon_{1,t}^{UIP} \quad (16)$$

$$z_{1,t} = \rho_1^z z_{1,t-1} + \sigma_1^z \epsilon_{1,t}^z \quad (17)$$

$$z_{2,t} = \rho_2^z z_{2,t-1} + \sigma_2^z \epsilon_{2,t}^z. \quad (18)$$

2.2 Model Solution

We solve a linear approximation of the model around a symmetric deterministic steady state with $\omega_1^c = \omega_2^c$ and balanced trade, i.e., $\tilde{T}_1 = 0$, $\delta_1 = 1$, $B_1 = 0$. As shown in Appendix A, we can simplify the model to the following system of equations:

$$(z_{1,t} - E_t z_{1,t+1}) - (z_{2,t} - E_t z_{2,t+1}) - \left(\hat{\delta}_{1,t} - E_t\hat{\delta}_{1,t+1}\right) = \chi\tilde{B}_{1,t} + \xi_{1,t}^{UIP} \quad (19)$$

$$\beta\tilde{B}_{1,t} = \tilde{T}_{1,t} + \tilde{B}_{1,t-1} \quad (20)$$

$$\tilde{T}_{1,t} = \frac{\omega_1^c}{1-\omega_1^c}\xi_{1,t}^{trade} - \frac{\omega_1^c}{1-\omega_1^c}\xi_{2,t}^{trade} - z_{1,t} + z_{2,t} + \varpi\hat{\delta}_{1,t}, \quad (21)$$

where $\varpi = 1 + 2\frac{\omega_1^c}{\rho^c}$. The terms trade, $\hat{\delta}_{1,t}$, are measured in log deviation from the steady state. The normalized trade balance, $\tilde{T}_{1,t}$, and the normalized domestic bond position (or NFA position), $\tilde{B}_{1,t} = \frac{B_{1,t}}{e_t P_{2,t}^m M_{2,t}}$, are measured in absolute deviation from their steady-state values of zero.

Equation 19 is the linearized risk-sharing condition under incomplete markets (or uncovered interest rate parity condition). The terms $\chi\tilde{B}_{1,t}$ and $\xi_{1,t}^{UIP}$ introduce time-varying wedges in the Equation and play a key role in our analysis. Equation 20 is the linearized condition governing how the NFA position evolves. Finally, Equation 21 is the linearized definition of the trade balance. For completeness, notice that the real exchange rate is proportional to the terms of trade, $\hat{q}_{1,t} = (2\omega_1^c - 1)\,\hat{\delta}_{1,t}$. An improvement in the terms of trade of country 1, $\hat{\delta}_{1,t} < 0$, goes along with an appreciation of the consumption real exchange rate, $\hat{q}_{1,t} < 0$. We solve for the decision rules of the endogenous variables in Appendix A.

2.3 Importance of the Trade Balance

In the remainder of this section, we demonstrate how the trade balance dynamics interact with the exchange rate dynamics. Theorem 1 establishes that the use of trade (balance) data allows one to distinguish empirically between rebalancing and financial (UIP) shocks because the shocks move the trade balance in opposite directions whenever they move the real exchange rate (or term of trade) in the same direction. Theorem 3 and Theorem 4 document that the major exchange rate puzzles in the literature do not contain sufficient information to distinguish empirically between rebalancing and financial (UIP) shocks. Both types of shocks have identical predictions about the exchange rate disconnect puzzle, the consumption-real-exchange-correlation puzzle, see Backus and Smith (1993), and the forward premium puzzle, see Fama (1984) and Engel (2011).

2.3.1 Trade Balance and Shock Identification

Theorem 1 *A rebalancing shock that improves the home country's terms of trade (appreciates the real exchange rate), $\xi_{1,t}^{trade} > 0$ and/or $\xi_{2,t}^{trade} < 0$, is associated with an improvement of the trade balance. By contrast, a financial (UIP) shock that improves the terms of trade (appreciates the real exchange rate), $\xi_{1,t}^{UIP} > 0$, is associated with a deterioration of the trade balance. If financial markets provide less risk sharing, i.e., χ assumes a higher value, the terms of trade are more (less) sensitive to the rebalancing (financial) shock, and the trade balance is less sensitive to both the rebalancing and the financial shock.*

The proof of Theorem 1 is provided in Appendix B.1. We use the equilibrium decision rules for the endogenous variables associated with the linear system in equations 19-21. In the decision rule for the terms of trade, the coefficients associated with the financial (UIP) and rebalancing shocks have the same sign. In the decision rule for the trade balance, the associated coefficients have opposite signs. The sensitivity of the effects to changes in the degree of risk sharing is measured by the derivatives of the coefficients with respect to χ.

Intuitively, according to the risk sharing condition, Equation 19, a positive financial (UIP) shock, $\xi_{1,t}^{UIP} > 0$, induces an expected worsening of the terms of trade (depreciation of the real exchange rate) implying an initial improvement of the terms of trade (appreciation of the real exchange rate). Yet, this initial improvement of the terms of trade causes the trade balance to deteriorate, Equation 21, and reduces country 1's NFA position, Equation 20. In turn, the less favorable NFA position of country 1 dampens the initial impact on the terms of trade in Equation 19. The larger χ, the stronger this dampening effect, and the smaller the terms of trade response for a given magnitude of the shock $\xi_{1,t}^{UIP}$.

The rebalancing shock works primarily on the trade balance, as seen in Equation 21. A shock that increases demand for country 1's goods, i.e., $\xi_{1,t}^{trade} > 0$ or $\xi_{2,t}^{trade} < 0$, causes the trade balance and the NFA position of country 1 to improve. From Equation 19, the improvement in the NFA position requires an initial improvement of the terms of trade (and an expected future worsening), dampening the trade balance's initial reaction. This dampening effect is stronger the larger the value of χ.

Appendix B.1 also shows that this intuition is consistent with the unconditional covariance between the growth rate of the terms of trade and the growth rate of the trade balance.

Corollary 2 *The financial (UIP) shock induces a positive covariance between the growth rate of the terms of trade and the growth rate of the trade balance. The rebalancing shock induces a negative covariance between the two growth rates. When both shocks are present in the model, the overall covariance is determined by the extent of international financial risk sharing as measured by* χ.

2.3.2 Exchange Rate Puzzles and Lack of Identification

In the data, the real exchange rate experiences large swings without being associated with swings of comparable magnitude in other macroeconomic variables (real exchange rate disconnect). In addition, the correlation between the real exchange rate and relative consumption is low and often negative (Backus-Smith puzzle). Standard models of the international business cycle struggle to replicate exchange rate moments when they rely on shocks to technology and monetary policy. Under standard parameterization, a model with technology shocks only implies that the volatility of the real exchange rate is of similar magnitude as that of consumption and the correlation between relative consumption and the real exchange rate is very close to 1.[6]

Theorem 3 *Abstracting from technology shocks, the ratio of the standard deviation of the real exchange rate, $\hat{q}_{1,t}$, and consumption, $\hat{C}_{1,t}$, is independent of the relative variances of the rebalancing and the financial (UIP) shock,*

$$\frac{std\left(\hat{q}_{1,t}\right)}{std\left(\hat{C}_{1,t}\right)} = \frac{std\left(\Delta\hat{q}_{1,t}\right)}{var\left(\Delta\hat{C}_{1,t}\right)} = \frac{2\omega_1^c - 1}{1-\omega_1^c}. \tag{22}$$

The correlation between relative consumption, $\hat{C}_{1,t} - \hat{C}_{2,t}$, and the real exchange rate is equal to -1 regardless of the relative variances of the rebalancing and the financial (UIP) shock,

$$corr\left(\hat{C}_{1,t} - \hat{C}_{2,t}, \hat{q}_{1,t}\right) = -1. \tag{23}$$

Notice that for sufficient home bias, i.e., ω_1^c close to 1, the relative volatility of the real exchange rate can exceed the volatility of consumption multiple times.

The proof in Appendix B.2 uses the first-order approximations (see Appendix A.2) to aggregate consumption and the (consumption) real exchange rate (both in log-deviations)

$$\hat{C}_{1,t} = z_{1,t} - (1-\omega_1^c)\,\hat{\delta}_{1,t} \tag{24}$$

$$\hat{C}_{2,t} = z_{2,t} + (1-\omega_1^c)\,\hat{\delta}_{1,t} \tag{25}$$

$$\hat{q}_{1,t} = (2\omega_1^c - 1)\,\hat{\delta}_{1,t} \tag{26}$$

which relate consumption and the real exchange rate to the terms of trade and the

[6] Exceptions are Benigno and Thoenissen (2008) and Corsetti, Dedola, and Leduc (2008). The latter shows that if the wealth effects from technology shocks dominate the substitution effects, their model can explain the two puzzles.

technology shocks, $z_{1,t}$ and $z_{2,t}$. Neither the financial (UIP) nor the rebalancing shock enter directly into the equation 24-26 that determines the real exchange rate and consumption. Both shocks enter only indirectly through the terms of trade. Hence, the computed moments do not depend on the relative rebalancing and the financial (UIP) shock variances.

Turning to the forward premium puzzle, Fama (1984) finds that the hypothesis of uncovered interest rate parity is violated in the data. While this theory predicts that the coefficient in the regression of the exchange rate on the interest rate differential is equal to 1, empirical work finds negative coefficient estimates. Engel (2016) shows that the forward premium puzzle documented in Fama (1984) also holds in real terms.

Using the conditions for pricing a non-state-contingent bond in each country, Equation 19 can be written in terms of the real interest rate differential, $r_{1,t} - r_{2,t}$,

$$r_{1,t} - r_{2,t} = E_t \left(\hat{q}_{1,t+1} - \hat{q}_{1,t} \right) - \chi \tilde{B}_{1,t} - \xi_{1,t}^{UIP}. \tag{27}$$

The sum of terms $\chi \tilde{B}_{1,t} + \xi_{1,t}^{UIP}$ constitutes the one-period-ahead expected excess return, ι_t. If the UIP condition holds, $\iota_t = 0$. The financial (UIP) shock breaks the uncovered interest rate parity condition by introducing exogenous departures from UIP by assumption. When $\chi > 0$, the rebalancing shock also breaks the condition. The NFA position governs the extent of the endogenous UIP departures, $\tilde{B}_{1,t}$.

Theorem 4 *Suppose the model admits only rebalancing and financial (UIP) shocks. In that case, the Fama coefficient is constant and negative independent of the degree of international financial risk sharing as measured by χ, as long as $\chi \neq 0$:*

$$\hat{\beta}^{Fama} = \frac{cov \left(E_t \Delta \hat{q}_{1,t+1}, r_{1,t} - r_{2,t} \right)}{var \left(r_{1,t} - r_{2,t} \right)} = -\frac{2\omega_1^c - 1}{2 \left(1 - \omega_1^c \right)} = 1 - \frac{1}{2 \left(1 - \omega_1^c \right)} < 0. \tag{28}$$

See Appendix B.2 for the proof.

An alternative formulation of the forward premium puzzle regresses the excess return, $\iota_t = E_t \Delta \hat{q}_{1,t+1} - (r_{1,t} - r_{2,t})$, on the interest rate differential. In this case, we obtain for our model the regression coefficient

$$\hat{\beta}^{Fama,\iota} = \frac{cov\left(\iota_t, r_{1,t} - r_{2,t}\right)}{var\left(r_{1,t} - r_{2,t}\right)} = \hat{\beta}^{Fama} - 1 = -\frac{1}{2\left(1 - \omega_1^c\right)}. \tag{29}$$

2.4 Discussion

Both UIP and rebalancing shocks can help align the observations from international macro models with those in the data, in particular about exchange rate dynamics. However, as shown by Theorems 3 and 4, celebrated exchange rate puzzles do not provide relevant information to distinguish the driving forces behind exchange rate movements.

Our main result is that the trade balance dynamics interact with exchange rate dynamics. Theorem 1 shows that the trade balance response helps distinguish UIP shocks from other potential drivers of exchange rate movements, which in our case are the rebalancing shocks. Moreover, the transmission of both UIP and rebalancing shocks depends on the degree of risk-bearing capacity captured in the parameter χ, whose role is typically underappreciated in the literature. These findings do not imply that conventional shocks are not important in explaining the various features in the data. Still, without rebalancing and financial (UIP) shocks, it is nearly impossible to address all the major exchange rate puzzles and capture the role of endogenous deviations in UIP.

3 Quantitative Assessment in the Analytical Model

To complement the findings of the previous section, we document how well the simple model with flexible prices and wages captures the key quantitative and qualitative features of exchange rate moments in the data.

3.1 Data and Calibration

Using data covering the period 1985Q1-2019:Q2 on the real exchange rate (q), real interest rates (r), real economic GDP (Y), and consumption (C), as well as the trade-balance-to-exports ratio ($\tilde{T}$), we compute key empirical moments against which we assess our theory. The U.S. is treated as the home country (country 1). Data for the foreign country (country 2) are obtained as the trade-weighted average of the respective time series of 34 countries. The countries included represent 85 percent of 2019 world GDP on a PPP basis. Appendix D provides details.

Table 2 lists the empirical moments of interest: the standard deviation of the exchange rate relative to that of output, $\sigma(\Delta\hat{q})/\sigma(\Delta\hat{Y})$, the persistence of the real exchange rate, $\rho(\hat{q})$, the correlation of output across countries, $\rho(\Delta\hat{Y}_1, \Delta\hat{Y}_2)$, the correlation between output and consumption, $\rho(\Delta\hat{Y}, \Delta\hat{C})$, the correlation between the real exchange rate and relative consumption (Backus-Smith puzzle), $\rho(\Delta\hat{q}, \Delta\hat{C}_1 - \Delta\hat{C}_2)$, and Fama regression coefficient (forward premium puzzle), $\hat{\beta}$, the correlation between the trade balance and the real exchange rate, $\rho(\Delta\tilde{T}, \Delta\hat{q})$, and the standard deviation of the trade balance relative to that of the real exchange rate, $\sigma(\Delta\tilde{T})/\sigma(\Delta\hat{q})$—where "$\Delta$" denotes the change in a variable rather than its level. For each moment, we report GMM standard errors in parentheses.

Before comparing these data moments to our model, we detail our parameter choices in Table 1. Some parameters are set in line with existing estimates while others are chosen so the model loosely matches the empirical features in Table 2. We set the home and foreign parameters at equal values whenever appropriate. The elasticity of substitution between the domestic and foreign good, $\theta \equiv \frac{1+\rho^c}{\rho^c}$, is set at 1.5 which is consistent with long-run estimates for the U.S. at the macro level—see Alessandria and Choi (2021), MacMullen and Woo (2023), and Boehm, Levchenko, and Pandalai-Nayar (2023). The discount factor, β, is set at 0.995 to be consistent with a 2 percent real interest rate. The "home-bias" parameter ω^c of 0.9 matches the cross-country average of domestic sourcing shares of final consumption goods in the World Input-Output Tables.[7]

[7] We measure domestic sourcing shares from 2000-2014 for 44 countries using the World Input-Output Database. In the U.S., a direct reading of the BEA Input-Output tables shows that the domestic sourcing share is slightly higher, averaging 0.94 over the same period. However, since our calibration is symmetric between the U.S. and the foreign block, we prefer the cross-country average estimate, including the U.S. The degree of home bias is tightly linked to the tradability of goods and services. The services sector is almost entirely non-tradable, with a domestic sourcing share of 0.98. By contrast, the manufacturing and agricultural sectors in the economy are substantially more tradable, with a combined sourcing share of about 0.8. For the U.S., the high home bias in the data reflects the large share of the service sector in the U.S. economy.

Table 1: Parameters - Analytical Model

Structural Parameters			
Discount Factor	Trade Elasticity	Home Bias	Extent of Risk Sharing
β	θ	ω^c	χ
0.995	1.5	0.9	0.1
Shocks: Persistence			
ρ_z	$\rho_{\xi^{trade}}$	$\rho_{\xi^{UIP}}$	$\rho(z_1, z_2)$
0.9	0.9	0.9	0.98
Shocks: Standard Deviation			
$100\sigma_z$	$100\sigma_{\xi^{trade}}$	$100\sigma_{\xi^{UIP}}$	
1.05	1.1	1.97	

Notes: Parameters chosen to replicate key exchange rate moments reported in Table 2. Unless otherwise specified, we set the same parameters for the home and foreign economies.

To pin down the key parameter in our exercise χ, which governs the extent of risk sharing through international markets in the model, we target an auto-correlation of 0.98 for the home country's NFA position, thereby roughly matching the observed auto-correlation of the NFA position of the U.S. with respect to the rest of the world. We explore the role of home bias, ω_1^c, and the extent of risk sharing, χ, in Section 4 in greater depth.

Turning to the exogenous shocks, we set the standard deviation of the technology shock relative to the UIP shock as $\frac{\sigma_z}{\sigma_{\zeta^{UIP}}} = 0.56$ to match the relative standard deviations of the exchange rate and output in the data. In addition, the standard deviation of the rebalancing shock relative to the UIP shock is $\frac{\sigma_{\zeta^{trade}}}{\sigma_{\zeta^{UIP}}} = 0.53$ to bring the model close to the relative standard deviations of the exchange rate and the trade balance in the data. We also set the correlation of the innovations to the home and foreign technology shocks equal to $\rho_{\epsilon_z,\epsilon_z^*} = 0.98$ to match the GDP co-movement observed in the data. Finally, the auto-regressive persistence parameters of all shock processes are set to 0.9.

3.2 Data versus Model

Table 2 compares the selected data and model unconditional moments. We distinguish four groups of moments. First, under the header "Disconnect and PPP," we summarize the excess volatility of the real exchange rate and its near-unit-root behavior. Second, we focus on international business cycle moments under the header "IRBC." The third set corresponds to the Backus-Smith and Forward premium puzzles. In broad terms, the vast majority of the literature studying exchange rate dynamics focuses on these three groups of moments and their nominal counterparts (Corsetti, Dedola, and Leduc 2008; Itskhoki and Mukhin 2021; Itskhoki and Mukhin 2023). We extend this set of statistics by a fourth one, which captures the relationship between the real exchange rate and the trade balance. These moments have received hardly any attention in the context of the exchange rate disconnect analysis.[8] However, as our theoretical results show, these moments provide key identification restrictions to distinguish between financial (UIP) and the rebalancing shocks.

Table 2: Empirical Moments

	Disconnect and PPP		IRBC	
	a.	**b.**	**c.**	**d.**
	$\sigma(\Delta\hat{q})/\sigma(\Delta\hat{Y})$	$\rho(\hat{q})$	$\rho(\Delta\hat{Y}_1, \Delta\hat{Y}_2)$	$\rho(\Delta\hat{Y}, \Delta\hat{C})$
Data	4.23	0.96	0.46	0.63
	(0.43)	(0.01)	(0.15)	(0.11)
Model	4.21	0.9	0.42	0.66
	Backus-Smith and Forward Premium		RER and Trade Balance	
	e.	**f.**	**g.**	**h.**
	$\rho(\Delta\hat{q}, \Delta\hat{C}_1 - \Delta\hat{C}_2)$	Fama $\hat{\beta}$	$\rho(\Delta\tilde{T}, \Delta\hat{q})$	$\sigma(\Delta\tilde{T})/\sigma(\Delta\hat{q})$
Data	0.10	-3.90	0.20	1.24
	(0.16)	(1.22)	(0.14)	(0.08)
Model	-0.99	-3.95	0.21	1.27

Notes: Empirical moments computed using quarterly data from 1985Q1 to 2019Q2. GMM standard errors in parenthesis.

[8] The paper of MacMullen and Woo (2023) explores the exchange rate disconnect puzzle using a dynamic trade model calibrated to U.S. trade data.

Although we did not target most of the moments in Table 2 as part of our calibration strategy, the model performs remarkably well compared to the data. In particular, it almost perfectly matches the moments relating to the trade balance. Although our model qualitatively captures the failure of the perfect risk-sharing benchmark, the Backus-Smith correlation is substantially weaker in the data—pointing to the need to introduce additional model features as discussed in Section 5.

