Exchange Rate Disconnect and the Trade Balance*

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Abstract

We show that shifts in demand between domestic and foreign goods are a central driver of exchange rate dynamics. In a framework with costly international financial intermediation, trade rebalancing shocks—arising from tariffs, transport costs, or preferences—account for roughly half of real exchange rate fluctuations. Exogenous deviations from uncovered interest parity explain only a smaller share. Incorporating trade flow data into the analysis allows us to link exchange rate movements more closely to macroeconomic fundamentals and to provide a unified explanation for classic exchange rate puzzles as well as the co-movement of exchange rates with the trade balance.

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1 Introduction

The real exchange rate is a central variable in international macroeconomics. The literature on exchange rate determination typically relies on mechanisms featuring a stark separation between the exchange rate and the remainder of the macroeconomy to resolve long-standing puzzles between economic theory and the data.[1] We propose a setup that does not impose such a stark separation. Using a model with costly international financial intermediation in the spirit of Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021), we link exchange rate movements to fundamental shifts in the demand for domestically produced goods relative to the demand for imported goods, which we label as *trade rebalancing*. Our model is not only consistent with classic facts of exchange rate dynamics, but also with key empirical relationships between the exchange rate and the trade balance, which are typically overlooked in the literature. In a quantitative assessment, trade rebalancing shocks explain about 50 percent of exchange rate fluctuations, whereas exogenous deviations from the uncovered interest rate parity (UIP)—the primary source of exchange rate fluctuations in most of the existing literature—account for just 15 percent.

Both trade rebalancing shocks and costly international financial intermediation are necessary to align the model with the data. The notion of trade rebalancing dates back to Mundell (1961): at given prices, a trade rebalancing shock raises the demand for domestically-produced goods relative to the demand for imported goods, causing both a trade balance improvement and a real currency appreciation. Rebalancing in our model can be caused by changes in trade costs, tariffs, trade boycotts, and sanctions, as well as shocks that alter household preferences over domestic and foreign goods. The magnitude of the responses of the trade balance and the real exchange rate depend critically on the extent of consumption risk sharing across countries. Following a rebalancing shock, higher costs of financial intermediation reduce international borrowing and lending, restrain the trade balance response, and thus require a larger real exchange rate adjustment to maintain goods market equilibrium. Under moderate financial intermediation costs, the movements of the real exchange rate relative to the trade balance in the model are

[1] The three most prominent puzzles are (1) the exchange rate disconnect/volatility puzzle (Obstfeld and Rogoff 1996; Obstfeld and Rogoff 2000), i.e., the fact that exchange rates are significantly more volatile than aggregate consumption and output; (2) the Backus-Smith puzzle (Backus and Smith 1993), i.e., the fact that the correlation between the real exchange rate and relative consumption is low and often negative; and (3) the forward-premium puzzle (Fama 1984), i.e., the fact that contrary to the uncovered interest rate parity condition, high interest rate currencies are expected to appreciate.

consistent with the qualitative and quantitative patterns in the data. The use of trade flows data is therefore key in our work to inform on the limits of international financial intermediation and to discipline the stochastic properties of the trade rebalancing shocks.

Although models that rely on exogenous departures from the UIP condition (Itskhoki and Mukhin 2021; Eichenbaum, Johannsen, and Rebelo 2021) as the main driver of the real exchange rate also address major real exchange rate puzzles, we show that this approach generally implies an excessive correlation and relative volatility between the trade balance and the real exchange rate. The introduction of costly financial intermediation in our model helps resolve the mismatch between the data and international macro models regarding the trade balance by restraining the financial net flows associated with a UIP shock.

We develop our argument in a stylized, real, two-country model with international trade in goods and assets. Each country uses labor to produce a country-specific good. These goods are traded internationally, but they are imperfect substitutes for one another. In addition, we postulate that household preferences are biased towards domestically produced goods. Households can trade in one international non-state-contingent bond that is in zero-net supply but international financial intermediation is costly. This friction limits the extent of consumption risk sharing across countries, as originally introduced in Turnovsky (1985) and micro-founded by Gabaix and Maggiori (2015).[2] The model features three types of shocks. Trade rebalancing shocks shift household preferences over the two traded goods. Technology shocks alter total factor productivity. UIP shocks lead to *exogenous* deviations from the UIP condition. In the presence of costly international financial intermediation, all three shocks induce *endogenous* departures from the UIP condition via their impact on international borrowing and lending.