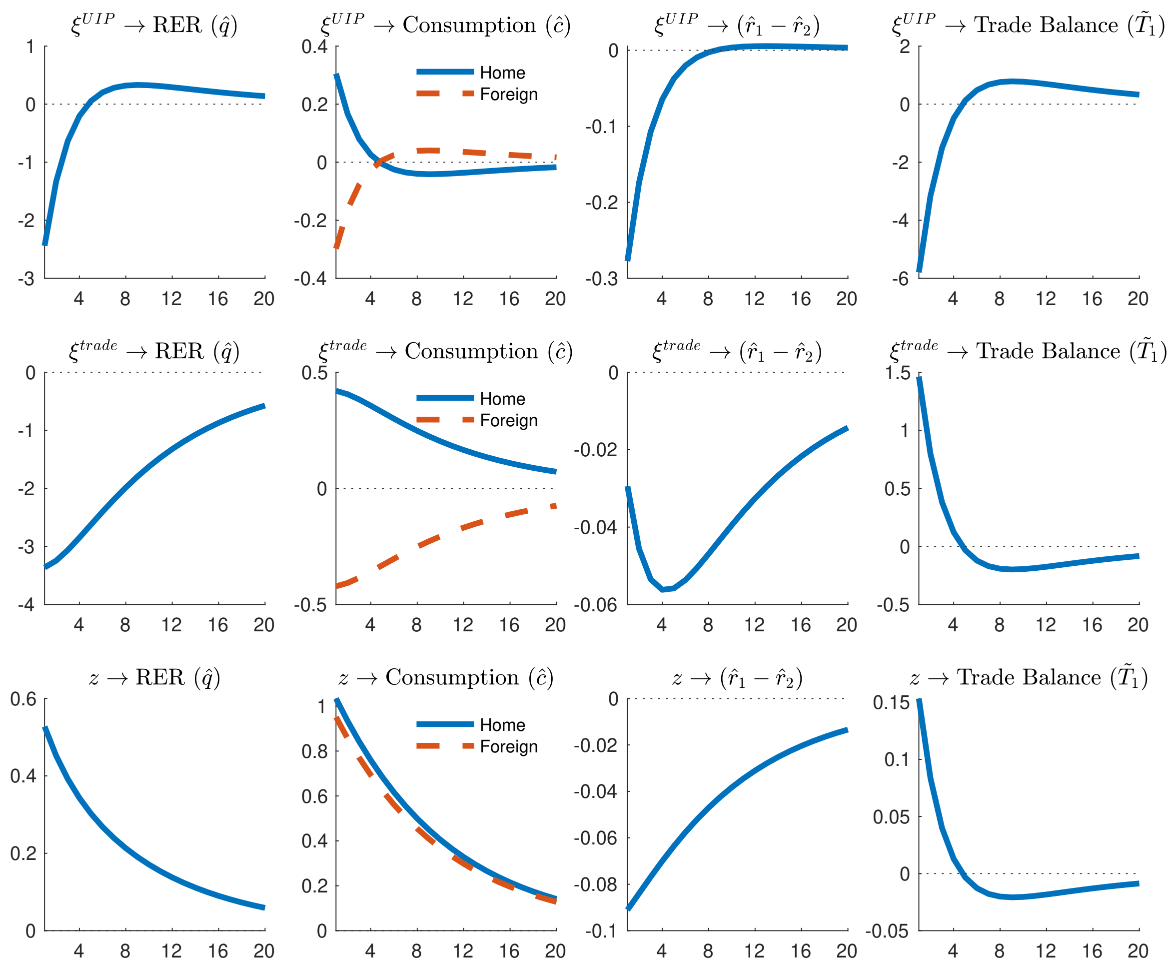
3.3 Exchange Rate Puzzles

To understand the exchange rate dynamics associated with each group of shocks, Figure 1 plots the impulse responses to a UIP shock ξ_1^{UIP}, a trade rebalancing shock towards home goods ξ_1^{trade}, and a home country technology shock z_1 under our preferred calibration.

The impulse responses confirm that the financial (UIP) shock helps address the major exchange rate puzzles. The financial (UIP) shock increases the foreign households' demand for the home country's bond. The resulting drop in the home country's NFA position is reflected in the deterioration of the home country's trade balance.[9] With aggregate foreign consumption, $\hat{C}_2$, being postponed into the future, foreign demand for the foreign good relative to the home good falls at given prices due to home bias in consumption. To equilibrate the goods markets, the terms of trade (measured from the perspective of the home country) improve, and the real exchange rate appreciates on impact so aggregate consumption in the home country, $\hat{C}_1$, can expand via increased demand for the foreign good relative to the home good. As a result, the real exchange rate and relative consumption, $\hat{C}_1 - \hat{C}_2$, are negatively correlated (Backus-Smith puzzle). Although the UIP shock changes the relative demand for two goods, the response of aggregate consumption is an order of magnitude smaller than the response of the real exchange rate (exchange rate disconnect puzzle). Finally, in addition to the expected depreciation of the real exchange rate following the initial appreciation, the foreign country's increased demand for the home country's bond causes a relative drop in the interest rate paid on the domestic bond so that $r_1 - r_2$

[9] This intuition is fully in line with a micro foundation of the UIP shock as a valuation shock that alters the effective time preference of households discussed in Section 2.

Figure 1: Impulse Responses



Notes: Impulse responses to one standard deviation shocks. First row: UIP shock. Second row: trade rebalancing shock. Third row: TFP shock. Blue solid lines correspond to the response of variables in the home country. Red dashed lines correspond to the response of variables in the foreign country.

turns negative (forward premium puzzle). Over time, as the direct effects of the shock dissipate, the NFA position determines the dynamics. As the foreign country sells off its accumulated assets, the home country's trade balance turns positive, and the real exchange rate depreciates relative to the steady state.

The trade rebalancing shock also addresses the three exchange rate puzzles; however, it moves the trade balance (and the NFA position) in the opposite direction. The shock raises the home country's appetite for its own goods and shifts home demand away from foreign goods towards home goods at given prices. Thus, the home country's terms of trade must improve for the goods market to clear. Again, the real exchange rate appreciates whereas relative consumption rises (Backus-Smith puzzle), and the increase in the real exchange rate is an order of magnitude larger than the movements in consumption (exchange rate disconnect puzzle). Contrary to the financial (UIP) shock, the exogenous boost to the demand for the home goods dominates the terms of trade effects and pushes the trade balance into surplus. As the foreign country increases its borrowing, intermediation costs rise, which in turn increases the overall cost of borrowing in the domestic bond. In equilibrium, the interest rate paid on the domestic bond falls so that $r_1 - r_2$ turns negative, whereas the real exchange rate is expected to depreciate after its initial appreciation (forward premium puzzle).

Technology shocks impact the economy in a fundamentally different manner. While the financial (UIP) and rebalancing shock primarily reallocates goods between the two countries, the technology shock increases the amount of goods available to both countries. A positive technology shock increases the production of the home goods. To equilibrate the goods market, the price of the home goods has to fall, causing the real exchange rate to depreciate. With aggregate consumption in the home country increasing by more than in the foreign country and the real exchange rate depreciating, the technology shock induces a counterfactual positive comovement between $\hat{q}$ and $\hat{C}_1 - \hat{C}_2$. Similarly, the model delivers the wrong comovement in the interest rate differential. Finally, the real exchange rate moves by less than aggregate consumption, and the technology shock also fails to reproduce the exchange rate disconnect.

4 Exploring the Disconnect Mechanism

We now use our model to explore the interplay of trade and financial integration to account for the exchange rate puzzles and investigate the main drivers of the real exchange rate.

4.1 The Role of Financial Integration

As discussed in Theorem 1, the portfolio adjustment cost parameter, χ, plays a central role in the theoretical predictions of our model. We now illustrate which exchange rate moments are affected as we vary the value of χ. Figure 2 shows theoretical moments under our benchmark calibration. Results for our preferred calibration of $\chi = 0.1$ are depicted with a red circle. Each panel corresponds to the corresponding moment in Table 2, and the red line indicates the associated value computed in our dataset. The blue line depicts the model's theoretical moments are calculated for different values of χ that range from near-perfect financial integration ($\chi = 0.001$) to configurations in which portfolio adjustment costs severely hamper the flow of financial assets ($\chi = 0.5$). In the language of Gabaix and Maggiori (2015), the parameter χ controls the capacity of the financial intermediary sector to absorb risk when taking foreign bond positions on behalf of households. Hence, in this experiment, we can assess the role that risk absorption plays in accounting for the different moments in the data.

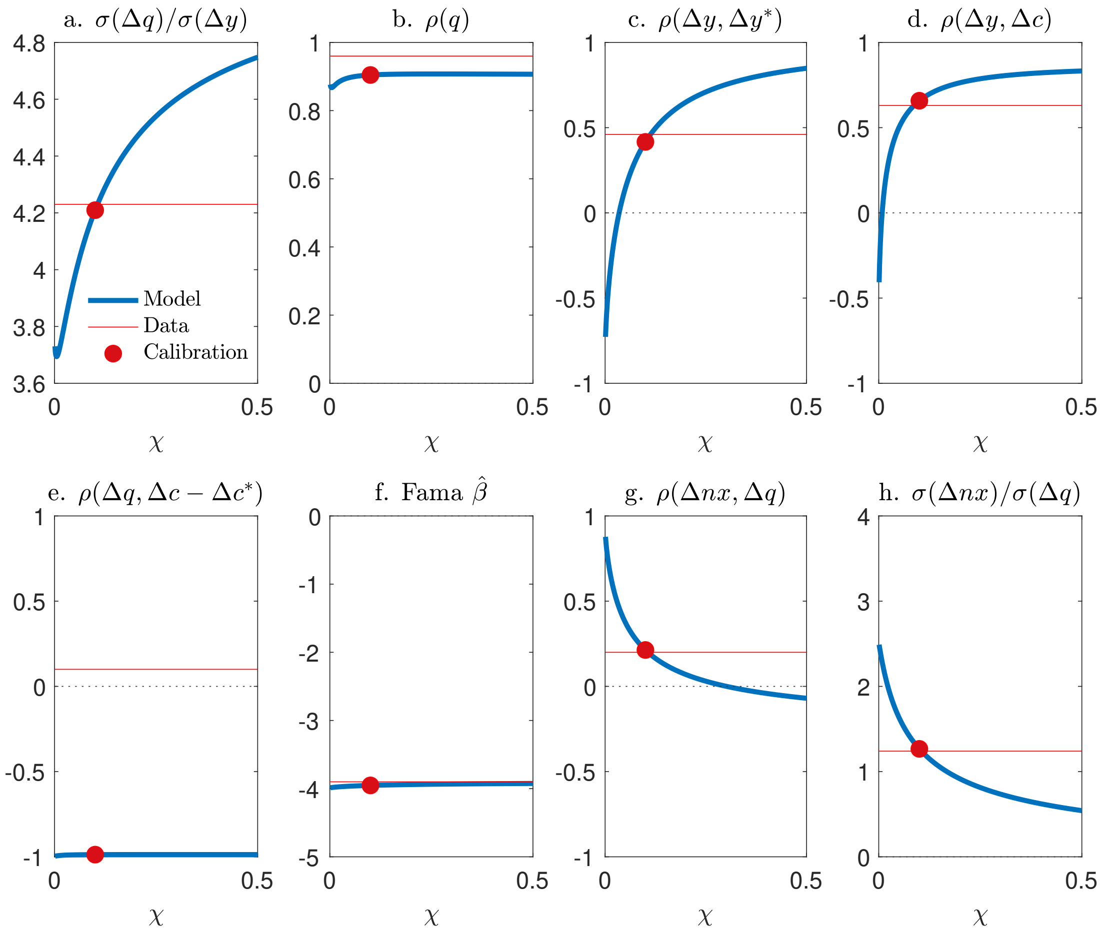
In Section 2, we showed that the Backus-Smith correlation and the Fama coefficient are independent of the parameter χ. This result extends to our quantitative model in which TFP shocks are also active.[10] Turning attention to other moments, the top row shows that reducing the risk-bearing capacity in the economy amplifies the volatility of the real exchange rate, exacerbating the macroeconomic disconnect (Panel a.), and strengthening the international comovement output and consumption (Panels c. and d.).

The intuition for this result is that higher portfolio adjustment costs reduce the flows of the only internationally traded asset, which reduces the ability of the economy to smooth out shocks that would require a trade balance adjustment. For example, if the preference for domestic goods increases at home, this would require improving the trade balance and the NFA at home. However, because of the higher cost of trading the international bond, the real exchange rate becomes more sensitive to changes in the NFA, triggering a much larger appreciation of the exchange rate.

Turning to the bottom row panels in Figure 2, a new result in our model is that we can account for the relation between the exchange rate and trade flows. In particular, our model matches the weak correlation between the trade balance and

[10] However, this outcome is calibration dependent. In the Appendix, we illustrate that TFP shocks produce positive Backus-Smith and Fama coefficients, which could potentially offset the negative values for these moments implied by the UIP and trade rebalancing shocks.

Figure 2: Financial Integration and Exchange Rate Moments



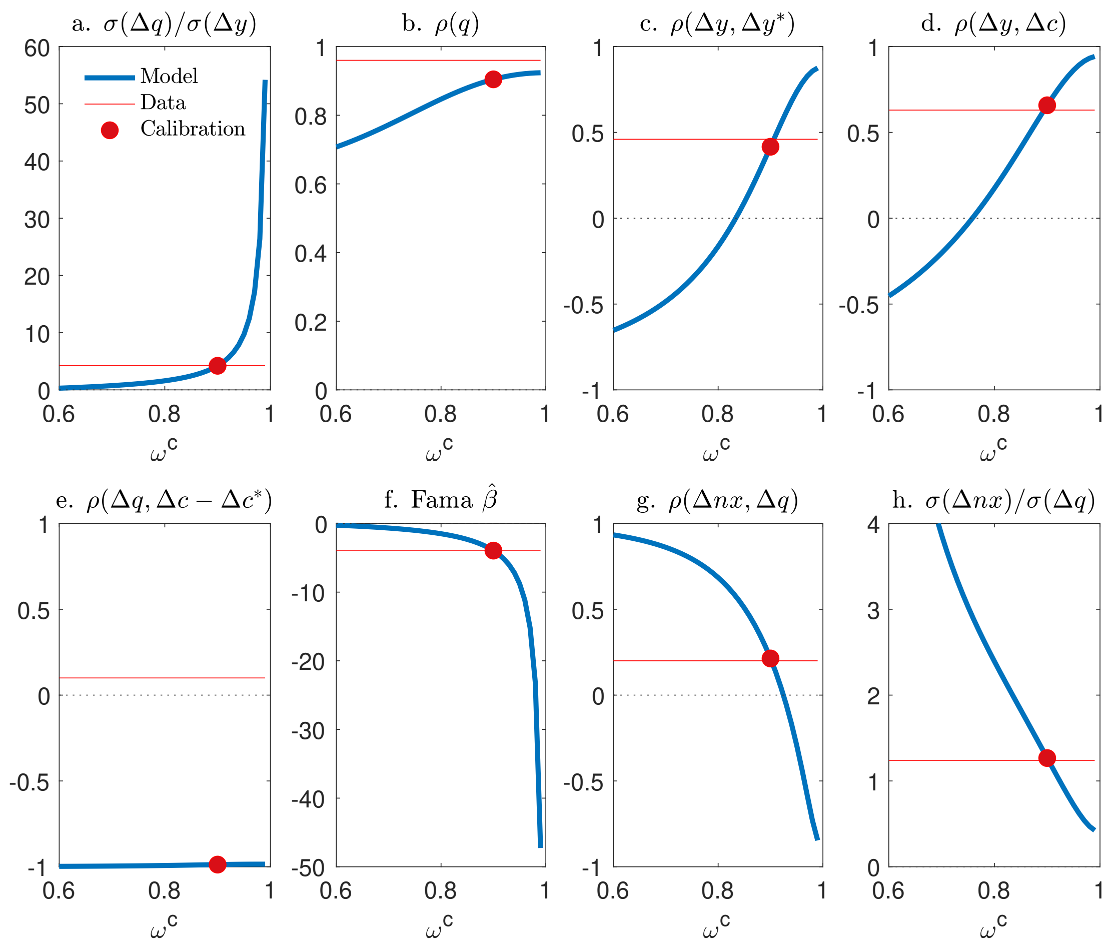
Notes: The blue line shows theoretical moments for different values of χ. The red horizontal line indicates the corresponding empirical moment in each panel. The red dot shows our preferred calibration.

the real exchange rate (g.) and the larger volatility of trade flows relative to the real exchange rate (h.). As we show in Section 4.4, the relationship between the trade balance and the real exchange rate plays a central role in distinguishing different models of exchange rate behavior and determining the drivers of the real exchange rate.

4.2 The Role of Trade Integration

How does trade integration affect the exchange rate puzzles? This question is important to understanding the transmission mechanism of shocks. For instance, (Itskhoki and Mukhin 2023) analyzes the limit in which an open economy converges towards (trade) autarky. In our setting, this corresponds to $\omega^c \to 1$, which implies domestic households have a nearly absolute preference for home goods, collapsing our model to that of a closed economy.

Figure 3: Trade Integration and Exchange Rate Moments



Notes: The blue line shows theoretical moments for different values of χ. The red horizontal line indicates the corresponding empirical moment in each panel. The red dot shows our preferred calibration.

Figure 3 shows exchange rate moments in our model as a function of the home-bias parameter. Lower values of ω^c correspond to a more open economy with greater trade integration. Higher values of ω^c correspond to an economy with lesser trade integration. The disconnect puzzles (panel a. and panel b.) become more stark as the economy moves towards trade autarky. Similarly, the IRBC correlation also increases with the home-bias parameter.

These results follow from the intertemporal approach of the current account pioneered in Obstfeld and Rogoff (1995). In all modern open economy models, the trade balance is the adjustment mechanism that helps smooth consumption over time. In our model, the consumption-smoothing mechanism is embedded in Equation 21, which we reproduce below:

$$\tilde{T}_{1,t} = \frac{\omega_1^c}{1-\omega_1^c}\xi_{1,t}^{trade} - \frac{\omega_1^c}{1-\omega_1^c}\xi_{2,t}^{trade} - z_{1,t} + z_{2,t} + \varpi\hat{\delta}_{1,t}$$

For example, consider a trade rebalancing shock that improves the home country's trade balance. Holding the terms of trade unchanged, the trade balance response is increasing in the degree of home bias. However, as the economy becomes more closed, the aggregate resource constraint implies $\Delta\tilde{T}_{1,t} \approx 0$. For the trade balance to remain unchanged, the terms of trade must appreciate. The required real exchange rate appreciation increases with the degree of home bias. Near the trade-autarky limit $r\hat{e}r_{1,t} = \hat{\delta}_{1,t}$, which implies that the volatility of the real exchange rate increases one-to-one with the volatility of the terms of trade. In other words, the volatility of the exchange rate increases without bounds to induce enough expenditure-switching toward home goods such that the trade balance is unchanged and equal to zero.

The intuition behind the response of the IRBC-related moments stems from the resource constraint of a closed economy. As home bias increases, the real exchange neutralizes any movement in the trade balance, and productivity shocks become the only ones affecting domestic output and consumption. In the trade-autarky limit, the correlation between domestic and foreign output will reflect our assumption about the cross-country correlation of total factor productivity shocks (panel c.). Similarly, the consumption-output correlation (panel d.) increases with the extent of home bias. In the close economy limit, the resource constraint implies a perfect correlation because $Y_{1,t} = C_{1,t}$.

The Backus-Smith correlation (panel e.) is negative and nearly equal to one, consistent with the predictions of Theorem 3 for the model without productivity disturbances. The degree of trade integration is irrelevant to this correlation. The forward premium puzzle (panel f.) is heavily influenced by trade integration. The Fama coefficient becomes increasingly negative as the home-bias parameter approaches the closed-economy limit. The intuition for this result is driven entirely by the covariance between the real exchange rate and interest rate differentials, $cov(rer, r_1 - r_2)$.