Based on this framework, we make three distinct contributions. First, we derive analytical solutions that characterize key exchange rate puzzles and the role of trade rebalancing in determining the exchange rate. We begin by illustrating the differences between the UIP and trade rebalancing shocks in terms of their impact on the trade balance. A trade rebalancing shock that causes a real appreciation is associated with a trade balance surplus. By contrast, UIP shocks that cause a real appreciation are associated with a trade balance deficit. If international financial intermediation is more costly and financial markets provide less risk sharing,

[2] See also Schmitt-Grohé and Uribe (2003), Bodenstein (2011), or, more recently, Fukui, Nakamura, and Steinsson (2023) and Guo, Ottonello, and Perez (2023) for related approaches.

the real exchange rate appreciation is larger for the rebalancing shocks but smaller for the UIP shock. The trade balance response is smaller for both shocks in this case. Importantly, these features are key to bring the model into alignment with the data. Additionally, they reduce the importance of the UIP shock in accounting for real exchange rate volatility in our model. While information on trade balance dynamics enables us to distinguish between the UIP shock and trade rebalancing shocks, the exchange rate puzzles typically cited in the literature are less relevant in identifying these two shocks. We show analytically that both types of shocks satisfy the excess volatility puzzle (Obstfeld and Rogoff 2000), the consumption-real-exchange-rate puzzle (Backus and Smith 1993), and the forward premium puzzle (Fama 1984). We also show that in our model, costly financial intermediation allows us to account for the Mussa facts documented in Itskhoki and Mukhin (2025).

Second, we show that our simple model captures key features of the data, both qualitatively and quantitatively. The model requires a moderate level of intermediation costs to match selected data moments on the international business cycle, the real exchange rate, and the trade balance. For our preferred parameterization, trade rebalancing shocks account for a significantly larger share of exchange rate volatility than the UIP shock. In line with previous research, if intermediation costs are set close to zero, the UIP shock is the dominant driver of exchange rate volatility. In that case, however, the trade balance becomes far too volatile and too tightly correlated with the real exchange rate.

Third, we extend our analytical insights to a medium-scale DSGE model of the U.S. and the Rest of the World that we estimate using Bayesian methods. In this setting, we find that trade rebalancing shocks account for approximately 50 percent of the variance in the real exchange rate. By contrast, the UIP shock explains just 15 percent. Our quantitative model confirms that our costly financial intermediation mechanism and trade data—required to discipline the limits to international risk sharing—are crucial to obtain these results.

Our work relates to three strands in the literature. First, we allow financial-type shocks to contribute to exchange rate fluctuations, in line with a large body of work building on Kollmann (2002) and Gabaix and Maggiori (2015). Relative to these papers, we demonstrate that data on trade flows are informative about the importance of different shocks in explaining the exchange rate and, through implications for international borrowing and lending, for UIP deviations. This sets us apart from, for example, Itskhoki and Mukhin (2021), Itskhoki and Mukhin (2025), and

Eichenbaum, Johannsen, and Rebelo (2021), who assign a dominant role to exogenous deviations from UIP as the driver of the real exchange rate. These papers either omit trade rebalancing shocks directly or study the autarky limit, which eliminates the role of goods trade. However, as our analysis shows, trade rebalancing shocks can be consistent with exchange rate volatility patterns. Alternatively, Kekre and Lenel (2024) show in a calibrated endowment economy that country-specific discount factor shocks—affecting a country's overall demand for goods as opposed to the relative demand for domestic versus imported goods—can lead to sizable exchange rate movements without deviating from UIP. However, when we embed their shock into our analysis we find that it generates a counterfactually high positive correlation between the real exchange rate and the trade balance. This finding underscores again the importance of trade related moments in distinguishing between different drivers of the real exchange rate. Hence our result regarding the importance of trade rebalancing shocks as a primary source of exchange rate fluctuations remains unchallenged.

Second, we build on insights from quantitative models using trade data to inform on the extent of frictions affecting international risk sharing or drivers of the real exchange rate. We show both analytically and through a quantitative model that trade data and its correlation with other series, such as the real exchange rate, also inform on financial frictions. Our paper is, therefore, close to Fitzgerald (2012), who uses bilateral trade data in a dynamic gravity model based on Anderson and van Wincoop (2003) to learn about the degree of trade and asset market frictions between countries.[3] In the context of lower frequency movements of the exchange rate, Alessandria and Choi (2021) find a sizable role for changes in trade costs for the path of the U.S. trade balance and real exchange rate after 1980. Gornemann, Guerrón-Quintana, and Saffie (2025) and MacMullen and Woo (2025) point to the importance of trade data and trade frictions in capturing the dynamics of the real exchange rate in quantitative international DSGE models.[4] All these papers take as given a low degree of friction in global financial markets. Our contribution is to show that trade rebalancing shocks and trade data help identify the degree of asset market frictions, allowing us to draw a sharper inference about their role. Our strategy also enables us to address the various puzzles presented in the international literature directly.

[3] While Fitzgerald (2012) derives a set of test statistics to probe for these frictions between countries using trade data, we utilize the co-movement of many macro-variables to draw inference about the risk-sharing frictions and shocks, more in line with work using quantitative DSGE models.

[4] Trade data also plays a crucial role in disciplining the dynamics of the trade balance and the transmission of shocks in estimated RBC models of small open economies (García-Cicco, Pancrazi, and Uribe 2010).

Third, we contribute to the literature studying the drivers of the exchange rate more broadly. In this sense, we complement Itskhoki and Mukhin (2021) in putting less emphasis on technology and technology-news based stories. This sets our analysis apart from, for example, Corsetti, Dedola, and Leduc (2008), Bodenstein (2008), Colacito and Croce (2011), Heathcote and Perri (2014), and Chahrour, Cormun, De Leo, Guerron-Quintana, and Valchev (2021). We focus on the various drivers of the real exchange rate in concert with UIP shocks in line with theoretical and empirical work like Schmitt-Grohé and Uribe (2022), Fukui, Nakamura, and Steinsson (2023), Kekre and Lenel (2024), and Miyamoto, Nguyen, and Oh (2025).