Suppose the real exchange rate depreciates. The expenditure-switching mechanism of an open economy would imply an improvement in the trade balance as cheaper domestic goods are exported. However, as the economy becomes more closed to trade, the trade balance needs to remain unchanged and equal to zero. The higher foreign demand for domestic goods can only be offset if domestic households can borrow to increase current consumption, which is biased towards home goods. Thus, the domestic real interest rate declines to boost domestic consumption. The closer the economy is to the autarky limit, the interest rate response to movements in the real exchange rate becomes stronger, and the Fama coefficient turns more negative, in line with Theorem B.2.

The correlation of the trade balance and the exchange rate (panel g.) and the volatility of the trade balance relative to the exchange rate (panel h.) decreases with the degree of home bias. As we show in the Appendix, the covariance between the trade balance and the exchange rate, absent TFP shocks, depends on:

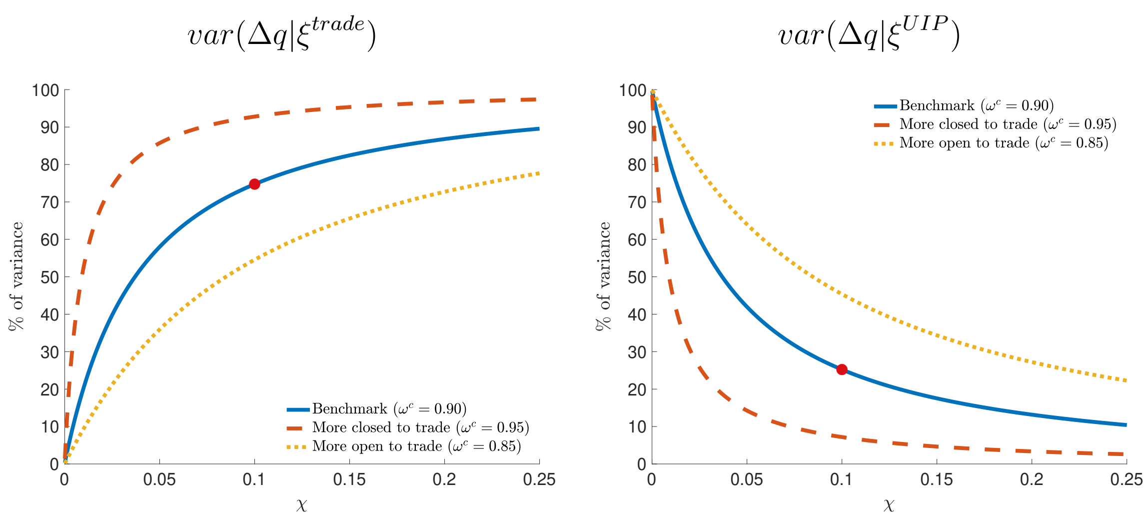
$$cov\left(\Delta\hat{\delta}_{1,t}, \Delta\tilde{T}_{1,t}\right) = \frac{\omega_1^c}{1-\omega_1^c} cov\left(\Delta\hat{\delta}_{1,t}, \Delta\xi_{1,t}^{trade}\right) + \varpi var\left(\Delta\hat{\delta}_{1,t}\right), \tag{30}$$

Where the covariance between the terms of trade and the rebalancing shocks is negative, as previously discussed, with greater home bias, the first term on the right-hand side becomes more negative at a geometric rate, whereas the second term on the right-hand side becomes positive at a linear rate. This difference implies that even if the variance of the terms of trade grows without bounds as we approach a closed-economy configuration, the correlation between the terms of trade and the trade balance will be more negative with greater home bias. Equation 30 also captures the limitation of models that abstract from trade rebalancing shocks. In such case, $cov\left(\Delta\hat{\delta}_{1,t}, \Delta\xi_{1,t}^{trade}\right) = 0$, implying that the correlation between the trade balance and terms of trade will always be positive, and in fact, it will always be equal to one.

4.3 What Drives the Exchange Rate?

We are now ready to explore which shocks drive the exchange rate. Figure 4 plots the contribution of trade rebalancing and UIP shock to the variance of the real exchange rate. The left panel shows the share of variance of the real exchange rate accounted for by the rebalancing shocks in our calibrated model. The right panel shows the corresponding variance share due to the UIP shock. Solid lines depict the unconditional variance decomposition as a function of the bond-adjustment cost parameter for a home bias parameter of $\omega^c = 0.9$. Our baseline calibration is shown with a red circle. The red dashed line presents the variance decomposition in an economy more closed to trade, and the yellow dotted line shows the variance decomposition in the case of an economy more open to trade.

Figure 4: Variance Decomposition of the Real Exchange Rate



Notes: Unconditional variance decomposition computed in the theoretical model. The blue line is our baseline model. The red dot shows our preferred calibration. Dashed and dashed-dotted lines correspond to alternative calibrations for different values of the home-bias parameter.

The main result is that trade rebalancing shocks explain nearly 75 percent of the variance of the real exchange rate in the benchmark calibration. Shocks to UIP explain about 25 percent of the total variance, and the contribution of TFP shocks is negligible. With greater financial integration, as measured by a lower value of the parameter χ, the variance contribution of trade rebalancing shocks declines, and the importance of IP shocks increases. Intuitively, when intertemporal substitution is frictionless, as would be the case under $\chi \to 0$, any movement in the trade balance can be accommodated with the corresponding change in the country's NFA position. In other words, if countries can borrow freely, the variation of the trade balance decouples from real shocks. Whether the economy is open or closed to trade is irrelevant when trade in financial assets is frictionless despite the absence of complete markets. In the limit, when $\chi \to 0$, the NFA asset position inherits a unit-root behavior $\tilde{B}_{1,t} = \tilde{B}_{1,t-1}$.

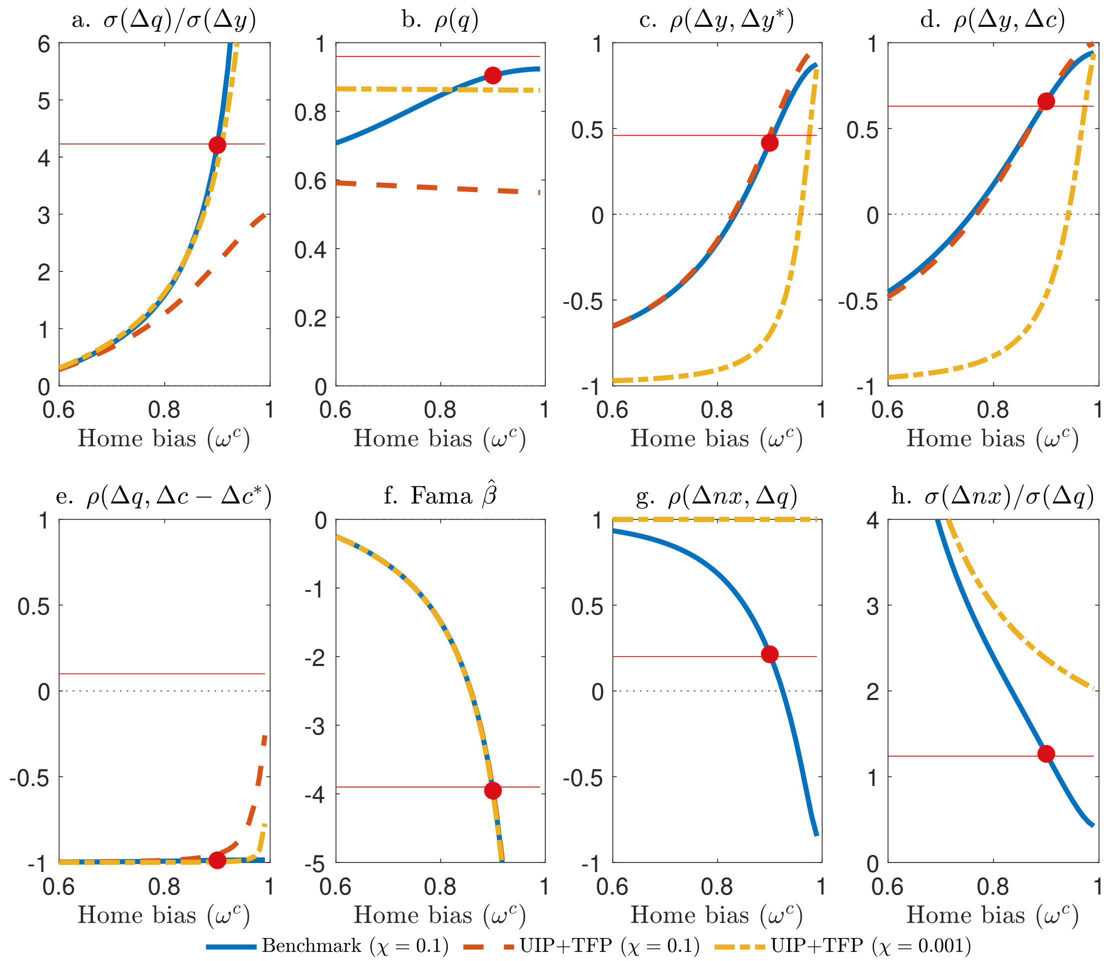
Aside from frictionless capital markets, the level of financial integration and trade openness significantly affect the contribution of shocks to the total variance of the real exchange rate. When trade integration is low, as illustrated by the calibration with $\omega^c = 0.95$, trade rebalancing shocks dominate the variance of the real exchange rate. The intuition for this result follows from the interaction between intertemporal substitution and expenditure switching. In a world where trading international bonds becomes costly, it entails a more sensitive real exchange rate to induce expenditure switching to limit trade flows and eliminate the need to engage in expensive international borrowing or lending.

4.4 What About Models without Trade Rebalancing?

In our setting, trade rebalancing shocks are important to account for exchange rate puzzles and are the main drivers of the real exchange rate. How do we reconcile our results with the literature emphasizing the role of financial shocks? In such models, financial shocks create UIP deviations consistent with the exchange rate disconnect and are the main drivers of the real exchange rate (Itskhoki and Mukhin 2021; Eichenbaum, Johannsen, and Rebelo 2021).

Figure 5 shows the different exchange rate moments we analyzed in previous sections. Alongside our benchmark model, shown in the solid blue line, we show two additional specifications. First, we consider a model without trade rebalancing shocks and moderate portfolio adjustment costs, shown in the dashed red line. Second, we also consider a model without trade rebalancing shocks but with greater financial integration, as demonstrated by the gold-dashed line. We plot the moments for different values of the home-bias parameter.

Figure 5: Abstracting from Trade Rebalancing



Notes: The blue line shows theoretical moments for different values of ω^c. The red horizontal line indicates the corresponding empirical moment in each panel. The red dot shows our preferred calibration.

We highlight a positive and a negative result. The positive outcome is that the models without trade rebalancing can replicate many exchange rate moments, but not all. In particular, these models perform well for the moments shown in panels a. to f. The negative result is that the models without trade rebalancing shocks cannot reproduce the moments relating to the real exchange rate and the trade balance (panels g. and h.). This result speaks to the importance of trade-related moments and the use of trade rebalancing shocks to replicate exchange rate moments and distinguish amongst competing models.

In the next section, we bring our insights into a fully-fledged medium-scale model to assess the quantitative relevance of our mechanism once we confront a wider range of macroeconomic data.

5 Quantitative Assessment

In what follows, we propose a two-bloc DSGE model that we estimate using Bayesian methods to verify that our conclusions carry over to a richer setting exposed to more data. We use a rich data set on macro-economic aggregates, interest rates, inflation, trade, and the RER. Having estimated the model, we use it to highlight the importance of trade shocks in explaining the dynamics of the RER and net exports and in informing the degree of frictions in international risk sharing.

5.1 Model Description

The model consists of two country blocs, which we index with the subscript $j = \{1, 2\}$. The home country, which we associated with the U.S., is indexed as $j = 1$, and a foreign block, which we associate with the rest of the World, is indexed as $j = 2$. Country sizes are given by the parameter $n_j \in (0, 1)$, with the restriction that $n_1 + n_2 = 1$. Each country is populated by households, wholesale retailers, and intermediate goods producers.

Households. Each country has a continuum of risk-averse households of measure one. Each household consists of a continuum of workers who supply differentiated labor to firms through an employment agency. We assume perfect risk-sharing within the household. Households derive utility from consumption and bond holdings and disutility from labor.

Let $C_{j,t}$ denote households' consumption of the final good in country j, $B^k_{j,t}$ their holdings of the bonds denominated in the currency of country k, with price $P^{b,k}_{j,t}$, for $k = \{1, 2\}$. Also, $\Pi_{j,t}$ are the profits received from firms, $T_{j,t}$ are lump-sum taxes collected by the government, $n_{j,t}(i)$ the labor supply of differentiated labor variety $i \in (0, 1)$, and $w_{j,t}(i)$ its the associated nominal wage. Then households in country j choose $C_{j,t}, B^k_{j,t}$, and $\{n_{j,t}(i), w_{j,t}(i)\}$ to maximize their expected lifetime utility given by[11]

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \log\left(C_{j,t} - b\tilde{C}_{j,t-1}\right) - \frac{\psi_N}{1+\eta}\int n_{j,t}(i)^{1+\eta}di + \exp(\zeta_{j,t}^{RP})\sum_{k=1}^{2} U(B_{j,t}^k) \right\}. \tag{31}$$

where $\eta > 0$ is the Frisch elasticity of labor supply, ψ_N controls the disutility from labor, $\zeta_{j,t}^{RP}$ is a risk-premium shock that shifts the demand for bonds, and $\tilde{C}_{j,t}$ is aggregate consumption in country j, which implies that households exhibit external habits on their consumption decisions.[12] The households' budget constraint is then given by:

$$P_{j,t}C_{j,t} + \sum_{k=1}^{2} \frac{P_{j,t}^{b,k} B_{j,t}^k}{e_{j,t}^k \phi_{j,t}^k} = \int w_{2,t}(i) n_{2,t}(i) di + \sum_{k=1}^{2} \frac{B_{j,t-1}^k}{e_{j,t}^k} + \Pi_{j,t} + T_{j,t},$$

where $e_{j,t}^k$ is the price of currency from country k in units of currency of country j, the nominal exchange rate, and obviously $e_{j,t}^k = 1$ for $k = j$. As in our analytical model, $\phi_{j,t}^k$ captures the financial intermediation cost of trading bonds from country k in country j. The budget constraint states that purchasing final consumption and new bonds must equal total labor income, proceeds from existing bond holdings, and firms' profits, net of lump-sum taxes.

Asset Markets. International asset markets are incomplete, and the only internationally-traded asset is the non-state contingent bond denominated in the currency of country 1. Thus, international borrowing and lending happen only in U.S. dollars, which implies that $B_{1,t}^2 = 0$. We assume that financial intermediation costs affect bond trade across borders but not within borders, thus $\phi_{j,t}^k = 1$ for $k = j$. With these restrictions the only non-trivial intermediation cost is $\phi_{2,t}^1$, which we assume takes the functional form $\phi_{2,t}^1 \equiv \exp(\zeta_{2,t}^{UIP})\left(1 - \chi \frac{\tilde{B}_{2,t}^1}{e_{2,t}^1 Y_{2,t} P_{2,t}}\right)$, with $\chi > 0$. The term $\zeta_{2,t}^{UIP}$ is the UIP shock that scales the return of the dollar bonds for foreign households.

Labor markets. Workers supply differentiated labor through employment agen-

[11] Later, when we log-linearize the equilibrium conditions, we will assume that in the non-stochastic steady state $\frac{U}{[C(1-b)]^{-1}} = 1$.

[12] For notation, we use X to denote the allocation by an individual agent whereas $\tilde{X}$ denotes the allocation by the representative agent.

cies, which bundle the differentiated varieties into a homogeneous labor input $N_{j,t}$ and sell it to intermediate producers at a wage $W_{j,t}$. Therefore, the demand for the labor varieties is given by $n_{1,t}(i) = \left(\frac{w_{1,t}(i)}{W_{1,t}}\right)^{-(1+\mu^w_{1,t})/\mu^w_{1,t}} N_{1,t}$, where $\mu^w_{1,t} = \mu^w \exp(\zeta^w_{1,t})$ and $\zeta^w_{1,t}$ is a wage-markup shock. As in (Erceg, Henderson, and Levin 2000), the employment agency sets the wages for each labor variety subject to Calvo-type frictions. The opportunity to reset the wage occurs with probability $1-\theta_w$ every period. There is no indexation when wages are not reset.

Final Consumption and Investment Goods. The production of the final consumption and investment goods is conducted by perfectly competitive firms and is symmetric between the two countries. We proceed with describing a generic country $j \in \{1,2\}$. The final consumption good $\mathcal{C}_{j,t}$ is produced after combining the final intermediate good $C^d_{j,t}$ and imports $M^c_{j,t}$ of the final intermediate from the other economy using an Armington technology

$$\mathcal{C}_{j,t} = \left[\omega_t^{1/\theta} C^{d\,\frac{\theta-1}{\theta}}_{j,t} + (1-\omega_t)^{1/\theta}\left((1-\psi^C_{j,t})M^c_{j,t}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

where $\theta > 1$ is the elasticity of substitution between home and foreign intermediate goods, with $\omega_t \equiv \omega \exp(\zeta^\omega_{j,t})$, where $\omega \in (0.5, 1]$ is the home-bias parameter, and $\zeta^\omega_{j,t}$ is a shock to the home bias in domestic consumption.

Similarly, the final investment good $I_{j,t}$ is produced as a composite of final intermediate good $I^d_{j,t}$ and imports $M^i_{j,t}$ of the final intermediate from the other economy according to

$$I_{j,t} = \left[\omega_t^{i\,1/\theta} I^{d\,\frac{\theta-1}{\theta}}_{j,t} + \left(1-\omega^i_t\right)^{1/\theta}\left((1-\psi^i_{j,t})M^i_{j,t}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

with $\omega^i_t \equiv \omega^i \exp(\zeta^\omega_{j,t})$ where $\omega^i \in (0.5, 1]$ is the home-bias parameter for investment goods and $\zeta^{\omega^i}_{j,t}$ is a shock to the home bias in domestic investment.

The import adjustment costs for consumption ($\psi^c_{j,t}$) and for investment ($\psi^i_{j,t}$) attenuate the response of imports to changes in relative prices in the short run, allowing for differences between the short-term and the long-term trade elasticities. We assume these adjustment costs take the quadratic form proposed in Erceg, Guerrieri, and Gust (2005).

Domestic and imported intermediate goods are bundles of a continuum of intermediate varieties aggregated according to the following technology:

$$Y^d_{j,t} = \left(\int_0^1 (Y^d_{j,t}(h))^{\frac{1}{1+\mu^d_{j,t}}} dh \right)^{1+\mu^d_{j,t}} \qquad M_{j,t} = \left(\int_0^1 (M_{j,t}(h))^{\frac{1}{1+\mu^M_{j,t}}} dh \right)^{1+\mu^M_{j,t}}$$

Where $\mu^k_{j,t}$ are time-varying markups, defined as $\mu^k_{j,t} = \mu^k \exp(\zeta^{\mu^k_j}_{j,t})$, for $k \in \{d, M\}$, with shocks $\zeta^{\mu^k_j}_{j,t}$ following ARMA(1,1) processes.

In equilibrium, the supply of final consumption goods has to equal the consumption demand by households and the government, $\mathcal{C}_{j,t} = C_{j,t} + G_{j,t}$. The market clearing conditions for the demand for domestic and imported goods are $Y^d_{j,t} = C^d_{j,t} + I^d_{j,t}$, and $M_{j,t} = M^c_{j,t} + M^i_{j,t}$.

Intermediate Goods Producers. A continuum of perfectly competitive firms produces a homogeneous intermediate good sold to intermediate retailers. Intermediate producers rent labor from the employment agency and rent capital from capital goods producers to operate a Cobb-Douglas production technology $Y_{j,t} = \exp(\zeta^A_{j,t})\bar{K}^\alpha_{j,t} N^{1-\alpha}_{j,t}$, where $\alpha \in (0,1)$, $\zeta^A_{j,t}$ is an aggregate TFP shock, and $\bar{K}_{j,t}$ is effective units of capital. We allow variable capital utilization, with $u_{j,t}$ the utilization rate. Therefore, effective capital is related to installed capital as follows: $\bar{K}_{j,t} = K_{j,t-1}u_{j,t}$. We assume that adjusting the utilization rate is costly and proportional to the level of capital, $\mathcal{A}(u_{j,t})K_{j,t-1}$, where $\mathcal{A}(u_{j,t})$ has the following functional form $\mathcal{A}(u_{j,t}) = r^K \frac{\exp(\xi(u_{j,t}-1)-1)}{\xi}$, where $\xi > 0$ and r^K is the steady-state rental rate of capital.