The rest of the paper is organized as follows. Section 2 presents the analytical model and derives the main theoretical results. Section 3 revisits the exchange rate disconnect in the data and our simple model. Section 4 explores the role of trade and financial integration to account for the exchange rate disconnect and related puzzles observed in the data. The section also discusses several model extensions and the robustness of our results. Section 5 estimates a medium-scale model featuring real and nominal rigidities to quantify the main drivers of the real exchange rate in the data. We offer conclusions in Section 6.

2 Analytical Model

We develop our argument in a simple model before turning to an estimated medium-scale DSGE model that encompasses the features of the analytical model in Section 5. A continuum of agents, each with mass one, lives in each of two equally sized countries. Each country produces one good using labor as the only input into production. Both prices and wages are fully flexible. Home and foreign goods are imperfect substitutes and traded across borders. International financial markets are incomplete, as captured by restricting financial flows to a single non-state-contingent bond. In addition, we assume that there are limits to the amount of debt intermediated internationally. This feature gives rise to *endogenous* departures from the uncovered interest rate parity (UIP) condition similar to the financial intermediation model of Gabaix and Maggiori (2015) or the convenience yield model of Valchev (2020).

The model features three sources of uncertainty: technology shocks that alter total factor productivity, rebalancing shocks that alter the relative demand for the domestically-produced and the imported goods, and a financial shock that gives rise to *exogenous* departures from the

UIP condition as in Itskhoki and Mukhin (2025).[5]

2.1 Assumptions

The intertemporal preferences of the representative household in country 1, the home country, are

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln\left(C_{1,t+j}\right) - L_{1,t+j} \right\}. \tag{1}$$

The felicity function in period $t+j$ depends on consumption, $C_{1,t+j}$, as well as hours worked, $L_{1,t+j}$. As discussed in Section 4.6, the assumption that preferences are quasi-linear is not essential for our key findings, but it simplifies the exposition. The household chooses consumption, labor supply, and asset holdings to maximize intertemporal utility given the sequence of budget constraints

$$P^c_{1,t+j} C_{1,t+j} + \frac{P^b_{1,t+j} B_{1,t+j}}{\phi^b_{1,t+j}} = W_{1,t+j} L_{1,t+j} + B_{1,t-1+j} + U_{1,t+j}. \tag{2}$$

The difference between consumption expenditures, $P^c_{1,t+j} C_{1,t+j}$, on the left side of the budget constraint and income from wages, $W_{1,t+j} L_{1,t+j}$, and transfers, $U_{1,t+j}$, on the right side is accounted for by trade in and holdings of financial assets, $B_{1,t-1+j}$. If $B_{1,t-1+j} > 0$, then country 1 is a net lender in period $t+j$. These assets are acquired at the cost $P^b_{1,t+j}$.

As in Turnovsky (1985), households face a cost of financial intermediation (measured in terms of country 1's good) reflected by the term $\phi^b_{1,t+j}$. This feature of the model limits the extent of international financial intermediation and consumption risk sharing across countries. Section 4.5 presents external evidence in favor of the endogenous UIP wedge that emerges from this mechanism. Gabaix and Maggiori (2015) offer a micro-foundation for this approach in a model with international financiers who are constrained in their ability to bear risks from international imbalances. Alternative approaches to costly financial intermediation that also yield endogenous UIP deviations include convex portfolio costs and borrowing constraints (Appendix C.1), preferences as in Uzawa (1968) (Appendix C.5), or overlapping generations models (Bodenstein 2011).

[5] Itskhoki and Mukhin (2025) discuss several micro-foundations for exogenous UIP deviation shocks, including shocks to the utility from holding specific assets, noise traders, and time-varying risk premia.

In the following, we assume intermediation costs to follow $\phi^b_{1,t} = \exp\left(-\frac{\chi}{2}\frac{B^*_{1,t}}{P^d_{1,t}M^*_{2,t}}\right)$, where $B^*_{1,t}$ denotes the aggregate amount of bonds issued (not the individual household's holdings) normalized by the aggregate value of exports, $P^d_{1,t}M^*_{2,t}$. In equilibrium, the aggregate net-foreign asset position (NFA) is equal to the debt position of the representative household, $B^*_{1,t} = B_{1,t}$. In other words, atomistic households do not internalize the effects of their asset choices on the intermediation cost. The parameter χ governs the intermediation cost: a given value of the NFA position is associated with higher overall bond prices from the perspective of the country if χ is larger, which in turn reduces households' inclination to borrow. This feature introduces endogenous departures from the UIP condition, resulting in excess returns on foreign assets. If χ approaches infinity, international risk sharing is shut down, and countries exist in financial autarky. Setting $\chi = 0$ shuts down *endogenous* deviations from the UIP condition.[6] We assume a similar intermediation function for the foreign country with $\phi^b_{2,t} = \exp\left(-\frac{\chi}{2}\frac{\frac{1}{e_{1,t}}B^*_{2,t}}{P^d_{2,t}M^*_{1,t}} + \xi^{UIP}_{1,t}\right)$.[7] Notice that in the foreign country's intermediation function we assume the presence of the stochastic component $\xi^{UIP}_{1,t}$ which introduces *exogenous* departures from the UIP condition as discussed in Itskhoki and Mukhin (2021).[8]