Capital Goods Producers. Every period, perfectly competitive firms, investment goods to augment the undepreciated capital stock $K_{j,t} = (1-\delta)K_{j,t-1} + \exp(\zeta^I_{j,t})F(I_{j,t}, I_{j,t-1})$, where $\zeta^I_{j,t}$ is a shock to the marginal efficiency of investment as in Justiniano, Primiceri, and Tambalotti (2010), and $F(I_{j,t}, I_{j,t-1}) = I_{j,t}\left[1 - S\left(\frac{I_{j,t}}{I_{j,t-1}}\right)\right]$ represents flow adjustments costs, and $S(.)$ is a convex adjustment-cost function, as in Christiano, Eichenbaum, and Evans (2005).

Intermediate Retailers. Intermediate retailers purchase the homogenous intermediate goods and produce a differentiated variety at no cost. Retailers operate in monopolistically competitive markets and sell the intermediate varieties to the wholesale retailers described earlier. We assume that intermediate retailers set prices as in Calvo (1983).

Each country has two types of retailers: domestic retailers who sell in local markets and exporters. With probability $1-\theta_{p,j}$ the domestic retailer chooses optimal reset prices $P^o_{j,t}(h)$ to maximize their profits given by

$$\mathbb{E}_t \sum_{i=0}^{\infty} (\theta_{p,j})^i \Lambda_{j,t,t+i} \left(\frac{P^{o,d}_{j,t}(h)}{P^c_{j,t+i}} - MC_{j,t+i} \right) Y^d_{j,t+i}(h)$$

where $MC_{j,t}$ is the price of the homogeneous intermediate input sold to retailers, and $\Lambda_{j,t,t+1} \equiv \beta \frac{\tilde{C}_{j,t} - b\tilde{C}_{j,t-1}}{\tilde{C}_{j,t+1} - b\tilde{C}_{j,t}}$ is the households' stochastic discount factor in country j. Also, let $P^d_{j,t}$, and $P^M_{j,t}$ be the prices of the domestic and non-domestic consumption goods in the country j. There is no indexation when prices are not reset.

Exporters set their prices in the currency where the good is sold, therefore featuring "local-currency pricing". They are also subject to price-setting frictions and reoptimize their price with probability $1-\theta^x_{p,j}$, with $\theta^x_{p,j}$ potentially different than $\theta_{p,j}$. Their optimal export price $P^{o,x}_{j,t}(h)$ is chosen to maximize profits given by

$$\mathbb{E}_t \sum_{i=0}^{\infty} (\theta^x_{p,j})^i \Lambda_{j,t,t+i} \left(\frac{P^{o,x}_{j,t}(h)}{P^c_{j,t+i}} \frac{1}{e_{j,t+i}} - MC_{j,t+i} \right) X_{j,t+i}(h),$$

where $X_{j,t}(h) \equiv M_{-j,t}(h)$ is the quantity of variety h exported from country j into the country different than j. From the LCP assumption, $P^x_{j,t}(h) \equiv P^M_{-j,t}(h)$, is the price of imported goods from country j into the country different than j.

Monetary and Fiscal Policy. The government issues no debt and runs a balanced budget by financing its expenditures with lump-sum taxes levied on the households, that is, $T_{j,t} = G_{j,t}$. Government expenditures are given by $G_{j,t} = \exp(\zeta^G_{j,t})G$, where G is the steady-state level and $\zeta^G_{j,t}$ is a government expenditure shock.

The monetary authority in each country sets nominal interest rates following a

Taylor-like rule which reacts to inflation and the output gap:

$$\frac{R_{j,t}}{R} = \left(\frac{R_{j,t-1}}{R}\right)^{\varphi_R} \left[\left(\frac{\bar{\pi}_{j,t}}{\exp(\zeta^{\pi}_{j,t})\bar{\pi}_j}\right)^{\varphi_\pi} \left(\frac{Y_{j,t}}{Y^{flex}_{j,t}}\right)^{\varphi_Y}\right]^{1-\varphi_R} \exp(\zeta^R_{j,t}) \tag{32}$$

where, $\bar{\pi}_{j,t} = \left(\prod_{s=0}^{3} \pi_{j,t-s}\right)^{1/4}$ is the 4-quarter average inflation with $\pi_{j,t} \equiv \frac{P^c_{j,t}}{P^c_{j,t-1}}$, and $Y^{flex}_{j,t}$ is aggregate output in the flexible-price version of the economy, $\zeta^{\pi}_{j,t}$ is a shock to the inflation target $\bar{\pi}_j$, and $\zeta^R_{j,t}$ is a monetary policy shock. The inflation target follows an AR(1) process common across blocks.

5.2 Calibrated Parameters

We calibrate most model parameters to standard values in the literature. Table 3 reports the calibrated parameters. On the household side, we set the (inverse) Frisch elasticity to 1 and target total hours of 0.33 in steady state. External habit is set at 0.8, and we target an interest rate of 4 percent annually.[13] The wage stickiness parameter, θ_w, is set to 0.85, and we target an average wage markup of ten percent. Regarding production-related parameters, we target a labor share of 65 percent and a depreciation rate of 2.5 percent per quarter and set the curvature parameters of the cost of capital utilization and investment adjustment to 1 and 5, respectively. The "Calvo" parameter controlling the frequency of domestic price changes is set to 0.85, which implies a frequency of price adjustment of about seven quarters. The Calvo parameter for internationally traded goods is set to 0.85 to balance estimates for short and medium-term pass-through.

For the parameters affecting trade flows, we set $\theta = 1.5$ and $\psi^C = \psi^i = 10$, implying a long run and a short run trade elasticity of 1.5 and around 0.5, respectively. Regarding country size, we assume that the U.S. is 25 percent of the world economy. We set the share of imported goods in U.S. consumption to 5 percent and 50 percent for U.S. investment. For the rest of the world, we re-scale these two moments proportionally to obtain balanced trade in a steady state.

[13] Our measurement equation for the interest rate adjusts the intercept to be consistent with average short-term interest rates.

Concerning policy parameters, we set the government expenditure-to-GDP ratio to 22 percent and assume that the Taylor rule responds to the lagged interest rate with a weight of 0.8, to 4-quarter average inflation with a weight of $1.5 \times (1 - 0.8)$, and to the output gap with a weight of $0.125 \times (1 - 0.8)$.

Table 3: Calibrated Parameters

Parameter	**Description**	**Value**
η	Inverse Frisch Elasticity	1
b	Consumption Habit	0.8
$\bar{R}$	Nominal Interest Rate Steady State	1.01
θ	Elasticity of Substitution Domestic Foreign Good	1.5
ψ^C, ψ^i	Trade Adjustment Costs	10
θ_w	Calvo Wage	0.85
θ_p	Calvo Domestic	0.85
θ_p^x	Calvo Export	0.85
α	Capital Elasticity Production	0.29
δ_k	Depreciation Capital	0.025
S''	Investment Adjustment Costs	5
ξ	Slope Utilization	1
$\bar{g}$	Government Expenditures Share GDP	0.22
φ_R	Inertia Taylor Rule	0.8
φ_π	Taylor Rule Inflation Response	1.5
φ_Y	Taylor Rule Output Gap Response	0.125

Notes: This table lists the parameters that are calibrated to values shown here. See the text for the details on the calibration targets. We omit scale parameters like $\bar{\psi}_1$, as they depend on estimated parameters and vary across different exercises.

5.3 Data and Inference

We estimate the model using quarterly data for real growth in GDP, consumption, and investment, GDP deflator inflation, and policy rates for the U.S. and the rest of the world. For the U.S., we also use data on the broad real dollar index, real wage growth, labor gap, export and import to GDP ratios, and inflation expectations. All series run from 1985Q1 to 2019Q2 and are constructed as in Bodenstein, Cuba-Borda, Gornemann, Presno, Prestipino, Queraltó, and Raffo (2023). In matching the data to the model, we allow for intercept terms in the measurement equations and measurement error in all the observable time series of the rest of the world. Details on the data construction are provided in Appendix D.[14]

We use Bayesian methods to estimate the parameters governing the shock processes and the curvature of the bond adjustment costs foreigners face for holding dollar-denominated bonds. Table 4 gives details of the prior specification for the persistence and standard deviations of the exogenous processes. When estimating the parameter χ, we either use a wide uniform prior or re-estimate the model along a grid, fixing the values of χ a-priori. We discuss the results of both estimation strategies in the next section.

5.4 Estimation Results

Table 4 shows the posterior distribution of parameter estimates. The estimated persistence of UIP shocks is close to unity, while the home bias and productivity shocks have significantly lower persistence. Notably, UIP shocks are not as volatile as in our calibrated example, and the rebalancing shocks, both at home and abroad, are over five times more volatile than UIP shocks and about two times more volatile than TFP shocks. Nonetheless, as we later show in this section, UIP shocks still significantly contribute to the exchange rate. This is consistent with our theoretical prediction that puts weight on both exogenous and endogenous UIP deviations to match key moments in the data.

As previously discussed, the transmission of endogenous UIP deviations hinges on the degree of financial integration. The top row in Table 4 shows the prior and posterior distribution of the parameter χ. Although there is substantial uncertainty in the estimation, the posterior distribution peaks at an estimated value of $\chi = 0.09$, remarkably close to our calibrated parameter of the stylized model in Section 4. We note that this estimated value of χ is crucial to recovering the endogenous deviations of UIP from the data. Using χ to induce first-order stationary dynamics results in a substantial deterioration of model fit.

To illustrate the importance of combining trade data to identify the role of fric-

[14] Measurement error is set to equal five percent of the in-sample variance of the underlying series. We use DYNARE to implement a standard RWMH algorithm for our estimation. See Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto, and Villemot (2011) for details.

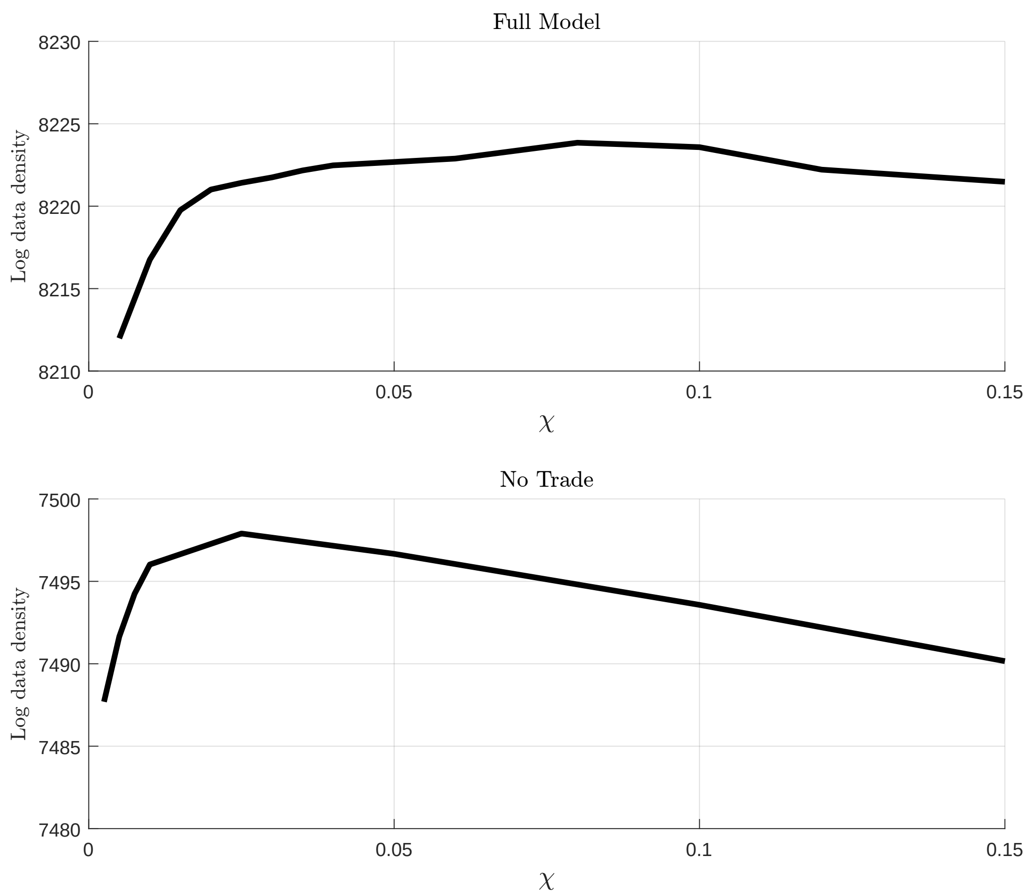
Table 4: Estimated Parameters - Medium Scale Model

Parameter	**Description**	**Prior Distribution**		**Posterior Distribution**	
		Family	[P(1), P(2)]	U.S.	Rest of World
Financial Integration					
χ	Bond adjustment cost	$\mathcal{U}$	$[0, 0.2]$	–	0.09 [0.05,0.14]
Standard Deviations					
$100 \times \sigma_R$	Monetary Policy Shock	$\mathcal{IG}$	$[0.1, 0.1]$	0.09 [0.08,0.10]	0.12 [0.09,0.16]
$100 \times \sigma_G$	Government Policy Shock	$\mathcal{IG}$	$[1, 5]$	2.04 [1.83,2.24]	0.63 [0.34,0.89]
$100 \times \sigma_I$	MEI Shock	$\mathcal{IG}$	$[1, 5]$	2.75 [2.37,3.11]	3.44 [2.68,4.19]
$100 \times \sigma_{RP}$	Risk Premium Shock	$\mathcal{IG}$	$[1, 1]$	0.88 [0.68,1.07]	0.24 [0.17,0.31]
$100 \times \sigma_A$	TFP Shock	$\mathcal{IG}$	$[1, 5]$	0.48 [0.43,0.53]	1.39 [1.05,1.72]
$100 \times \sigma_\omega$	Home Bias Shock	$\mathcal{IG}$	$[1, 5]$	1.93 [1.69,2.18]	0.91 [0.78,1.03]
$100 \times \sigma_\mu$	Markup Shock	$\mathcal{IG}$	$[1, 5]$	2.43 [2.16,2.66]	1.42 [1.14,1.71]
$100 \times \sigma_W$	Wage Markup Shock	$\mathcal{IG}$	$[1, 5]$	1.43 [1.29,1.59]	–
$100 \times \sigma_\pi$	Inflation Trend Shock	$\mathcal{IG}$	$[0.01, 0.1]$	0.05 [0.04,0.05]	0.05 [0.04,0.05]
$100 \times \sigma_{UIP}$	UIP Shock	$\mathcal{IG}$	$[1, 5]$	–	0.22 [0.16,0.27]
Persistence Parameters					
ρ_R	Monetary Policy Shock	$\mathcal{B}$	$[0.4, 0.125]$	0.68 [0.63,0.74]	0.50 [0.35,0.66]
ρ_G	Government Policy Shock	$\mathcal{B}$	$[0.6, 0.125]$	exploring	0.77 [0.59,0.93]
ρ_I	MEI Shock	$\mathcal{B}$	$[0.6, 0.125]$	0.87 [0.83,0.91]	0.47 [0.32,0.63]
ρ_{RP}	Risk Premium Shock	$\mathcal{B}$	$[0.6, 0.125]$	0.74 [0.69,0.80]	0.95 [0.92,0.97]
ρ_A	TFP Shock	$\mathcal{B}$	$[0.6, 0.125]$	0.96 [0.94,0.98]	0.90 [0.82,0.98]
ρ_ω	Home Bias Shock	$\mathcal{B}$	$[0.6, 0.125]$	0.90 [0.88,0.92]	0.80 [0.74,0.86]
ρ_μ	Markup Shock	$\mathcal{B}$	$[0.6, 0.125]$	0.93 [0.91,0.95]	0.62 [0.47,0.78]
θ_μ	MA Price Markup Shock	$\mathcal{B}$	$[0.5, 0.125]$	0.52 [0.41,0.63]	0.41 [0.22,0.59]
ρ_W	Wage Markup Shock	$\mathcal{B}$	$[0.6, 0.125]$	0.55 [0.38,0.71]	–
θ_W	MA Wage Markup Shock	$\mathcal{B}$	$[0.5, 0.125]$	0.53 [0.39,0.69]	–
ρ_π	Inflation Trend Shock	$\mathcal{B}$	$[0.995, 0.002]$	0.62 [0.47,0.78]	0.62 [0.47,0.78]
ρ_{UIP}	UIP Shock	$\mathcal{B}$	$[0.6, 0.125]$	–	0.99 [0.99,1.00]

Notes: $\mathcal{U}$ is Uniform distribution; $\mathcal{IG}$ is Inverse Gamma distribution; $\mathcal{B}$ is Beta distribution. P(1) and P(2) are the mean and standard deviations for Beta and Inverse Gamma distributions. For Uniform distributions, P(1) and P(2) represent the hyper-parameters determining the lower and upper bound of the support of the distribution. The table reports the posterior mean and the 90% credible set in square brackets. We omit the level shifters in the measurement equations. We rounded to the second decimal. As a result, ρ_{UIP} lies between 0.99 and 1.00. Using greater decimal precision, the interval is $[0.9876, 0.9951]$.

tions in international risk sharing, we vary χ on a grid between 0 and 0.15 and re-estimate the rest of the model parameters. Figure 6 shows the resulting log data density for each of these estimations in the top panel. The curve peaks around 0.08[15], well above the values used for χ when considering bond adjustment costs mainly as a device to induce stationarity–at or below 0.01. The data favors endogenous UIP deviations that respond to changes in the country's NFA position. As shown in the bottom panel of Figure 6, we can see that using trade data is key for this result. If we exclude exports and impints from the estimation, the log data density peaks around 0.025, a much lower value.

Figure 6: Identifying the Financial Integration Parameter (χ)



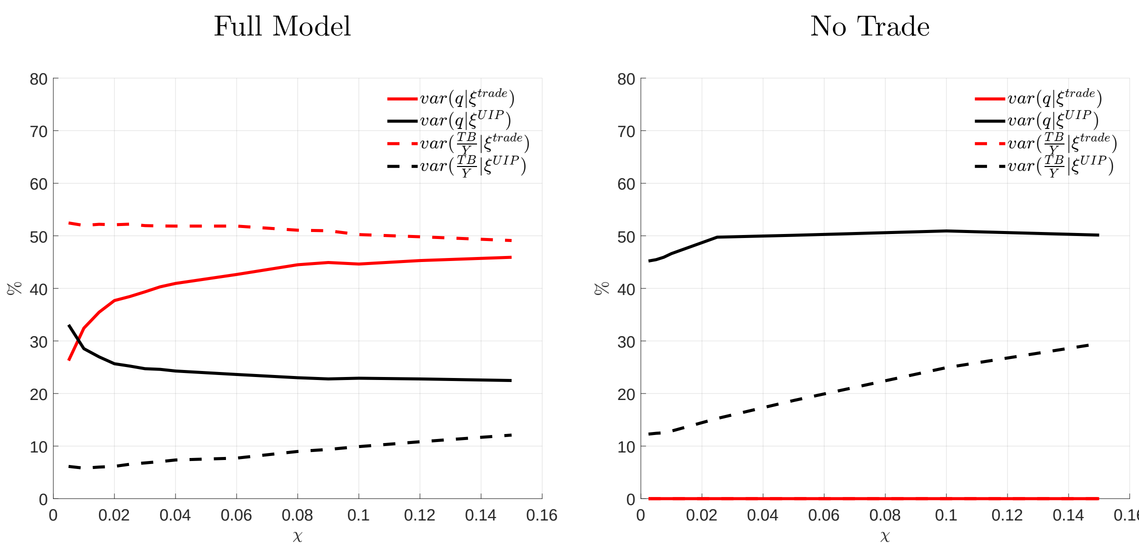
Note: The top panel shows the log-data density of the estimated model for different values of the parameter χ when including U.S. exports and imports data in estimation. The bottom panel shows the log-data density in an estimated model without trade data.