The final consumption good, $C_{1,t}$, is an aggregate of the home good, $C^d_{1,t}$, and imports of the foreign good, $M_{1,t}$,

$$C_{1,t} = \left(\left(\omega^c_{1,t}\right)^{\frac{\rho^c}{1+\rho^c}}\left(C^d_{1,t}\right)^{\frac{1}{1+\rho^c}} + \left(1-\omega^c_{1,t}\right)^{\frac{\rho^c}{1+\rho^c}}\left(M_{1,t}\right)^{\frac{1}{1+\rho^c}}\right)^{1+\rho^c}. \tag{3}$$

The elasticity of substitution between the domestic and foreign goods is measured by $\theta \equiv \frac{1+\rho^c}{\rho^c}$. The parameter $\omega^c_{1,t} \equiv \omega^c_1 \exp\left(\xi^{trade}_{1,t}\right)$ is time-varying to allow for shifts in the relative demand for the home and foreign goods, that are not induced by a change in the relative price. Accordingly, we label shocks to $\omega^c_{1,t}$ trade rebalancing shocks.[9] As discussed in Section 4.6, the rebalancing

[6] The parameter χ not only controls the amount of borrowing and lending in response to shocks but also the speed with which the NFA position unwinds over time. If $\chi = 0$, the NFA position follows a unit-root process to a first-order approximation. See Bodenstein (2011) for an extensive discussion.

[7] Aggregate bond holdings abroad, $B^*_{2,t}$ are normalized by foreign exports, $P^d_{2,t}M^*_{1,t}$.

[8] In the linearized model, there is no difference whether the UIP shock enters only in one of the functions. We retain the subscript of the home country because we refer to a positive UIP shock as an increase in the demand for country 1's bonds.

[9] Trade rebalancing shocks have a long tradition in international macroeconomic models dating back to the works of Mundell (1961) and Dornbusch, Fischer, and Samuelson (1977). More recently, Blanchard, Giavazzi, and Sa (2005) studied relative demand shocks in a simple model of the U.S. exchange rate and the trade balance. Devereux (2004) invokes the presence of such shocks to understand the role of the exchange rate as a shock absorber. Coeurdacier, Kollmann, and Martin (2007) introduced relative demand shocks for goods in a two-

shock can be obtained as a shock to import tariffs, export subsidies, transportation costs, or a combination thereof. The rebalancing shock also captures boycotts of foreign goods. Finally, we focus strictly on the case $\omega_1^c < 1$, as in the autarky limit, trade rebalancing shocks are not well defined.

We denote the price of the home good by $P_{1,t}^d$ and the price of the imported foreign good by $P_{1,t}^m$. The price of the imported foreign good satisfies $P_{1,t}^m = e_{1,t} P_{2,t}^d$, where $e_{1,t}$ is the nominal exchange rate and $P_{2,t}^d$ is the price of the foreign good in the foreign country. The terms of trade, $\delta_{1,t}$, are the ratio of import prices of the two countries expressed in common currency

$$\delta_{1,t} \equiv \frac{e_{1,t} P_{2,t}^d}{P_{1,t}^d}. \tag{4}$$

Relatedly, the real consumption exchange rate is defined as

$$q_{1,t} \equiv \frac{e_{1,t} P_{2,t}^c}{P_{1,t}^c}. \tag{5}$$

We assume that prices and wages are flexible. Production of each country's good is linear in the country's labor, which is priced at the wage $W_{1,t}$. Total output, $Y_{1,t}$, in country 1 is

$$Y_{1,t} = exp(z_{1,t}) L_{1,t}. \tag{6}$$

The assumptions for preferences and production in country 2 mirror those of country 1 in Equations (1)-(3) and (6). The rebalancing shock in country 2 follows $\omega_{2,t}^c = \omega_2^c \exp\left(\xi_{2,t}^{trade}\right)$ and, as already noted, the financial intermediation costs in the foreign country are $\phi_{2,t}^b = \exp\left(-\frac{\chi}{2} \frac{\frac{1}{e_{1,t}} B_{2,t}^*}{P_{2,t}^d M_{1,t}} + \xi_{1,t}^{UIP}\right)$ with the exogenous UIP shock $\xi_{1,t}^{UIP}$.

Market clearance in goods and financial markets requires

$$Y_{1,t} = C_{1,t}^d + M_{2,t} \tag{7}$$

$$Y_{2,t} = C_{2,t}^d + M_{1,t} \tag{8}$$

$$0 = B_{1,t} + B_{2,t}. \tag{9}$$

country open economy model to generate home bias in equity holdings and valuation effects consistent with the data. Pavlova and Rigobon (2008) use domestic demand shifters to examine the comovement between exchange rates and stock prices.