[15] This is also close to the value we estimate when including χ directly in the estimation.

Figure 7 shows that different values for χ matter for the contribution of UIP and home bias shocks to the variance of the RER and trade balance at business cycle frequency. For this exercise, we also re-estimate the model for different values of χ and record the variance shares at the posterior mean. The left of Figure 4 shows that the share of the RER explained by the home bias shocks grows in the degree of friction in international risk sharing in the bond market, rising from as low as 25 percent to over 45 percent.

The contribution of the UIP shock falls as the parameter χ increases. We also see that the home bias shocks explain over 50 percent of the variation in trade. The right panel of Figure 4 shows that the model without trade data assigns a high share of the variance of both variables to the UIP shock for any value of χ. Together with our previous results, we conclude that accounting for trade data and trade shocks is important to understand exchange rate determination in this class of open economy models.

Figure 7: Variance Share - Business Cycle Frequency



Notes: Solid lines depict the variance decomposition of the real exchange rate. Dashed lines correspond to variance decomposition of the trade balance-to-output ratio. Red lines correspond to the share of variance explained by rebalancing shocks. Black lines correspond to the share of variance explained by UIP shocks. All model implied series are bandpass filtered at 6-32 quarters.

5.5 Exchange Rate Moments

Our last exercise is to show the ability of our estimated model to fit key exchange rate disconnect moments. Contrary to our calibrated strategy in Section 3, the estimation of the medium scale model does not explicitly target the exchange rate disconnect moments. Instead, the likelihood function trades-off model fit along multiple dimensions, including the covariances and cross-correlations with additional series we provide for estimation. Moreover, the medium-scale model imposes additional cross-equation restrictions that may penalize the moments we are interested in. Thus, we are not guaranteed to recover the exchange rate disconnect.

Table 5: Exchange Rate Moments - Medium Scale Model

		Full Model	
	Data	Mean	Std. Error
Disconnect and PPP puzzles			
$\sigma\Delta q/\sigma\Delta y$	4.23	3.67	(0.36)
$\rho(q)$	0.96	0.91	(0.04)
International Co-movement			
$\rho(\Delta y, \Delta y^*)$	0.46	0.14	(0.12)
$\rho(\Delta y, \Delta c)$	0.63	0.57	(0.08)
Backus-Smith and Forward Premium			
$\rho(\Delta q, \Delta c - \Delta c^*)$	0.1	0.22	(0.08)
Fama (real) $\hat{\beta}$	-3.9	-0.70	(0.43)
RER and NX			
$\rho(\Delta nx, \Delta q)$	0.2	0.28	(0.09)
$\sigma(\Delta nx)/\sigma(\Delta q)$	1.24	1.01	(0.08)

Notes: Medium scale models computed using 2000 simulations drawn from the estimated innovations at the posterior mean. Each simulation has a length of 138 quarters to match the observations in our data sample from 1985Q1 to 2019Q2.

Table 5 shows that despite the additional restrictions in matching additional time series, our medium-scale model with trade rebalancing shocks leans toward the moments related to exchange rate disconnect. For comparison, we present the moments in Table 2.

6 Conclusion

In this paper, we developed an exchange rate determination model consistent with key exchange rate moments. We show, analytically and numerically, that key exchange rate puzzles, such as excess exchange rate volatility, lack of international risk sharing, and the forward-premium puzzle, do not necessarily imply that the exchange rate is disconnected from the macroeconomy. Trade rebalancing and endogenous UIP deviations from imperfect risk-bearing capacity are crucial for exchange rate determination.

Trade rebalancing shocks allow our model to replicate the observed relation between the real exchange rate and the trade balance. In particular, rebalancing shocks are necessary to match the mute volatility of the trade balance and to deliver the weak correlation between the real exchange rate and the trade balance. Theories of exchange rate determination that rely mainly on exogenous UIP shocks are inconsistent with these empirical patterns. We show that correctly capturing the degree of risk sharing of international financial markets plays a crucial role in reconciling the exchange rate disconnect with the trade balance dynamics.

Our results extend to a New Keynesian open economy model of the U.S. and the Rest of the World that we confront with a broader set of macroeconomic data. In our estimated model, we quantify the main shocks that drive the exchange rate and find that trade rebalancing shocks account for close to 50 percent of the variance of the real exchange rate and the trade balance at business cycle frequencies.

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A Appendix: Model Solution, Decision Rules, and Theoretical Moments of the Simple Model

A.1 First-order and equilibrium conditions

We present the first-order and equilibrium conditions of the model. Optimality of the household's decisions imply

$$C_{1,t} = \frac{P^d_{1,t}}{P^c_{1,t}} \frac{W_{1,t}}{P^d_{1,t}} \tag{A.1}$$

$$P^b_{1,t} = \phi^b_{1,t} \beta E_t \left\{ \frac{C_{1,t}}{C_{1,t+1}} \frac{P^c_{1,t}}{P^c_{1,t+1}} \right\}. \tag{A.2}$$

$$C^d_{1,t} = \omega^c_{1,t} \left(\frac{P^c_{1,t}}{P^d_{1,t}} \right)^{\frac{1+\rho^c}{\rho^c}} C_{1,t} \tag{A.3}$$

$$M_{1,t} = (1-\omega^c_{1,t}) \left(\frac{P^c_{1,t}}{P^m_{1,t}} \right)^{\frac{1+\rho^c}{\rho^c}} C_{1,t}. \tag{A.4}$$

These conditions and the consumption aggregator imply for the relative prices

$$\frac{P^c_{1,t}}{P^d_{1,t}} = \left[\omega^c_{1,t} + (1-\omega^c_{1,t}) \delta_{1,t}^{-\frac{1}{\rho^c}} \right]^{-\rho^c} = F_{1,t}^{-\rho^c} \tag{A.5}$$

where the terms of trade $\delta_{1,t}$ is the ratio of import prices expressed in common currency

$$\delta_{1,t} = \frac{e_{1,t} P^d_{2,t}}{P^d_{1,t}}. \tag{A.6}$$

Similar conditions are obtained for country 2, (keeping in mind that the sole internationally traded bond pays in the currency of country 1:

$$C_{2,t} = \frac{P^d_{2,t}}{P^c_{2,t}} \frac{W_{2,t}}{P^d_{2,t}} \tag{A.7}$$

$$P^b_{1,t} = \phi^b_{2,t} \beta E_t \left\{ \frac{C_{2,t}}{C_{2,t+1}} \frac{P^c_{2,t}}{P^c_{2,t+1}} \frac{e_{1,t}}{e_{1,t+1}} \right\} \tag{A.8}$$

$$C^d_{2,t} = \omega^c_{2,t} \left(\frac{P^c_{2,t}}{P^d_{2,t}} \right)^{\frac{1+\rho^c}{\rho^c}} C_{2,t} \tag{A.9}$$

$$M_{2,t} = (1-\omega^c_{2,t}) \left(\frac{P^c_{2,t}}{P^m_{2,t}} \right)^{\frac{1+\rho^c}{\rho^c}} C_{2,t} \tag{A.10}$$

Appendix

$$\frac{P^c_{2,t}}{P^d_{2,t}} = \left[\omega^c_{2,t} + (1-\omega^c_{2,t})\delta_{1,t}^{\frac{1}{\rho^c}}\right]^{-\rho^c} = F_{2,t}^{-\rho^c}. \tag{A.11}$$

As we assume that prices and wages are flexible and that the production of each country's good is linear in the use of the country's labor,

$$Y_{1,t} = exp(z_{1,t})L_{1,t} \tag{A.12}$$

the production real wage equals the productivity level

$$\frac{W_{1,t}}{P^d_{1,t}} = exp(z_{1,t}). \tag{A.13}$$

Similarly, for country 2, we obtain

$$Y_{2,t} = exp(z_{2,t})L_{2,t} \tag{A.14}$$

$$\frac{W_{2,t}}{P^d_{2,t}} = exp(z_{2,t}). \tag{A.15}$$

Recall from the main text that market clearance in goods and financial markets requires

$$Y_{1,t} = C^d_{1,t} + M_{2,t} \tag{A.16}$$

$$Y_{2,t} = C^d_{2,t} + M_{1,t} \tag{A.17}$$

$$0 = B_{1,t} + B_{2,t}. \tag{A.18}$$

We also define in the main text that the trade balance (normalized by the value of exports) is

$$T_{1,t} \equiv e_t P^m_{2,t} M_{2,t} - P^m_{1,t} M_{1,t} \tag{A.19}$$

$$\tilde{T}_{1,t} = \frac{T_{1,t}}{e_t P^m_{2,t} M_{2,t}}. \tag{A.20}$$

which implies that the consolidated budget constraint of households in country 1 can be writte as

$$\frac{P^b_{1,t} B_{1,t}}{\phi^b_{1,t}} = T_{1,t} + B_{1,t-1}. \tag{A.21}$$

Finally, the law of one price for the internationally traded bond implies the risk-sharing condition

$$\phi^b_{1,t} E_t \left\{ \frac{C_{1,t}}{C_{1,t+1}} \frac{P^c_{1,t}}{P^c_{1,t+1}} \right\} = \phi^b_{2,t} E_t \left\{ \frac{C_{2,t}}{C_{2,t+1}} \frac{P^c_{2,t}}{P^c_{2,t+1}} \frac{e_{1,t}}{e_{1,t+1}} \right\}. \tag{A.22}$$

A.2 Simplifying the Nonlinear Model

Noticing that consumption in the two countries can be expressed as

$$C_{1,t} = exp(z_{1,t}) F_{1,t}^{\rho^c} \tag{A.23}$$

$$C_{2,t} = exp(z_{2,t}) F_{2,t}^{\rho^c} \tag{A.24}$$

and the consumption real exchange rate is given

$$rer_{1,t} = \frac{e_{1,t} P^c_{2,t}}{P^c_{1,t}} = \left(\frac{F_{1,t}}{F_{2,t}} \right)^{\rho^c} \delta_{1,t} \tag{A.25}$$

we can write the trade balance as

$$\begin{aligned} \tilde{T}_{1,t} &= 1 - \delta_{1,t} \frac{M_{1,t}}{M_{2,t}} \\ &= 1 - \frac{1-\omega^c_{1,t}}{1-\omega^c_{2,t}} \left(\frac{F_{1,t}}{F_{2,t}} \right)^{-1} \frac{exp(z_{1,t})}{exp(z_{2,t})} \delta_{1,t}^{1-2\frac{1+\rho^c}{\rho^c}}. \end{aligned} \tag{A.26}$$

Similarly, we write the risk sharing condition and the evolution of the NFA position as

$$E_t \left\{ \frac{P^c_{1,t}}{F^{\rho^c}_{1,t+1} P^c_{1,t+1}} \left[\phi^b_{1,t} \frac{exp(z_{1,t})}{exp(z_{1,t+1})} - \phi^b_{2,t} \frac{exp(z_{2,t})}{exp(z_{2,t+1})} \frac{\delta_{1,t}}{\delta_{1,t+1}} \right] \right\} = 0 \tag{A.27}$$

$$\frac{P^b_{1,t} \tilde{B}_{1,t}}{\phi^b_{1,t}} = \tilde{T}_{1,t} + \frac{e_{1,t-1} P^m_{2,t-1} M_{2,t-1}}{e_{1,t} P^m_{2,t} M_{2,t}} \tilde{B}_{1,t-1} \tag{A.28}$$

where $\tilde{B}_{1,t} = \frac{B_{1,t}}{e_{1,t} P^m_{2,t} M_{2,t}}$.

We next express all variables entering in equations A.26, A.27, and A.28 and in terms of $\tilde{T}_{1,t}$, $\delta_{1,t}$, $\tilde{B}_{1,t}$. The rebalancing shocks follow $\omega^c_{i,t} = \omega^c_i \exp\left(\xi^{trade}_{i,t}\right)$, for $i = 1, 2$ and we assume that $\phi^b_{1,t} = \exp\left(-\frac{\chi}{2} \frac{B^*_{1,t}}{P^d_{1,t} M^*_{2,t}}\right)$ whereas $\phi^b_{2,t} = \exp\left(-\frac{\chi}{2} \frac{\frac{1}{e_{1,t}} B^*_{2,t}}{P^d_{2,t} M_{1,t}} + \xi^{UIP}_{1,t}\right)$. Then, linearization around the symmetric deterministic steady state with $\omega^c_1 = \omega^c_2$

and balanced trade, i.e. $\tilde{T}_1 = 0$, $\delta_1 = 1$, $B_1 = 0$, yields the linear system

$$\left(z_{1,t} - E_t z_{1,t+1}\right) - \left(z_{2,t} - E_t z_{2,t+1}\right) - \left(\hat{\delta}_{1,t} - E_t \hat{\delta}_{1,t+1}\right) = \chi \tilde{B}_{1,t} + \xi^{UIP}_{1,t} \quad \text{(A.29)}$$

$$\beta \tilde{B}_{1,t} = \tilde{T}_{1,t} + \tilde{B}_{1,t-1} \quad \text{(A.30)}$$

$$\tilde{T}_{1,t} = \frac{\omega^c_1}{1-\omega^c_1}\xi^{trade}_{1,t} - \frac{\omega^c_1}{1-\omega^c_1}\xi^{trade}_{2,t} - z_{1,t} + z_{2,t} + \varpi \hat{\delta}_{1,t}, \quad \text{(A.31)}$$

where $\varpi = 1 + 2\frac{\omega^c_1}{\rho^c}$. The terms trade, $\hat{\delta}_{1,t}$, are measured in log-deviation from the steady state, $\tilde{T}_{1,t}$ and $\tilde{B}_{1,t}$ is in absolute deviation., If the term $\chi \neq 0$, the NFA dynamics are stationary.

For later reference, the remaining model variables can be recovered from the following linear relationships

$$\hat{C}_{1,t} = z_{1,t} - \left(1-\omega^c_1\right)\hat{\delta}_{1,t} \quad \text{(A.32)}$$

$$\hat{C}_{2,t} = z_{2,t} + \left(1-\omega^c_1\right)\hat{\delta}_{1,t} \quad \text{(A.33)}$$

$$\hat{q}_{1,t} = \left(2\omega^c_1 - 1\right)\hat{\delta}_{1,t}. \quad \text{(A.34)}$$

$$\hat{M}_{1,t} = -\frac{\omega^c_1}{1-\omega^c_1}\xi^{trade}_{1,t} + z_{1,t} - \frac{\varpi+1}{2}\hat{\delta}_{1,t} \quad \text{(A.35)}$$

$$\hat{M}_{2,t} = -\frac{\omega^c_2}{1-\omega^c_2}\xi^{trade}_{2,t} + z_{2,t} + \frac{\varpi+1}{2}\hat{\delta}_{1,t} \quad \text{(A.36)}$$

$$\hat{Y}_{1,t} = \omega^c_1\left(\xi^{trade}_{1,t} - \xi^{trade}_{2,t}\right) + \omega^c_1 z_{1,t} + \left(1-\omega^c_1\right) z_{2,t} + \varpi\left(1-\omega^c_1\right)\hat{\delta}_{1,t} \quad \text{(A.37)}$$

$$\hat{Y}_{2,t} = -\omega^c_1\left(\xi^{trade}_{1,t} - \xi^{trade}_{2,t}\right) + \omega^c_1 z_{2,t} + \left(1-\omega^c_1\right) z_{1,t} - \varpi\left(1-\omega^c_1\right)\hat{\delta}_{1,t} \quad \text{(A.38)}$$

Defining the real interest rate for country i as the real return on a bond that pays one unit of consumption in country i regardless of the state of the world, i.e., $r_{i,t} = \hat{C}_{i,t+1} - \hat{C}_{i,t}$, we can express Equation A.29 in terms of the differential of real interest rates, $r_{1,t} - r_{2,t}$, between countries

$$r_{1,t} - r_{2,t} = E_t\left(\hat{q}_{1,t+1} - \hat{q}_{1,t}\right) - \chi \tilde{B}_{1,t} - \xi^{UIP}_{1,t}. \quad \text{(A.39)}$$

A.3 Applying the Method of Undetermined Coefficients

To compute the solution of the dynamic linear system, we employ the method of undetermined coefficients. Starting from the conjecture that in equilibrium the terms of trade evolve according to

$$\hat{\delta}_{1,t} = \gamma_1 \xi_{1,t}^{trade} + \gamma_2 \xi_{2,t}^{trade} + \gamma_3 \xi_{1,t}^{UIP} + \gamma_4 z_{1,t} + \gamma_5 z_{2,t} + \gamma_b \tilde{B}_{1,t-1} \tag{A.40}$$

we compute the values of the unknown coefficients γ_1 through γ_5 and γ_b by substituting the conjectured solution into the dynamic system (A.29)-(A.31). Using Equation A.31, the trade balance follows

$$\begin{aligned} \tilde{T}_{1,t} &= \left(\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_1\right)\xi_{1,t}^{trade} + \left(-\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_2\right)\xi_{2,t}^{trade} \\ &\quad + \varpi\gamma_3\xi_{1,t}^{UIP} + \left(-1+\varpi\gamma_4\right)z_{1,t} + \left(1+\varpi\gamma_5\right)z_{2,t} + \varpi\gamma_b\tilde{B}_{1,t-1}. \end{aligned} \tag{A.41}$$

Turning to the risk sharing/UIP condition, Equation A.29, we first evaluate the term $\left(\hat{\delta}_{1,t} - E_t\hat{\delta}_{1,t+1}\right)$ using the equation for the evolution of the NFA position, Equation A.30, and Equation A.41 to substitute out for $\tilde{B}_{1,t}$:

$$\begin{aligned} \left(\hat{\delta}_{1,t} - E_t\hat{\delta}_{1,t+1}\right) &= \gamma_1\left(1-\rho_1^{trade}\right)\xi_{1,t}^{trade} + \gamma_2\left(1-\rho_2^{trade}\right)\xi_{2,t}^{trade} + \gamma_3\left(1-\rho_1^{UIP}\right)\xi_{1,t}^{UIP} \\ &\quad + \gamma_4\left(1-\rho_1^z\right)z_{1,t} + \gamma_5\left(1-\rho_2^z\right)z_{2,t} \\ &\quad + \gamma_b\left(\tilde{B}_{1,t-1} - \tilde{B}_{1,t}\right) \\ &= \gamma_1\left(1-\rho_1^{trade}\right)\xi_{1,t}^{trade} + \gamma_2\left(1-\rho_2^{trade}\right)\xi_{2,t}^{trade} + \gamma_3\left(1-\rho_1^{UIP}\right)\xi_{1,t}^{UIP} \\ &\quad + \gamma_4\left(1-\rho_1^z\right)z_{1,t} + \gamma_5\left(1-\rho_2^z\right)z_{2,t} \\ &\quad + \frac{\gamma_b}{\beta}\left(\left(\beta-1\right)\tilde{B}_{1,t-1} - \tilde{T}_{1,t}\right) \\ &= \left(\gamma_1\left(1-\rho_1^{trade}\right) - \frac{\gamma_b}{\beta}\left(\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_1\right)\right)\xi_{1,t}^{trade} \\ &\quad + \left(\gamma_2\left(1-\rho_2^{trade}\right) - \frac{\gamma_b}{\beta}\left(-\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_2\right)\right)\xi_{2,t}^{trade} \\ &\quad + \left(\gamma_3\left(1-\rho_1^{UIP}\right) - \frac{\gamma_b}{\beta}\varpi\gamma_3\right)\xi_{1,t}^{UIP} \\ &\quad + \left(\gamma_4\left(1-\rho_1^z\right) - \frac{\gamma_b}{\beta}\left(-1+\varpi\gamma_4\right)\right)z_{1,t} \\ &\quad + \left(\gamma_5\left(1-\rho_2^z\right) - \frac{\gamma_b}{\beta}\left(1+\varpi\gamma_5\right)\right)z_{2,t} \\ &\quad + \left(\frac{\gamma_b}{\beta}\left(\beta-1\right) - \frac{\gamma_b}{\beta}\varpi\gamma_b\right)\tilde{B}_{1,t-1}. \end{aligned} \tag{A.42}$$