We define the trade balance normalized by the value of exports, $P^d_{1,t}M_{2,t} = e_t P^m_{2,t} M_{2,t}$, as

$$\tilde{T}_{1,t} \equiv \frac{T_{1,t}}{e_t P^m_{2,t} M_{2,t}} = \frac{e_t P^m_{2,t} M_{2,t} - P^m_{1,t} M_{1,t}}{e_t P^m_{2,t} M_{2,t}}. \tag{10}$$

We choose the price of the domestic good to be the numeraire.

The exogenous technology, trade rebalancing, and UIP shocks follow autoregressive processes of order 1 with

$$\xi^{trade}_{1,t} = \rho^{trade}_1 \xi^{trade}_{1,t-1} + \sigma^{trade}_1 \epsilon^{trade}_{1,t} \tag{11}$$

$$\xi^{trade}_{2,t} = \rho^{trade}_2 \xi^{trade}_{2,t-1} + \sigma^{trade}_2 \epsilon^{trade}_{2,t} \tag{12}$$

$$\xi^{UIP}_{1,t} = \rho^{UIP}_1 \xi^{UIP}_{1,t-1} + \sigma^{UIP}_1 \epsilon^{UIP}_{1,t} \tag{13}$$

$$z_{1,t} = \rho^z_1 z_{1,t-1} + \sigma^z_1 \epsilon^z_{1,t} \tag{14}$$

$$z_{2,t} = \rho^z_2 z_{2,t-1} + \sigma^z_2 \epsilon^z_{2,t}. \tag{15}$$

2.2 Model Solution

We solve a linear approximation of the model around a symmetric deterministic steady state with $\omega^c_1 = \omega^c_2$ and balanced trade, i.e., $\tilde{T}_1 = 0$, $\delta_1 = 1$, $B_1 = 0$. As shown in Appendix A, we can simplify the model to the following system of three equations:

$$(z_{1,t} - E_t z_{1,t+1}) - (z_{2,t} - E_t z_{2,t+1}) - \left(\hat{\delta}_{1,t} - E_t \hat{\delta}_{1,t+1}\right) = \chi \tilde{B}_{1,t} + \xi^{UIP}_{1,t} \tag{16}$$

$$\beta \tilde{B}_{1,t} = \tilde{T}_{1,t} + \tilde{B}_{1,t-1} \tag{17}$$

$$\tilde{T}_{1,t} = \frac{\omega^c_1}{1-\omega^c_1}\xi^{trade}_{1,t} - \frac{\omega^c_1}{1-\omega^c_1}\xi^{trade}_{2,t} - z_{1,t} + z_{2,t} + \varpi \hat{\delta}_{1,t}, \tag{18}$$

where $\varpi = 1 + 2\frac{\omega^c_1}{\rho^c}$. Throughout the analysis, we assume that the trade elasticity, $\theta \equiv \frac{1+\rho^c}{\rho^c}$, is large enough to ensure that $\varpi > 0$.[10] The terms of trade, $\hat{\delta}_{1,t}$, are measured in log deviation from the steady state. The normalized trade balance, $\tilde{T}_{1,t}$, and the normalized NFA position, $\tilde{B}_{1,t} = \frac{B_{1,t}}{e_t P^m_{2,t} M_{2,t}}$, are measured in absolute deviation from their steady-state values of zero.

Equation (16) is the linearized risk-sharing or UIP condition under incomplete markets. The terms $\chi \tilde{B}_{1,t}$ and $\xi^{UIP}_{1,t}$ introduce time-varying wedges in the equation and play a key role in our analysis. Equation (17) governs the evolution of the NFA position. Finally, Equation (18) is the

[10] Under our assumptions, it is $\varpi > 0$ for values of the trade elasticity larger than 0.5. Most works on international business cycles assume values between 1 and 2.5.

linearized definition of the trade balance. For completeness, notice that the (consumption) real exchange rate is proportional to the terms of trade, $\hat{q}_{1,t} = (2\omega_1^c - 1)\,\hat{\delta}_{1,t}$. An improvement in the terms of trade of country 1, $\hat{\delta}_{1,t} < 0$, goes along with an appreciation of the real exchange rate, $\hat{q}_{1,t} < 0$. We solve for the decision rules of the endogenous variables in Appendix A.

2.3 Importance of the Trade Balance

In the remainder of this section, we first demonstrate how the trade balance interacts with the exchange rate. Theorem 1 establishes that the use of trade balance data allows us to distinguish empirically between trade rebalancing and UIP shocks because the two shocks move the trade balance in opposite directions whenever they move the real exchange rate (or the terms of trade) in the same direction. Theorem 2 and Theorem 3 document that the major exchange rate puzzles in the literature do not contain sufficient information to distinguish empirically between rebalancing and UIP shocks. Both types of shocks have identical predictions about the exchange rate disconnect puzzle (Obstfeld and Rogoff 2000), the consumption-real-exchange-correlation puzzle (Backus and Smith 1993), and the forward premium puzzle (Fama 1984; Engel 2016).