Note that the UIP condition, Equation A.29, can be written as

$$\begin{aligned}\left(\hat{\delta}_{1,t}-E_t\hat{\delta}_{1,t+1}\right) &= -\frac{\chi}{\beta}\left(\frac{\omega_1^c}{1-\omega_1^c}+\varpi\gamma_1\right)\xi_{1,t}^{trade}\\ &\quad -\frac{\chi}{\beta}\left(-\frac{\omega_1^c}{1-\omega_1^c}+\varpi\gamma_2\right)\xi_{2,t}^{trade}\\ &\quad -\left(\frac{\chi}{\beta}\varpi\gamma_3+1\right)\xi_{1,t}^{UIP}\\ &\quad -\left(\frac{\chi}{\beta}\left(-1+\varpi\gamma_4\right)-\left(1-\rho_1^z\right)\right)z_{1,t}\\ &\quad -\left(\frac{\chi}{\beta}\left(1+\varpi\gamma_5\right)+\left(1-\rho_2^z\right)\right)z_{2,t}\\ &\quad -\frac{\chi}{\beta}\left(\varpi\gamma_b+1\right)\tilde{B}_{1,t-1}. \end{aligned} \tag{A.43}$$

Combining the expressions A.42 and A.43 yields the following condition that determines the coefficients of the decision rule for the terms of trade:

$$\begin{aligned}
&\left(\gamma_1\left(1-\rho_1^{trade}\right)-\frac{\gamma_b}{\beta}\left(\frac{\omega_1^c}{1-\omega_1^c}+\varpi\gamma_1\right)\right)=-\frac{\chi}{\beta}\left(\frac{\omega_1^c}{1-\omega_1^c}+\varpi\gamma_1\right)\\
&\left(\gamma_2\left(1-\rho_2^{trade}\right)-\frac{\gamma_b}{\beta}\left(-\frac{\omega_1^c}{1-\omega_1^c}+\varpi\gamma_2\right)\right)=-\frac{\chi}{\beta}\left(-\frac{\omega_1^c}{1-\omega_1^c}+\varpi\gamma_2\right)\\
&\left(\gamma_3\left(1-\rho_1^{UIP}\right)-\frac{\gamma_b}{\beta}\varpi\gamma_3\right)=-\left(\frac{\chi}{\beta}\varpi\gamma_3+1\right)\\
&\left(\gamma_4\left(1-\rho_1^z\right)-\frac{\gamma_b}{\beta}\left(-1+\varpi\gamma_4\right)\right)=-\left(\frac{\chi}{\beta}\left(-1+\varpi\gamma_4\right)-\left(1-\rho_1^z\right)\right)\\
&\left(\gamma_5\left(1-\rho_2^z\right)-\frac{\gamma_b}{\beta}\left(1+\varpi\gamma_5\right)\right)=-\left(\frac{\chi}{\beta}\left(1+\varpi\gamma_5\right)+\left(1-\rho_2^z\right)\right)\\
&\left(\frac{\gamma_b}{\beta}\left(\beta-1\right)-\frac{\gamma_b}{\beta}\varpi\gamma_b\right)=-\frac{\chi}{\beta}\left(\varpi\gamma_b+1\right).
\end{aligned}$$

We present the final coefficients next.

A.4 Decision Rules

This subsection collects the decision roles of the main variables in our model, the terms of trade, the trade balance, and the NFA position. These three decision rules are used extensively obtaining the proofs of our theorems.

A.4.1 Terms of Trade, $\hat{\delta}_{1,t}$

The terms of trade are a linear function of the exogenous shocks and the inherited NFA position, $\tilde{B}_{1,t-1}$

$$\hat{\delta}_{1,t} = \gamma_1 \xi_{1,t}^{trade} + \gamma_2 \xi_{2,t}^{trade} + \gamma_3 \xi_{1,t}^{UIP} + \gamma_4 z_{1,t} + \gamma_5 z_{2,t} + \gamma_b \tilde{B}_{1,t-1} \tag{A.44}$$

with the parameters

$$\gamma_1 : \qquad \gamma_1 = \frac{\frac{\gamma_b - \chi}{\beta} \frac{\omega_1^c}{1-\omega_1^c}}{-\frac{\gamma_b-\chi}{\beta}\varpi + \left(1 - \rho_1^{trade}\right)} = -\frac{\tilde{\gamma}_b \frac{1}{\varpi} \frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1 - \rho_1^{trade}\right)} < 0 \tag{A.45}$$

$$\gamma_2 : \qquad \gamma_2 = \frac{-\frac{\gamma_b - \chi}{\beta} \frac{\omega_1^c}{1-\omega_1^c}}{-\frac{\gamma_b-\chi}{\beta}\varpi + \left(1 - \rho_2^{trade}\right)} = \frac{\tilde{\gamma}_b \frac{1}{\varpi} \frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1 - \rho_2^{trade}\right)} > 0 \tag{A.46}$$

$$\gamma_3 : \qquad \gamma_3 = \frac{-1}{-\frac{\gamma_b-\chi}{\beta}\varpi + (1 - \rho_1^{UIP})} = -\frac{1}{\tilde{\gamma}_b + (1 - \rho_1^{UIP})} < 0 \tag{A.47}$$

$$\gamma_4 : \qquad \gamma_4 = \frac{-\frac{\gamma_b-\chi}{\beta} + (1 - \rho_1^z)}{-\frac{\gamma_b-\chi}{\beta}\varpi + (1 - \rho_1^z)} = \frac{\tilde{\gamma}_b \frac{1}{\varpi} + (1 - \rho_1^z)}{\tilde{\gamma}_b + (1 - \rho_1^z)} > 0 \tag{A.48}$$

$$\gamma_5 : \qquad \gamma_5 = \frac{\frac{\gamma_b-\chi}{\beta} - (1 - \rho_2^z)}{-\frac{\gamma_b-\chi}{\beta}\varpi + (1 - \rho_2^z)} = -\frac{\tilde{\gamma}_b \frac{1}{\varpi} + (1 - \rho_2^z)}{\tilde{\gamma}_b + (1 - \rho_2^z)} < 0. \tag{A.49}$$

where the parameter $\tilde{\gamma}_b = -\frac{\gamma_b - \chi}{\beta}\varpi$ and the parameter γ_b is given by

$$\gamma_b \quad = \quad \frac{-(1 - \beta - \chi\varpi) - \sqrt{(1 - \beta - \chi\varpi)^2 + 4\chi\varpi}}{2\varpi} \tag{A.50}$$

which is the stable root associated with the quadratic equation

$$-\varpi\gamma_b^2 + (\beta - 1 + \chi\varpi)\,\gamma_b + \chi = 0. \tag{A.51}$$

The parameter γ_b is negative and decreasing in χ, i.e., $\frac{\partial \gamma_b}{\partial \chi} < 0$ for $0 < \beta < 1$. For the parameter $\tilde{\gamma}_b$ we therefore obtain $\frac{\partial \tilde{\gamma}_b}{\partial \chi} = \frac{\varpi}{\beta}\left(1 - \frac{\partial \gamma_b}{\partial \chi}\right) > 0$.

A.4.2 Trade Balance, $\tilde{T}_{1,t}$

Using Equation A.41, the trade balance is a linear function of the exogenous shocks and the inherited NFA position, $\tilde{B}_{1,t-1}$

$$\tilde{T}_{1,t} \quad = \quad \alpha_1 \xi_{1,t}^{trade} + \alpha_2 \xi_{2,t}^{trade} + \alpha_3 \xi_{1,t}^{UIP} + \alpha_4 z_{1,t} + \alpha_5 z_{2,t} + \alpha_b \tilde{B}_{1,t-1} \tag{A.52}$$

with the parameters

$$\alpha_1: \qquad \alpha_1 = \frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_1 = \frac{\left(1-\rho_1^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_1^{trade}\right)} > 0 \tag{A.53}$$

$$\alpha_2: \qquad \alpha_2 = -\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_2 = -\frac{\left(1-\rho_2^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_2^{trade}\right)} < 0 \tag{A.54}$$

$$\alpha_3: \qquad \alpha_3 = \varpi\gamma_3 = -\frac{\varpi}{\tilde{\gamma}_b + (1-\rho_1^{UIP})} < 0 \tag{A.55}$$

$$\alpha_4: \qquad \alpha_4 = -1 + \varpi\gamma_4 = \frac{(1-\rho_1^z)(\varpi-1)}{\tilde{\gamma}_b + (1-\rho_1^z)} \tag{A.56}$$

$$\alpha_5: \qquad \alpha_5 = 1 + \varpi\gamma_5 = \frac{-(1-\rho_2^z)(\varpi-1)}{\tilde{\gamma}_b + (1-\rho_2^z)} \tag{A.57}$$

$$\alpha_b: \qquad \alpha_b = \varpi\gamma_b < 0. \tag{A.58}$$

A.4.3 Net Foreign Assets, $\tilde{B}_{1,t}$

The trade balance is a linear function of the exogenous shocks and the inherited NFA position, $\tilde{B}_{1,t-1}$

$$\tilde{B}_{1,t} = \beta_1\xi_{1,t}^{trade} + \beta_2\xi_{2,t}^{trade} + \beta_3\xi_{1,t}^{UIP} + \beta_b\tilde{B}_{1,t-1} + \beta_4 z_{1,t} + \beta_5 z_{2,t} \tag{A.59}$$

with the parameters

$$\beta_1: \qquad \beta_1 = \frac{\alpha_1}{\beta} = \frac{1}{\beta}\frac{\left(1-\rho_1^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_1^{trade}\right)} > 0 \tag{A.60}$$

$$\beta_2: \qquad \beta_2 = \frac{\alpha_2}{\beta} = -\frac{1}{\beta}\frac{\left(1-\rho_2^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_2^{trade}\right)} < 0 \tag{A.61}$$

$$\beta_3: \qquad \beta_3 = \frac{\alpha_3}{\beta} = -\frac{1}{\beta}\frac{\varpi}{\tilde{\gamma}_b + (1-\rho_1^{UIP})} < 0 \tag{A.62}$$

$$\beta_4: \qquad \beta_4 = \frac{\alpha_4}{\beta} = \frac{1}{\beta}\frac{(1-\rho_1^z)(\varpi-1)}{\tilde{\gamma}_b + (1-\rho_1^z)} \tag{A.63}$$

$$\beta_5: \qquad \beta_5 = \frac{\alpha_5}{\beta} = -\frac{1}{\beta}\frac{(1-\rho_2^z)(\varpi-1)}{\tilde{\gamma}_b + (1-\rho_2^z)} \tag{A.64}$$

$$\beta_b: \qquad \beta_b = \frac{\alpha_b + 1}{\beta} = \frac{\gamma_b}{\gamma_b - \chi}. \tag{A.65}$$

A.5 Statistical Moments

Using the decision rules of the model, we compute analytical expressions for statistical moments displayed in the main text and the proofs. To ease notation just within this subsection we define

$$\rho_1 = \rho_1^{trade} \quad \text{(A.66)}$$

$$\rho_2 = \rho_2^{trade} \quad \text{(A.67)}$$

$$\rho_3 = \rho_1^{UIP} \quad \text{(A.68)}$$

$$\rho_4 = \rho_1^{z} \quad \text{(A.69)}$$

$$\rho_5 = \rho_2^{z} \quad \text{(A.70)}$$

and similarly for the variances σ_i. Although we generally assume shock processes to be uncorrelated, we do allow for this possibility in the following. The correlations coefficients between two shocks are denote by ρ_{ij}.

From the decision role for $\tilde{B}_{1,t}$ in Equation A.59, the unconditional variance of $\tilde{B}_{1,t}$ is

$$E_t\left(\tilde{B}_{1,t}^2\right) = \sum_i \sum_j \left\{ \frac{\beta_i \beta_j}{1-\beta_b^2} \frac{1+\rho_i \beta_b}{1-\rho_i \beta_b} \frac{\rho_{ij}\sigma_i \sigma_j}{1-\rho_i \rho_j} \right\} \quad \text{(A.71)}$$

where i and $j \in \{1,2,3,4,5\}$. The covariance between two exogenous shocks is given by

$$E_t\left(\xi_{i,t}\xi_{j,t}\right) = \frac{\rho_{ij}\sigma_i\sigma_j}{1-\rho_i\rho_j} \quad \text{(A.72)}$$

and the covariance between $\tilde{B}_{1,t}$ and shock $\xi_{i,t}$ is given by

$$E_t\left(\xi_{i,t}\tilde{B}_{1,t}\right) = \sum_j \left\{ \frac{\beta_j}{1-\rho_i\beta_b} \frac{\rho_{ij}\sigma_i\sigma_j}{1-\rho_i\rho_j} \right\}. \quad \text{(A.73)}$$

Let the coefficients in the decision rules for variables x_t and y_t are denoted by γ and α, respectively. The variance/covariance of these two variables (expressed in deviations from the steady state) is

$$E_t(x_t y_t) = \sum_i \sum_j \left\{ \left(\gamma_i \alpha_j + \Omega_i \beta_j\right) \frac{\rho_{ij}\sigma_i\sigma_j}{1-\rho_i\rho_j} \right\} \tag{A.74}$$

where

$$\Omega_i = \left(\gamma_i \alpha_b + \gamma_b \alpha_i\right) \frac{\rho_i}{1-\rho_i\beta_b} + \gamma_b \alpha_b \frac{\beta_i}{1-\beta_b^2} \frac{1+\rho_i\beta_b}{1-\rho_i\beta_b}. \tag{A.75}$$

Here the coefficients β_i or β_b are those in the decision rules of the NFA position.

Similarly, to compute the moments for $\Delta x_{t,t-1} = x_t - x_{t-1}$ and $\Delta y_{t,t-1} = y_t - y_{t-1}$ (variables in expressed in growth rates), it is

$$E_t\left(\Delta x_{t,t-1} \Delta y_{t,t-1}\right) = \sum_i \sum_j \left\{ \left(d\gamma_i d\alpha_j + \gamma_i \alpha_j \left(1-\rho_i\rho_j\right) + \Gamma_i \beta_j\right) \frac{\rho_{ij}\sigma_i\sigma_j}{1-\rho_i\rho_j} \right\} \tag{A.76}$$

where the coefficients for the decision rules $\Delta x_{t,t-1}$ and $\Delta y_{t,t-1}$ relate to those of the original rules associated with x_t and y_t as follows

$$\Gamma_i = \left(d\gamma_i d\alpha_b + d\gamma_b d\alpha_i\right) \frac{\rho_i}{1-\rho_i\beta_b} + d\gamma_b d\alpha_b \frac{\beta_i}{1-\beta_b^2} \frac{1+\rho_i\beta_b}{1-\rho_i\beta_b} \tag{A.77}$$

$$d\gamma_i = \gamma_i\left(\rho_i - 1\right) + \gamma_b \beta_i \tag{A.78}$$

$$d\gamma_b = \gamma_b\left(\beta_b - 1\right) = \chi\beta_b \tag{A.79}$$

$$d\alpha_i = \alpha_i\left(\rho_i - 1\right) + \alpha_b \beta_i \tag{A.80}$$

$$d\alpha_b = \alpha_b\left(\beta_b - 1\right). \tag{A.81}$$

B Appendix: Proofs of Theorems

B.1 Proof of Theorem 1 and Corollary 2

Theorem 1 *A rebalancing shock that improves the home country's terms of trade (appreciates the real exchange rate), $\xi_{1,t}^{trade} > 0$ and/or $\xi_{2,t}^{trade} < 0$, is associated with an improvement of the trade balance. By contrast, a financial (UIP) shock that improves the terms of trade (appreciates the real exchange rate), $\xi_{1,t}^{UIP} > 0$, is associated with a deterioration of the trade balance. If financial markets provide less risk sharing, i.e., χ assumes a higher value, the terms of trade are more (less) sensitive to the rebalancing (financial) shock, and the trade balance is less sensitive to both the rebalancing and the financial shock.*

Proof. We split the proof into two parts.

Claim 1: The home country's terms of trade improve (i.e., the real exchange rate appreciates) after a positive rebalancing shock towards the home country's good, $\xi_{1,t}^{trade} > 0$ and/or $\xi_{2,t}^{trade} < 0$, and a positive financial (UIP) shock $\xi_{1,t}^{UIP} > 0$. The magnitude of the terms of trade response to a given-sized shock is increasing in the value of χ for rebalancing shocks, but decreasing for the financial (UIP) shock.

Consider the decision rules for the terms of trade computed in Appendix A.4, where

$$\hat{\delta}_{1,t} = \gamma_1 \xi_{1,t}^{trade} + \gamma_2 \xi_{2,t}^{trade} + \gamma_3 \xi_{1,t}^{UIP} + \gamma_4 z_{1,t} + \gamma_5 z_{2,t} + \gamma_b \tilde{B}_{1,t-1}. \tag{B.1}$$

The coefficients of interest rate are γ_1 and γ_3, repeated here for convenience:

$$\gamma_1 = -\frac{\tilde{\gamma}_b \frac{1}{\varpi} \frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1 - \rho_1^{trade}\right)} < 0 \tag{B.2}$$

$$\gamma_3 = -\frac{1}{\tilde{\gamma}_b + (1 - \rho_1^{UIP})} < 0 \tag{B.3}$$

where $\tilde{\gamma}_b = -\frac{\gamma_b - \chi}{\beta} \varpi$.

Recall that γ_b is negative and decreasing in χ, i.e., $\frac{\partial \gamma_b}{\partial \chi} < 0$ for $\beta > 0$. For the parameter $\tilde{\gamma}_b$ we therefore obtain $\frac{\partial \tilde{\gamma}_b}{\partial \chi} = \frac{\varpi}{\beta} \left(1 - \frac{\partial \gamma_b}{\partial \chi}\right) > 0$. Hence, the derivatives of the coefficients γ_1 and γ_3 with respect to χ are

$$\frac{\partial \gamma_1}{\partial \chi} = -\frac{\frac{1}{\varpi}\frac{\omega_1^c}{1-\omega_1^c}\left(1-\rho_1^{trade}\right)}{\left(\tilde{\gamma}_b+\left(1-\rho_1^{trade}\right)\right)^2}\frac{\partial \tilde{\gamma}_b}{\partial \chi} < 0 \quad (B.4)$$

$$\frac{\partial \gamma_3}{\partial \chi} = \frac{1}{\left(\tilde{\gamma}_b+(1-\rho_1^{UIP})\right)^2}\frac{\partial \tilde{\gamma}_b}{\partial \chi} > 0. \quad (B.5)$$

Equations B.2 - B.5 establish Claim 1.

Claim 2: The trade and UIP shocks move the terms of trade (the real exchange rate) in the same direction, but move the trade balance in opposite directions. The higher the bond price elasticity to the NFA position, χ, the smaller are the effects on the trade balance of given-size shocks. The effects on the terms of trade is larger (smaller) for the trade (UIP) shock, the larger the value of χ.

Consider the decision rules for the trade balance computed in Appendix A.4, where

$$\tilde{T}_{1,t} = \alpha_1\xi_{1,t}^{trade} + \alpha_2\xi_{2,t}^{trade} + \alpha_3\xi_{1,t}^{UIP} + \alpha_4 z_{1,t} + \alpha_5 z_{2,t} + \alpha_b\tilde{B}_{1,t-1} \quad (B.6)$$

The coefficients of interest rate are α_1 and α_3, repeated here for convenience:

$$\alpha_1 = \frac{\left(1-\rho_1^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b+\left(1-\rho_1^{trade}\right)} > 0 \quad (B.7)$$

$$\alpha_3 = -\frac{\varpi}{\tilde{\gamma}_b+(1-\rho_1^{UIP})} < 0. \quad (B.8)$$

The derivatives of the coefficients α_1 and α_3 with respect to χ are

$$\frac{\partial \alpha_1}{\partial \chi} = -\frac{\left(1-\rho_1^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\left(\tilde{\gamma}_b+\left(1-\rho_1^{trade}\right)\right)^2}\frac{\partial \tilde{\gamma}_b}{\partial \chi} < 0 \quad (B.9)$$

$$\frac{\partial \alpha_3}{\partial \chi} = \frac{\varpi}{\left(\tilde{\gamma}_b+(1-\rho_1^{UIP})\right)^2}\frac{\partial \tilde{\gamma}_b}{\partial \chi} > 0. \quad (B.10)$$

Equations B.7 - B.10 establish Claim 2.