2.3.1 Trade Balance and Shock Identification

Theorem 1 *A trade rebalancing shock that improves the home country's terms of trade (appreciates the real exchange rate), $\xi_{1,t}^{trade} > 0$ and/or $\xi_{2,t}^{trade} < 0$, is associated with an improvement of the trade balance. By contrast, a UIP shock that improves the terms of trade (appreciates the real exchange rate), $\xi_{1,t}^{UIP} > 0$, is associated with a deterioration of the trade balance. If financial markets provide less risk sharing due to higher intermediation costs, i.e., χ assumes a higher value, the terms of trade are more (less) sensitive to the trade rebalancing (UIP) shock, and the trade balance is less sensitive to both shocks.*

The proof of Theorem 1 is provided in Appendix B.1 and involves solving for the linear decision rules associated with the system in Equations (16)-(18). The signs of the coefficients and their derivatives with respect to χ inform on the relationship between the shocks and the endogenous variables.[11]

[11] Note that the effects of the trade rebalancing shock for country 2 always have the opposite sign of the effects of the trade rebalancing shock for country 1.

Intuitively, according to the risk sharing condition, Equation (16), a positive UIP shock, $\xi_{1,t}^{UIP} > 0$, induces an expected worsening of the terms of trade (a depreciation of the real exchange rate) implying an initial improvement of the terms of trade (an appreciation of the real exchange rate). Yet, this initial improvement of the terms of trade causes the trade balance to deteriorate, Equation (18), and reduces the home country's NFA position, Equation (17). In turn, the less favorable NFA position of the home country dampens the initial impact on the terms of trade in Equation (16). The larger χ, the stronger this dampening effect, and the smaller the terms of trade response for a given magnitude of the shock $\xi_{1,t}^{UIP}$.

Instead, the trade rebalancing shock primarily affects the trade balance, as shown in Equation (18). A shock that increases demand for country 1's goods, i.e., $\xi_{1,t}^{trade} > 0$ or $\xi_{2,t}^{trade} < 0$, causes the trade balance and the NFA position of country 1 to improve. From Equation (16), the improvement in the NFA position requires an initial improvement of the terms of trade (and an expected future worsening), dampening the initial reaction of the trade balance. This dampening effect is stronger the larger the value of χ.

Appendix B.1 also shows that this intuition is consistent with the unconditional covariance between the growth rate of the terms of trade and the growth rate of the trade balance.

Corollary 1 *The UIP shock induces a positive covariance between the growth rate of the terms of trade and the growth rate of the trade balance. The trade rebalancing shock induces a negative covariance between the two growth rates. When both shocks are present in the model, the overall covariance is determined by the costliness of financial intermediation as measured by χ.*

2.3.2 Exchange Rate Puzzles and Lack of Identification

In the data, the real exchange rate exhibits large swings that are not accompanied by comparable swings in other macroeconomic variables (the real exchange rate disconnect). In addition, the correlation between the real exchange rate and relative consumption is low and often negative (the Backus-Smith puzzle). Standard models of the international business cycle struggle to replicate exchange rate moments when they rely on shocks to technology and monetary policy. Under standard parameterization, a model with technology shocks only implies that the volatility of the real exchange rate is of similar magnitude as that of consumption and the correlation between relative consumption and the real exchange rate is very close to 1.[12] By contrast, the trade rebalancing and the UIP shocks bring the model better in line with the data.

Theorem 2 *Abstracting from technology shocks, the ratio of the standard deviation of the real exchange rate, $\hat{q}_{1,t}$, and consumption, $\hat{C}_{1,t}$, is independent of the relative variances of the trade rebalancing and the UIP shocks,*

$$\frac{std\left(\hat{q}_{1,t}\right)}{std\left(\hat{C}_{1,t}\right)} = \frac{std\left(\Delta\hat{q}_{1,t}\right)}{std\left(\Delta\hat{C}_{1,t}\right)} = \frac{2\omega_1^c - 1}{1-\omega_1^c}. \tag{19}$$

The correlation between relative consumption, $\hat{C}_{1,t} - \hat{C}_{2,t}$, and the real exchange rate is equal to minus one regardless of the relative variances of the trade rebalancing and the UIP shocks,

$$corr\left(\hat{C}_{1,t} - \hat{C}_{2,t}, \hat{q}_{1,t}\right) = -1. \tag{20}$$

Notice that for sufficient home bias, i.e., ω_1^c close to 1, the relative volatility of the real exchange rate can exceed the volatility of consumption multiple times. The proof in Appendix B.2 uses the first-order approximations to aggregate consumption and the real exchange rate (both in log-deviations)

$$\hat{C}_{1,t} = z_{1,t} - \left(1-\omega_1^c\right)\hat{\delta}_{1,t} \tag{21}$$

$$\hat{C}_{2,t} = z_{2,t} + \left(1-\omega_1^c\right)\hat{\delta}_{1,t} \tag{22}$$

$$\hat{q}_{1,t} = \left(2\omega_1^c - 1\right)\hat{\delta}_{1,t} \tag{23}$$

which relate consumption and the real exchange rate to the terms of trade and the technology shocks, $z_{1,t}$ and $z_{2,t}$. Neither the UIP nor the trade rebalancing shocks enter directly into Equations (21)-(23) that determine the real exchange rate and consumption. Rather, both shocks enter only indirectly through the terms of trade. Hence, the computed moments do not depend on the relative variances of the trade rebalancing and the UIP shocks.