The theorem follows directly from the two claims. ■

These features of the decision rules are also reflected in the unconditional moments of the trade balance and the terms of trade. Abstracting from technology shocks to simplify the exposition, the following corollary applies.

Appendix

Corollary 2 *The financial (UIP) shock induces a positive covariance between the growth rate of the terms of trade and the growth rate of the trade balance. The rebalancing shock induces a negative covariance between the two growth rates. When both shocks are present in the model, the overall covariance is determined by the extent of international financial risk sharing as measured by* χ.

Proof. The covariance between the growth rate of the terms of trade and the growth rate of the trade balance is

$$\begin{aligned} cov\left(\Delta\hat{\delta}_{1,t}, \Delta\tilde{T}_{1,t}\right) &= cov\left(\Delta\hat{\delta}_{1,t}, \frac{\omega_1^c}{1-\omega_1^c}\Delta\xi_{1,t}^{trade} + \varpi\Delta\hat{\delta}_{1,t}\right) \\ &= \frac{\omega_1^c}{1-\omega_1^c} cov\left(\Delta\hat{\delta}_{1,t}, \Delta\xi_{1,t}^{trade}\right) + \varpi var\left(\Delta\hat{\delta}_{1,t}\right) \end{aligned} \tag{B.11}$$

where, using Equation A.76,

$$\begin{aligned} cov\left(\Delta\hat{\delta}_{1,t}, \Delta\xi_{1,t}^{trade}\right) &= -\left(1-\rho_1^{trade}\right)\left[d\gamma_1 - \left(1+\rho_1^{trade}\right)\gamma_1 + \gamma_b\frac{\rho_1^{trade}\beta_1}{1-\rho_1^{trade}\beta_b}\right] var\left(\xi_{1,t}^{trade}\right) \\ &= \left[\left(1-\left(\rho_1^{trade}\right)^2\right)\gamma_1 - \frac{\left(1-\rho_1^{trade}\right)\chi\beta_1}{1-\rho_1^{trade}\beta_b}\right] var\left(\xi_{1,t}^{trade}\right) \\ &= -\chi\beta_1\left[\frac{1+\rho_1^{trade}}{1-\beta_b} + \frac{1-\rho_1^{trade}}{1-\rho_1^{trade}\beta_b}\right] var\left(\xi_{1,t}^{trade}\right) < 0 \end{aligned} \tag{B.12}$$

and

$$\begin{aligned} var\left(\Delta\hat{\delta}_{1,t}\right) &= \left[d\gamma_1^2 + 2d\gamma_1 d\gamma_b\frac{\rho_1^{trade}\beta_1}{1-\rho_1^{trade}\beta_b} + d\gamma_b^2\beta_1^2\frac{1}{1-\beta_b^2}\frac{1+\rho_1^{trade}\beta_b}{1-\rho_1^{trade}\beta_b}\right] var\left(\xi_{1,t}^{trade}\right) \\ &\quad + \left(1-\left(\rho_1^{trade}\right)^2\right)\gamma_1^2 var\left(\xi_{1,t}^{trade}\right) \\ &\quad + \left[d\gamma_3^2 + 2d\gamma_3 d\gamma_b\frac{\rho_1^{UIP}\beta_3}{1-\rho_1^{UIP}\beta_b} + d\gamma_b^2\beta_3^2\frac{1}{1-\beta_b^2}\frac{1+\rho_1^{UIP}\beta_b}{1-\rho_1^{UIP}\beta_b}\right] var\left(\xi_{1,t}^{UIP}\right) \\ &\quad + \left(1-\left(\rho_1^{UIP}\right)^2\right)\gamma_3^2 var\left(\xi_{1,t}^{UIP}\right) \\ &= \frac{\chi^2\beta_1^2}{1-\beta_b^2}\frac{1+\rho_1^{trade}\beta_b}{1-\rho_1^{trade}\beta_b} var\left(\xi_{1,t}^{trade}\right) + \gamma_1^2\left(1-\left(\rho_1^{trade}\right)^2\right) var\left(\xi_{1,t}^{trade}\right) \\ &\quad + \frac{\chi^2\beta_3^2}{1-\beta_b^2}\frac{1+\rho_1^{UIP}\beta_b}{1-\rho_1^{UIP}\beta_b} var\left(\xi_{1,t}^{UIP}\right) + \left(1+2\frac{\chi\beta_3}{1-\rho_1^{UIP}\beta_b}\right) var\left(\xi_{1,t}^{UIP}\right) \\ &\quad + \gamma_3^2\left(1-\left(\rho_1^{UIP}\right)^2\right) var\left(\xi_{1,t}^{UIP}\right) \\ &= var\left(\Delta\hat{\delta}_{1,t}|\xi_{1,t}^{trade}\right) + var\left(\Delta\hat{\delta}_{1,t}|\xi_{1,t}^{UIP}\right). \end{aligned} \tag{B.13}$$

Appendix

The overall variance of the terms of trade is the sum of the terms of trade variance that is due to the rebalancing shock and the terms of trade variance that is due to the financial (UIP) shock.

The variance of the change in the trade balance is given by

$$\begin{aligned} var\left(\Delta\tilde{T}_{1,t}\right) &= \left(\frac{\omega_1^c}{1-\omega_1^c}\right)^2 var\left(\Delta\xi_{1,t}^{trade}\right) + \varpi^2 var\left(\Delta\hat{\delta}_{1,t}\right) \\ &\quad +2\left(\frac{\omega_1^c}{1-\omega_1^c}\right)\varpi cov\left(\Delta\hat{\delta}_{1,t},\Delta\xi_{1,t}^{trade}\right). \end{aligned} \tag{B.14}$$

To understand the comovement between the changes in the terms of trade and the trade balance, we consider financial (UIP) and rebalancing shocks in turns. If the model admits financial (UIP) shocks only, it is $\frac{\omega_1^c}{1-\omega_1^c}cov\left(\Delta\hat{\delta}_{1,t},\Delta\xi_{1,t}^{trade}\right)=0$ and

$$cov\left(\Delta\hat{\delta}_{1,t},\Delta\tilde{T}_{1,t}|\xi_{1,t}^{UIP}\right) = \varpi var\left(\Delta\hat{\delta}_{1,t}|\xi_{1,t}^{UIP}\right) > 0. \tag{B.15}$$

Because $var\left(\Delta\tilde{T}_{1,t}\right)=\varpi^2 var\left(\Delta\hat{\delta}_{1,t}|\xi_{1,t}^{UIP}\right)$, the associated correlation coefficient is equal to 1.

If the model admits rebalancing shocks only,

$$\begin{aligned} cov\left(\Delta\hat{\delta}_{1,t},\Delta\tilde{T}_{1,t}|\xi_{1,t}^{trade}\right) &= \left[-\frac{\omega_1^c}{1-\omega_1^c}\frac{\left(1-\rho_1^{trade}\right)\chi\beta_1}{1-\rho_1^{trade}\beta_b} + \varpi\frac{\chi^2\beta_1^2}{1-\beta_b^2}\frac{1+\rho_1^{trade}\beta_b}{1-\rho_1^{trade}\beta_b}\right. \\ &\quad \left. + \frac{\omega_1^c}{1-\omega_1^c}\left(1-\rho_1^{trade}\right)\gamma_1 + \varpi\gamma_1^2\left(1-\left(\rho_1^{trade}\right)^2\right)\right] var\left(\xi_{1,t}^{trade}\right) \\ &= -\frac{\omega_1^c}{1-\omega_1^c}\chi\beta_1\left[\frac{\left(1-\rho_1^{trade}\right)^2}{1-\rho_1^{trade}\beta_b}\frac{\chi\beta_b+\frac{\beta}{\varpi}\left(1-\beta_b^2\right)}{\chi\frac{1+\beta_b}{1-\rho_1^{trade}}+\frac{\beta}{\varpi}\left(1-\beta_b^2\right)}\right. \\ &\quad \left. +\frac{1-\left(\rho_1^{trade}\right)^2}{1-\beta_b}\right] var\left(\xi_{1,t}^{trade}\right) < 0. \end{aligned} \tag{B.16}$$

The negative covariance implies that the associated correlation coefficient is also negative (but larger than -1).

With the rebalancing shock inducing negative correlation between $\Delta\hat{\delta}_{1,t}$ and $\Delta\tilde{T}_{1,t}$ and the financial (UIP) shock inducing positive correlation, the covariance in a model with both shocks being active depends, among other parameters, on the extent of international risk sharing as governed by the value of χ. ■

B.2 Proof of Theorem 3 and Theorem 4

Theorem 3 *Abstracting from technology shocks, the ratio of the standard deviation of the real exchange rate, $\hat{q}_{1,t}$, and consumption, $\hat{C}_{1,t}$, is independent of the relative variances of the rebalancing and the financial (UIP) shock,*

$$\frac{std\left(\hat{q}_{1,t}\right)}{std\left(\hat{C}_{1,t}\right)} = \frac{std\left(\Delta\hat{q}_{1,t}\right)}{var\left(\Delta\hat{C}_{1,t}\right)} = \frac{2\omega_1^c - 1}{1-\omega_1^c}. \tag{B.17}$$

The correlation between relative consumption, $\hat{C}_{1,t} - \hat{C}_{2,t}$, and the real exchange rate is equal to -1 regardless of the relative variances of the rebalancing and the financial (UIP) shock,

$$corr\left(\hat{C}_{1,t} - \hat{C}_{2,t}, \hat{q}_{1,t}\right) = -1. \tag{B.18}$$

Proof. Absent technology shocks the consumption-real-exchange-rate variance ratio is

$$\frac{std\left(\hat{q}_{1,t}\right)}{std\left(\hat{C}_{1,t}\right)} = \frac{\sqrt{\left(2\omega_1^c - 1\right)^2 var\left(\hat{\delta}_{1,t}\right)}}{\sqrt{\left(1-\omega_1^c\right)^2 var\left(\hat{\delta}_{1,t}\right)}} = \frac{2\omega_1^c - 1}{1-\omega_1^c}. \tag{B.19}$$

The same applies when expressing the variables in growth rates instead of levels.

For the correlation between relative consumption and the real exchange rate it is

$$corr\left(\hat{C}_{1,t} - \hat{C}_{2,t}, \hat{q}_{1,t}\right) = \frac{-2\left(1-\omega_1^c\right)\left(2\omega_1^c - 1\right)E\left(\hat{\delta}_{1,t}^2\right)}{\sqrt{4(1-\omega_1^c)^2 E\left(\hat{\delta}_{1,t}^2\right)}\sqrt{(2\omega_1^c - 1)^2 E\left(\hat{\delta}_{1,t}^2\right)}} = -1. \tag{B.20}$$

Neither the financial (UIP) nor the rebalancing shock enter directly into the equations determining the real exchange rate and consumption. Both shocks enter only indirectly through the terms of trade. Hence, the computed moments do not depend on the relative variances of the rebalancing and the financial (UIP) shock. ■

Theorem 4 *If the model admits only rebalancing and financial (UIP) shocks, the Fama coefficient is constant and negative independent of the degree of international* *financial risk sharing as measured by* χ*, as long as* $\chi \neq 0$*:*

$$\hat{\beta}^{Fama} = \frac{cov\left(E_t \Delta \hat{q}_{1,t+1}, r_{1,t} - r_{2,t}\right)}{var\left(r_{1,t} - r_{2,t}\right)} = -\frac{2\omega_1^c - 1}{2\left(1 - \omega_1^c\right)} < 0. \tag{B.21}$$

Proof. We assume that rebalancing shocks and financial (UIP) shocks are the only shocks in the model. All shocks are uncorrelated with each other. First, note that using the UIP condition, Equation A.39, it is

$$\begin{aligned} cov\left(\Delta \hat{q}_{1,t+1}, r_{1,t} - r_{2,t}\right) &= var\left(r_{1,t} - r_{2,t}\right) + \chi cov\left(\Delta \hat{q}_{1,t+1}, \tilde{B}_{1,t}\right) \\ &\quad + cov\left(\Delta \hat{q}_{1,t+1}, \xi_{1,t}^{UIP}\right) - var\left(\chi \tilde{B}_{1,t} + \xi_{1,t}^{UIP}\right) \end{aligned}$$

which, after applying the relationship $\hat{q}_{1,t} = \left(2\omega_1^c - 1\right) \hat{\delta}_{1,t}$, implies that the Fama coefficient can be stated as

$$\begin{aligned} \hat{\beta}^{Fama} &= 1 + \frac{\left(2\omega_1^c - 1\right)}{var\left(r_{1,t} - r_{2,t}\right)} \left[\chi cov\left(\Delta \hat{\delta}_{1,t+1}, \tilde{B}_{1,t}\right) + cov\left(\Delta \hat{\delta}_{1,t+1}, \xi_{1,t}^{UIP}\right)\right] \\ &\quad - \frac{1}{var\left(r_{1,t} - r_{2,t}\right)} \left[\chi^2 var\left(\tilde{B}_{1,t}\right) + 2\chi cov\left(\tilde{B}_{1,t}, \xi_{1,t}^{UIP}\right) + var\left(\xi_{1,t}^{UIP}\right)\right]. \end{aligned} \tag{B.22}$$

The decision rules presented in Appendix A.4, imply that the terms entering Equation B.22 can be expressed solely in terms of the underlying exogenous shocks:

$$\begin{aligned} & cov\left(\Delta \hat{\delta}_{1,t+1}, \tilde{B}_{1,t}\right) = E\left(\left[d\gamma_1 \xi_{1,t}^{trade} + d\gamma_3 \xi_{1,t}^{UIP} + d\gamma_b \tilde{B}_{1,t-1}\right] \tilde{B}_{1,t}\right) \\ & = \chi \beta_1 \left(1 + \rho_1^{trade} \beta_b\right) E\left(\xi_{1,t}^{trade} \tilde{B}_{1,t}\right) + \left[1 + \chi \beta_3 \left(1 + \rho_1^{UIP} \beta_b\right)\right] E\left(\xi_{1,t}^{UIP} \tilde{B}_{1,t}\right) + \chi \beta_b^2 E\left(\tilde{B}_{1,t}^2\right) \\ & = \chi \frac{\beta_1^2}{1 - \beta_b^2} \frac{1 + \rho_1^{trade} \beta_b}{1 - \rho_1^{trade} \beta_b} var\left(\xi_{1,t}^{trade}\right) + \left[\chi \frac{\beta_3^2}{1 - \beta_b^2} \frac{1 + \rho_1^{UIP} \beta_b}{1 - \rho_1^{UIP} \beta_b} + \frac{\beta_3}{1 - \rho_1^{UIP} \beta_b}\right] var\left(\xi_{1,t}^{UIP}\right) \end{aligned}$$

$$\begin{aligned} & cov\left(\Delta \hat{\delta}_{1,t+1}, \xi_{1,t}^{UIP}\right) = E\left(\left[d\gamma_1 \xi_{1,t}^{trade} + d\gamma_3 \xi_{1,t}^{UIP} + d\gamma_b \tilde{B}_{1,t-1}\right] \xi_{1,t}^{UIP}\right) \\ & = \left(1 + \frac{\chi \beta_3}{1 - \rho_1^{UIP} \beta_b}\right) var\left(\xi_{1,t}^{UIP}\right) \end{aligned}$$

$$\begin{aligned} & \chi^2 var\left(\tilde{B}_{1,t}\right) + 2\chi cov\left(\tilde{B}_{1,t}, \xi_{1,t}^{UIP}\right) + var\left(\xi_{1,t}^{UIP}\right) = \chi^2 \frac{\beta_1^2}{1 - \beta_b^2} \frac{1 + \rho_1^{trade} \beta_b}{1 - \rho_1^{trade} \beta_b} var\left(\xi_{1,t}^{trade}\right) \\ & + \left[\chi^2 \frac{\beta_3^2}{1 - \beta_b^2} \frac{1 + \rho_1^{UIP} \beta_b}{1 - \rho_1^{UIP} \beta_b} + \left(1 + \frac{2\chi \beta_3}{1 - \rho_1^{UIP} \beta_b}\right)\right] var\left(\xi_{1,t}^{UIP}\right) \end{aligned}$$

which equals the sum of the two preceding terms, $\Sigma_{\Delta\hat{\delta}_{1,t+1},\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)} = \chi cov\left(\Delta\hat{\delta}_{1,t+1},\tilde{B}_{1,t}\right) + cov\left(\Delta\hat{\delta}_{1,t+1},\xi_{1,t}^{UIP}\right)$.

Before turning to the variance of the interest rate differential, $r_{1,t}-r_{2,t}$, we first establish that the variance of the growth rate of the terms of trade, $\Delta\hat{\delta}_{1,t+1}$, is

$$\begin{aligned}
var\left(\Delta\hat{\delta}_{1,t+1}\right) &= E\left(\left[d\gamma_1\xi_{1,t}^{trade}+d\gamma_3\xi_{1,t}^{UIP}+d\gamma_b\tilde{B}_{1,t-1}\right]^2\right)\\
&= \frac{\chi^2\beta_1^2}{1-\beta_b^2}\frac{1+\rho_1^{trade}\beta_b}{1-\rho_1^{trade}\beta_b}var\left(\xi_{1,t}^{trade}\right)\\
&\quad+\frac{\chi^2\beta_3^2}{1-\beta_b^2}\frac{1+\rho_1^{UIP}\beta_b}{1-\rho_1^{UIP}\beta_b}var\left(\xi_{1,t}^{UIP}\right)\\
&\quad+\left(1+2\frac{\chi\beta_3}{1-\rho_1^{UIP}\beta_b}\right)var\left(\xi_{1,t}^{UIP}\right)\\
&= \Sigma_{\Delta\hat{\delta}_{1,t+1},\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)} \qquad \text{(B.23)}
\end{aligned}$$

Finally, we obtain for the variance of the interest rate differential, $r_{1,t}-r_{2,t}$, that

$$\begin{aligned}
var\left(r_{1,t}-r_{2,t}\right) &= var\left(\Delta\hat{q}_{1,t+1}\right)-2cov\left(\Delta\hat{q}_{1,t+1},\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)\\
&\quad+var\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)\\
&= \left(2\omega_1^c-1\right)^2 var\left(\Delta\hat{\delta}_{1,t+1}\right)-\left(4\omega_1^c-3\right)\Sigma_{\Delta\hat{\delta}_{1,t},\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)}\\
&= 4\left(1-\omega_1^c\right)^2\Sigma_{\Delta\hat{\delta}_{1,t},\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)}. \qquad \text{(B.24)}
\end{aligned}$$

Applying these findings in Equation B.22, the Fama coefficient reduces to

$$\hat{\beta}^{Fama} = -\frac{2\omega_1^c-1}{2\left(1-\omega_1^c\right)} = 1-\frac{1}{2\left(1-\omega_1^c\right)}. \qquad \text{(B.25)}$$

■

C Appendix: Model Extensions

C.1 Tariffs and Iceberg trade costs

In this section, we show that the rebalancing shock is closely related to shocks to tariffs and trade costs. In detail, we distinguish between import tariffs, export subsidies, and iceberg trade costs.

We denote the import price of country 1 by $P^m_{1,t}$ and the producer price in country 2 by $P^d_{2,t}$. The different trading frictions affect international prices as follows:

- under iceberg trade costs a share $\tau^{ice}_{1,t}$ of the shipped good is lost in the shipping process, implying an import price $P^m_{1,t}$ to be

$$P^m_{1,t} = \frac{1}{1-\tau^{ice}_{1,t}} e_{1,t} P^d_{2,t} \tag{C.1}$$

- under an import tariff $\tau^m_{1,t}$ the import price $P^m_{1,t}$ increases over the producer price according to

$$P^m_{1,t} = (1+\tau^m_{1,t}) e_{1,t} P^d_{2,t} \tag{C.2}$$

- under an export subsidy $\tau^x_{2,t}$ the import price $P^m_{1,t}$ falls below the producer price according to

$$P^m_{1,t} = \left(1-\tau^x_{2,t}\right) e_{1,t} P^d_{2,t} = e_{1,t} \tilde{P}^d_{2,t} \tag{C.3}$$

If all three elements are present, the following relationship applies between the import price of country 1 and the foreign production price of country 2

$$P^m_{1,t} = (1+\tau^m_{1,t}) \frac{1-\tau^x_{2,t}}{1-\tau^{ice}_{1,t}} e_{1,t} P^d_{2,t} = \frac{1+\tau^m_{1,t}}{1-\tau^{ice}_{1,t}} e_{1,t} \tilde{P}^d_{2,t}. \tag{C.4}$$

Similarly, import prices of country 2 and the foreign production price of country 1 are related via

$$P^m_{2,t} = (1+\tau^m_{2,t}) \frac{1-\tau^x_{1,t}}{1-\tau^{ice}_{2,t}} \frac{1}{e_{1,t}} P^d_{1,t} = \frac{1+\tau^m_{2,t}}{1-\tau^{ice}_{2,t}} \frac{1}{e_{1,t}} \tilde{P}^d_{1,t}. \tag{C.5}$$

Appendix

Using Equations A.3 and A.4, the relative prices $\frac{P^c_{1,t}}{P^d_{1,t}}$ and $\frac{P^c_{2,t}}{P^d_{2,t}}$ are shown to be

$$\begin{aligned}\frac{P^c_{1,t}}{P^d_{1,t}} &= \left[\omega^c_{1,t} + \left(1-\omega^c_{1,t}\right)\left(\frac{P^d_{1,t}}{P^m_{1,t}}\right)^{\frac{1}{\rho^c}}\right]^{-\rho^c} = \left[\omega^c_{1,t} + \left(1-\omega^c_{1,t}\right)\left((1+\tau^m_{1,t})\frac{1-\tau^x_{2,t}}{1-\tau^{ice}_{1,t}}\delta_{1,t}\right)^{-\frac{1}{\rho^c}}\right]^{-\rho^c} \\ &= \left[\omega^c_{1,t} + (1-\tilde{\omega}^c_{1,t})\delta_{1,t}^{-\frac{1}{\rho^c}}\right]^{-\rho^c} = F_{1,t}^{-\rho^c} \qquad \text{(C.6)}\end{aligned}$$

$$\begin{aligned}\frac{P^c_{2,t}}{P^d_{2,t}} &= \left[\omega^c_{2,t} + \left(1-\omega^c_{2,t}\right)\left(\frac{P^d_{2,t}}{P^m_{2,t}}\right)^{\frac{1}{\rho^c}}\right]^{-\rho^c} = \left[\omega^c_{2,t} + \left(1-\omega^c_{2,t}\right)\left((1+\tau^m_{2,t})\frac{1-\tau^x_{1,t}}{1-\tau^{ice}_{2,t}}\frac{1}{\delta_{1,t}}\right)^{-\frac{1}{\rho^c}}\right]^{-\rho^c} \\ &= \left[\omega^c_{2,t} + (1-\tilde{\omega}^c_{2,t})\delta_{1,t}^{\frac{1}{\rho^c}}\right]^{-\rho^c} = F_{2,t}^{-\rho^c}. \qquad \text{(C.7)}\end{aligned}$$

In the presence of iceberg trade costs the market clearing condition is given by

$$Y_{1,t} = C^d_{1,t} + \frac{1}{1-\tau^{ice}_{2,t}}M_{2,t} \qquad \text{(C.8)}$$

$$Y_{2,t} = C^d_{2,t} + \frac{1}{1-\tau^{ice}_{1,t}}M_{1,t} \qquad \text{(C.9)}$$

where $M_{1,t}$ and $M_{2,t}$ denote the final consumption of imports (net of iceberg costs).

The government's net receipts from import tariffs and export subsidies amount to

$$tariffs_{t,1} = \tau^m_{1,t}e_{1,t}\tilde{P}^d_{2,t}\frac{M_{1,t}}{1-\tau^{ice}_{1,t}} - \tau^x_{1,t}P^d_{1,t}\frac{M_{2,t}}{1-\tau^{ice}_{2,t}}. \qquad \text{(C.10)}$$

From the consolidated budget constraint we obtain

$$P^d_{1,t}C^d_{1,t} + P^m_{1,t}M_{1,t} + \frac{P^b_{1,t}}{\phi^b_{1,t}}B_{1,t} = P^d_{1,t}C^d_{1,t} + P^d_{1,t}\frac{M_{2,t}}{1-\tau^{ice}_{2,t}} + B_{1,t-1} + tariffs_{t,1} \qquad \text{(C.11)}$$

or

$$\frac{P^b_{1,t}B_{1,t}}{\phi^b_{1,t}} = T_{1,t} + B_{1,t-1} \qquad \text{(C.12)}$$

with

$$\begin{aligned}T_{1,t} &= P^d_{1,t}\frac{M_{2,t}}{1-\tau^{ice}_{2,t}} - P^m_{1,t}M_{1,t} + tariffs_{t,1} \qquad \text{(C.13)} \\ &= \frac{1-\tau^x_{1,t}}{1-\tau^{ice}_{2,t}}P^d_{1,t}M_{2,t} - \frac{1-\tau^x_{2,t}}{1-\tau^{ice}_{1,t}}e_{1,t}P^d_{2,t}M_{1,t} \qquad \text{(C.14)}\end{aligned}$$

Appendix

We define

$$\tilde{T}_{1,t} = \frac{T_{1,t}}{\frac{1-\tau^x_{1,t}}{1-\tau^{ice}_{2,t}} P^d_{1,t} M_{2,t}} = 1 - \frac{1-\tau^x_{2,t}}{1-\tau^x_{1,t}} \frac{1-\tau^{ice}_{2,t}}{1-\tau^{ice}_{1,t}} \delta_{1,t} \frac{M_{1,t}}{M_{2,t}} \tag{C.15}$$

As the first order conditions for consumption are unchanged, the model dynamics can be summarized by the same three equations as before:

$$E_t \left\{ \frac{P^c_{1,t}}{F^{\rho^c}_{1,t+1} P^c_{1,t+1}} \left[\phi^b_{1,t} \frac{exp(z_{1,t})}{exp(z_{1,t+1})} - \phi^b_{2,t} \frac{exp(z_{2,t})}{exp(z_{2,t+1})} \frac{\delta_{1,t}}{\delta_{1,t+1}} \right] \right\} = 0 \tag{C.16}$$

$$\frac{P^b_{1,t} \tilde{B}_{1,t}}{\phi^b_{1,t}} = \tilde{T}_{1,t} + \frac{\frac{1-\tau^x_{1,t-1}}{1-\tau^{ice}_{2,t-1}} P^d_{1,t-1} M_{2,t-1}}{\frac{1-\tau^x_{1,t}}{1-\tau^{ice}_{2,t}} P^d_{1,t} M_{2,t}} \tilde{B}_{1,t-1} \tag{C.17}$$

with

$$\tilde{T}_{1,t} = 1 - \frac{1-\tilde{\omega}^c_{1,t}}{1-\tilde{\omega}^c_{2,t}} \frac{1+\tau^m_{2,t}}{1+\tau^m_{1,t}} \left(\frac{F_{1,t}}{F_{2,t}} \right)^{-1} \frac{exp(z_{1,t})}{exp(z_{2,t})} \delta_{1,t}^{1-2\frac{1+\rho^c}{\rho^c}} \tag{C.18}$$

$$= 1 - \frac{1-\tilde{\omega}^c_{1,t}}{1-\tilde{\omega}^c_{2,t}} \frac{1+\tau^m_{2,t}}{1+\tau^m_{1,t}} \left(\frac{\omega^c_{1,t} + (1-\tilde{\omega}^c_{1,t}) \delta_{1,t}^{-\frac{1}{\rho^c}}}{\omega^c_{2,t} + (1-\tilde{\omega}^c_{2,t}) \delta_{1,t}^{\frac{1}{\rho^c}}} \right)^{-1} \frac{exp(z_{1,t})}{exp(z_{2,t})} \delta_{1,t}^{1-2\frac{1+\rho^c}{\rho^c}} \tag{C.19}$$

$$= 1 - \frac{1+\tau^m_{2,t}}{1+\tau^m_{1,t}} \left(\frac{\frac{\omega^c_{1,t}}{1-\tilde{\omega}^c_{1,t}} + \delta_{1,t}^{-\frac{1}{\rho^c}}}{\frac{\omega^c_{2,t}}{1-\tilde{\omega}^c_{2,t}} + \delta_{1,t}^{\frac{1}{\rho^c}}} \right)^{-1} \frac{exp(z_{1,t})}{exp(z_{2,t})} \delta_{1,t}^{1-2\frac{1+\rho^c}{\rho^c}} \tag{C.20}$$

Recall that we define $1-\tilde{\omega}^c_{1,t} = (1-\omega^c_{1,t}) \left((1+\tau^m_{1,t}) \frac{1-\tau^x_{2,t}}{1-\tau^{ice}_{1,t}} \right)^{-\frac{1}{\rho^c}}$ which replaces the term $1-\omega^c_{1,t}$ in the extended model.

C.1.1 Linearized Model

We assume a symmetric steady state with $\omega^c_1 = \omega^c_2 = \omega^c$, $\tau^i_1 = \tau^i_2 = \tau^i$ with $i \in \{m, x, ice\}$ and $\delta_1 = 1$, $\tilde{B}_1 = 0$, the dynamics around the steady state are approximated by the equations

$$(z_{1,t} - E_t z_{1,t+1}) - (z_{2,t} - E_t z_{2,t+1}) - \left(\hat{\delta}_{1,t} - E_t \hat{\delta}_{1,t+1} \right) = \chi \tilde{B}_{1,t} + \xi^{UIP}_{1,t} \tag{C.21}$$

$$\beta \tilde{B}_{1,t} = \tilde{T}_{1,t} + \tilde{B}_{1,t-1} \tag{C.22}$$

$$\tilde{T}_{1,t} = \frac{\bar{\omega}^c}{1-\bar{\omega}^c} \left(\xi^{trade}_{1,t} - \xi^{trade}_{2,t} \right) - (z_{1,t} - z_{2,t}) + \bar{\varpi} \hat{\delta}_{1,t} \tag{C.23}$$

where we now define $\varpi = 1 + 2\frac{\omega^c}{\rho^c}\frac{1}{1-(\tilde{\omega}^c-\omega^c)}$. The trade shock reflects movements in the four underlying shocks to preferences, import tariffs, export subsidies, and transportation costs:

$$\begin{aligned}
\frac{\bar{\omega}^c}{1-\bar{\omega}^c}\xi_{1,t}^{trade} &= \frac{1}{1-(\tilde{\omega}^c-\omega^c)}\frac{\omega^c}{1-\omega^c}\xi_{1,t}^c \\
&+\left(1+\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\right)\frac{\tau^m}{1-\tau^m}\xi_{1,t}^m \\
&-\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tau^x}{1-\tau^x}\xi_{2,t}^x \\
&+\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tau^{ice}}{1-\tau^{ice}}\xi_{1,t}^{ice}.
\end{aligned} \tag{C.24}$$

The solution for the terms of trade, the trade balance, and the NFA position in the model with differentiated trade shocks is isomorphic with the solution to the model with a single trade shock. By making the appropriate substitutions in the solution coefficients above, we can obtain the solution coefficients for each of the shock processes. For example, for the import tariff shock it is

$$\gamma_1^m: \qquad \gamma_1^m = \frac{\frac{\gamma_b-\chi}{\beta}\left(1+\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\right)\frac{\tau^m}{1-\tau^m}}{-\frac{\gamma_b-\chi}{\beta}\varpi+(1-\rho_1^m)} = -\frac{\tilde{\gamma}_b\frac{1+\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}}{1+2\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}}\frac{\tau^m}{1-\tau^m}}{\tilde{\gamma}_b+(1-\rho_1^m)} < 0. \tag{C.25}$$

Thus, an increase in the import tariff causes the terms of trade (net of tariffs) to improve and similarly for export subsidies, preferences, and iceberg costs. All lemmas and theorems apply subject to the appropriate modifications.

Other variables of interest are total consumption, imports, and the consumption real exchange rate. In the richer model, these variables experience experience direct effects from the trade shocks.

$$\begin{aligned}
\hat{C}_{1,t} &= z_{1,t} - \frac{1-\tilde{\omega}^c}{1-(\tilde{\omega}^c-\omega^c)}\hat{\delta}_{1,t} + \rho^c\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tilde{\omega}^c-\omega^c}{1-\omega^c}\xi_{1,t}^c \\
&-\frac{1-\tilde{\omega}^c}{1-(\tilde{\omega}^c-\omega^c)}\left[\frac{\tau^m}{1-\tau^m}\xi_{1,t}^m - \frac{\tau^x}{1-\tau^x}\xi_{2,t}^x + \frac{\tau^{ice}}{1-\tau^{ice}}\xi_{1,t}^{ice}\right]
\end{aligned} \tag{C.26}$$

$$\begin{aligned}
\hat{C}_{2,t} &= z_{2,t} + \frac{1-\tilde{\omega}^c}{1-(\tilde{\omega}^c-\omega^c)}\hat{\delta}_{1,t} + \rho^c\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tilde{\omega}^c-\omega^c}{1-\omega^c}\xi_{2,t}^c \\
&-\frac{1-\tilde{\omega}^c}{1-(\tilde{\omega}^c-\omega^c)}\left[\frac{\tau^m}{1-\tau^m}\xi_{2,t}^m - \frac{\tau^x}{1-\tau^x}\xi_{1,t}^x + \frac{\tau^{ice}}{1-\tau^{ice}}\xi_{2,t}^{ice}\right].
\end{aligned} \tag{C.27}$$

Appendix

As before, if a shock improves the terms of trade $\delta_{1,t}$, the shock provides an indirect boos to consumption. However, in this new formulation the trade shocks that induce a terms of trade improvement also have a direct negative effect on consumption. In the context of the import tariff, the total effect of an increase in the import tariff on consumption is negative:

$$
\begin{aligned}
-\frac{1-\tilde{\omega}^c}{1-(\tilde{\omega}^c-\omega^c)}\left(\gamma_1^m+\frac{\tau_1^m}{1+\tau_1^m}\right) &= \frac{1-\tilde{\omega}^c}{1-(\tilde{\omega}^c-\omega^c)}\left(\frac{\tilde{\gamma}_b\frac{1+\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}}{1+2\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}}}{\tilde{\gamma}_b+(1-\rho_1^m)}-1\right)\frac{\tau_1^m}{1+\tau_1^m} \\
&= -\frac{1-\tilde{\omega}^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tau_1^m}{1+\tau_1^m}\frac{\frac{\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}}{1+2\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}}\tilde{\gamma}_b+(1-\rho_1^m)}{\tilde{\gamma}_b+(1-\rho_1^m)}<0. \qquad \text{(C.28)}
\end{aligned}
$$

Similarly, we can show that real imports decline in response to an increase in the import tariff. Note that real imports are give by

$$
\begin{aligned}
\hat{M}_{1,t} &= z_{1,t}-\left(1+\frac{1}{\rho^c}\frac{\omega^c}{1-\omega^c}\frac{1-\tilde{\omega}^c}{1-(\tilde{\omega}^c-\omega^c)}\right)\hat{\delta}_{1,t}-\frac{1}{1-(\tilde{\omega}^c-\omega^c)}\frac{\omega^c}{1-\omega^c}\xi_{1,t}^c \\
&\quad -\left(1+\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\right)\left[\frac{\tau^m}{1-\tau^m}\xi_{1,t}^m-\frac{\tau^x}{1-\tau^x}\xi_{2,t}^x+\frac{\tau^{ice}}{1-\tau^{ice}}\xi_{1,t}^{ice}\right] \qquad \text{(C.29)}
\end{aligned}
$$

implying an overall response of imports to the import tariff of

$$
\begin{aligned}
&-\left(1+\frac{1}{\rho^c}\frac{\omega^c}{1-\omega^c}\frac{1-\tilde{\omega}^c}{1-(\tilde{\omega}^c-\omega^c)}\right)\gamma_1^m-\left(1+\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\right)\frac{\tau^m}{1-\tau^m} \\
&= -\left(1-\frac{\frac{1+\frac{1-\tilde{\omega}^c}{1-\omega^c}\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}}{1+2\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}}\tilde{\gamma}_b}{\tilde{\gamma}_4+(1-\rho_1^m)}\right)\left(1+\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\right)\frac{\tau^m}{1-\tau^m}<0 \qquad \text{(C.30)}
\end{aligned}
$$

For the real exchange rate, the following relationship obtains

$$
\begin{aligned}
\hat{q}_{1,t} &= \rho^c\left(\hat{F}_{1,t}-\hat{F}_{2,t}\right)+\hat{\delta}_{1,t} \\
&= \frac{2\omega^c-(1-(\tilde{\omega}^c-\omega^c))}{1-(\tilde{\omega}^c-\omega^c)}\hat{\delta}_{1,t}+\rho^c\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tilde{\omega}^c-\omega^c}{1-\omega^c}\left(\xi_{1,t}^c-\xi_{2,t}^c\right) \\
&\quad -\frac{1-\tilde{\omega}^c}{1-(\tilde{\omega}^c-\omega^c)}\left[\frac{\tau^m}{1+\tau^m}\left(\xi_{1,t}^m-\xi_{2,t}^m\right)+\frac{\tau^x}{1+\tau^x}\left(\xi_{1,t}^x-\xi_{2,t}^x\right)+\frac{\tau^{ice}}{1+\tau^{ice}}\left(\xi_{1,t}^{ice}-\xi_{2,t}^{ice}\right)\right] \qquad \text{(C.31)}
\end{aligned}
$$

The terms of trade pushes the real exchange rate in the directions of its own movement as before, but the trade shocks also play a role. In the case of an increase of the import tariff, the overall magnitude of the real exchange rate movement is given by the coefficient

$$\begin{aligned}
&\frac{2\omega^c-(1-(\tilde{\omega}^c-\omega^c))}{1-(\tilde{\omega}^c-\omega^c)}\gamma_1^m-\frac{1-\tilde{\omega}^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tau^m}{1+\tau^m}\\
=\quad&-\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tilde{\gamma}_b\frac{1+\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}}{1+2\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}}\frac{\tau^m}{1-\tau^m}}{\tilde{\gamma}_b+(1-\rho_1^m)}-\frac{1-\tilde{\omega}^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tau^m}{1+\tau^m}\left(1-\frac{\tilde{\gamma}_b\frac{1+\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}}{1+2\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}}}{\tilde{\gamma}_b+(1-\rho_1^m)}\right)
\end{aligned} \tag{C.32}$$

which is negative for $\tilde{\omega}^c$ sufficiently close to ω^c. Thus, an increase in the import tariff causes a real appreciation in the home country.

In deriving the above we used:

$$\hat{F}_{1,t} = -\frac{1}{\rho^c}\frac{(1-\tilde{\omega}^c)}{1-(\tilde{\omega}^c-\omega^c)}\hat{\delta}_{1,t}+\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\xi_{1,t}^c-\frac{1-\tilde{\omega}^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tilde{\omega}^c}{1-\tilde{\omega}^c}\hat{\tilde{\omega}}_{1,t}^c \tag{C.33}$$

$$\frac{\tilde{\omega}^c}{1-\tilde{\omega}^c}\hat{\tilde{\omega}}_{1,t}^c = \frac{\omega^c}{1-\omega^c}\xi_{1,t}^c+\frac{1}{\rho^c}\left\{\frac{\tau^m}{1+\tau^m}\xi_{1,t}^m-\frac{\tau^x}{1+\tau^x}\xi_{2,t}^x+\frac{\tau^{ice}}{1+\tau^{ice}}\xi_{1,t}^{ice}\right\} \tag{C.34}$$

D Appendix: Data

To be added.