Turning to the forward premium puzzle, Fama (1984) finds that the hypothesis of uncovered interest rate parity is violated in the data. While this theory predicts that the coefficient in the regression of the exchange rate on the interest rate differential equals one, empirical work

[12] Exceptions are Benigno and Thoenissen (2008) and Corsetti, Dedola, and Leduc (2008). The latter work shows that, if the wealth effects from technology shocks dominate the substitution effects, both puzzles can be explained.

estimates a negative value for this coefficient. Put differently, in the data, the high-interest-rate currency is expected to appreciate. Engel (2016) shows that the forward premium puzzle documented in Fama (1984) also holds in real terms.

To express Equation (16) in its conventional form, we make use of the real interest rate differential, $r_{1,t} - r_{2,t}$, to obtain

$$E_t\left(\hat{q}_{1,t+1} - \hat{q}_{1,t}\right) = r_{1,t} - r_{2,t} + \chi \tilde{B}_{1,t} + \xi_{1,t}^{UIP}. \tag{24}$$

The expected one-period-ahead excess return of the foreign bond over the home bond, $\iota_t = r_{2,t} - r_{1,t} + E_t\left(\hat{q}_{1,t+1} - \hat{q}_{1,t}\right)$, is equal to $\chi \tilde{B}_{1,t} + \xi_{1,t}^{UIP}$. The UIP condition holds if $\iota_t = 0$. The UIP shock introduces exogenous departures from UIP. If households are limited in their ability to share consumption risk, i.e., $\chi > 0$, any shock causes endogenous UIP departures through its impact on the NFA position, $\tilde{B}_{1,t}$. Whether the direction of the endogenous departures conforms with the data evidence is not guaranteed.

Theorem 3 *Suppose the model admits only rebalancing and UIP shocks. In that case, the Fama coefficient is constant and negative, independent of the degree of costly financial intermediation as measured by χ, as long as $\chi > 0$:*

$$\hat{\beta}^{Fama} = \frac{cov\left(E_t \Delta \hat{q}_{1,t+1}, r_{1,t} - r_{2,t}\right)}{var\left(r_{1,t} - r_{2,t}\right)} = -\frac{2\omega_1^c - 1}{2\left(1 - \omega_1^c\right)} = 1 - \frac{1}{2\left(1 - \omega_1^c\right)} < 0 \tag{25}$$

for $\omega_1^c > \frac{1}{2}$.

See Appendix B.2 for the proof. The presence of the wedge $\chi \tilde{B}_{1,t} + \xi_{1,t}^{UIP}$ allows for the expected depreciation of the home country's currency to be associated with a negative interest rate differential, i.e., $cov\left(E_t \Delta \hat{q}_{1,t+1}, r_{1,t} - r_{2,t}\right) < 0$. In alternative formulations of this puzzle, researchers regresses the excess return, ι_t, on the interest rate differential. In this case, the regression coefficient is $\hat{\beta}^{Fama,\iota} = \frac{cov(\iota_t, r_{1,t} - r_{2,t})}{var(r_{1,t} - r_{2,t})} = \hat{\beta}^{Fama} - 1 = -\frac{1}{2\left(1-\omega_1^c\right)} < 0$. For comparability, we use this formulation in the quantification of the model properties.

2.4 Discussion

Both UIP and trade rebalancing shocks can help to align the observations from international macro models with those in the data with regards to exchange rate dynamics. However, as shown by Theorems 2 and 3, celebrated exchange rate puzzles do not provide relevant information to distinguish between these driving forces of exchange rates in macroeconomic models.

By contrast, data on the interplay between the trade balance and the real exchange rate can deliver such distinction. Theorem 1 shows that the trade balance response helps distinguish UIP shocks from other potential drivers of exchange rate movements, in our case trade rebalancing shocks. Moreover, the transmission of both UIP and trade rebalancing shocks depends on the degree of risk-bearing capacity captured by the parameter χ, whose role is typically underappreciated in the literature. These findings do not imply that conventional shocks are not important in explaining the various features in the data. Still, without rebalancing and UIP shocks, it is nearly impossible to address all the major exchange rate puzzles.

3 Quantitative Assessment in the Analytical Model

To complement the findings of the previous section, we document how well our simple model captures the key qualitative features of exchange rate moments from a quantitative perspective.

3.1 Data and Calibration

We use data covering the period 1985Q1-2019:Q2 on the real exchange rate (q), realized real interest rates (r), real GDP (Y), and consumption (C), as well as the trade-balance-to-exports ratio ($\tilde{T}$) to compute key empirical moments against which we assess our theory. The U.S. is treated as the home country. Data for the foreign country (country 2) are obtained as the trade-weighted average of the respective time series of 34 countries. The countries included represent 85 percent of 2019 world GDP on a PPP basis. Appendix F provides details.

Table 2 lists the empirical moments of interest: the standard deviation of the exchange rate relative to that of output, $\sigma(\Delta\hat{q})/\sigma(\Delta\hat{Y})$, the persistence of the real exchange rate, $\rho(\hat{q})$, the correlation of output across countries, $\rho(\Delta\hat{Y}_1, \Delta\hat{Y}_2)$, the correlation between output and consumption, $\rho(\Delta\hat{Y}, \Delta\hat{C})$, the Backus-Smith correlation between the real exchange rate and relative consumption, $\rho(\Delta\hat{q}, \Delta\hat{C}_1 - \Delta\hat{C}_2)$, and the Fama correlation for real rates and the real exchange rate, $\hat{\beta}^{Fama}$[13], the correlation between the trade balance and the real exchange rate,

[13] To be precise, we approximate the expected change in the real exchange rate, $E_t\Delta\hat{q}_{1,t+1}$, with the realized change, both in the data and for model moments throughout the paper, an assumption commonly made in the

Table 1: Parameters - Analytical Model

Structural Parameters			
Discount Factor	Trade Elasticity	Home Bias	Extent of Risk Sharing
β	θ	ω^c	100*χ
0.995	1.5	0.9	1.92
Shocks: Persistence			
ρ_z	$\rho_{\xi^{trade}}$	$\rho_{\xi^{UIP}}$	$\rho(z_1, z_2)$
0.96	0.96	0.96	0.75
Shocks: Standard Deviation			
$100\sigma_z$	$100\sigma_{\xi^{trade}}$	$100\sigma_{\xi^{UIP}}$	
1.00	0.95	0.58	

Notes: Parameters chosen to replicate key exchange rate moments reported in Table 2. Unless otherwise specified, we set the same parameters for the home and foreign economies.

$\rho(\Delta\tilde{T}, \Delta\hat{q})$, and the standard deviation of the trade balance relative to that of the real exchange rate, $\sigma(\Delta\tilde{T})/\sigma(\Delta\hat{q})$—where "$\Delta$" denotes the change in a variable rather than its level.

Before comparing the data moments to their model counterparts, we detail our parameter choices in Table 1. Some parameters are set in line with prevailing estimates in the literature, while others are chosen so the model matches the empirical features in Table 2. We set the home and foreign parameters at equal values whenever appropriate. The elasticity of substitution between the domestic and foreign good, $\theta \equiv \frac{1+\rho^c}{\rho^c}$, is set at 1.5 which is consistent with long-run estimates for the U.S. at the macro level—see Alessandria and Choi (2021), MacMullen and Woo (2025), and Boehm, Levchenko, and Pandalai-Nayar (2023). The discount factor, β, is set at 0.995 to be consistent with a 2-percent real interest rate. The value of 0.9 for the "home-bias" parameter ω^c matches the cross-country average of domestic sourcing shares of final consumption goods in the World Input-Output Tables.[14]

To pin down the remaining five parameters in our exercise—χ, the relative standard deviations of the shocks, their common persistence, and the cross-coutry correlation of technology shocks—we target six moments using an equally weighted quadratic loss function. We target

literature invoking rational expectations.

[14] We measure domestic sourcing shares from 2000-2014 for 44 countries using the World Input-Output Database. In the U.S., a direct reading of the BEA Input-Output tables shows that the domestic sourcing share is slightly higher, averaging 0.94 over the same period. However, since our calibration is symmetric between the U.S. and the foreign block, we prefer the cross-country average estimate, including the U.S. The degree of home bias is tightly linked to the tradability of goods and services. The services sector is almost entirely non-tradable, with a domestic sourcing share of 0.98. By contrast, the manufacturing and agricultural sectors in the economy are substantially more tradable, with a combined sourcing share of about 0.8. For the U.S., the high home bias in the data reflects the large share of the service sector in the U.S. economy.

an auto-correlation of 0.99 for the home country's NFA position, thereby roughly matching the observed auto-correlation of the NFA position of the U.S. with respect to the rest of the world. We also target the relative standard deviations of real exchange rate growth to GDP growth and of the real exchange rate growth and the growth rate of the trade balance. In addition, we include the correlations of GDP growth between the countries and between the growth rates of the real exchange rate and the trade balance. Finally, we match the persistence of the real exchange rate in the data. Our moment matching exercise settles on $\chi = 0.019$ for the intermediation cost parameter. The relative volatilities of the shocks are $\frac{\sigma_{\xi^{UIP}}}{\sigma_z} = 0.58$ and $\frac{\sigma_{\xi^{trade}}}{\sigma_z} = 0.95$. Meanwhile, we obtain a cross-country correlation for the TFP shock of $\rho_{\epsilon_z, \epsilon_z^*} = 0.75$. Finally, the auto-regressive persistence parameters of all shock processes are set at 0.96. While we targeted six moments with five parameters the match is almost perfect as we will see next.

3.2 Data versus Model

Table 2 compares the selected unconditional model moments to the data. We distinguish four groups of moments. First, under the header "Disconnect and PPP," we summarize the excess volatility of the real exchange rate and its near-unit-root behavior. Second, we focus on international business cycle moments under the header "IRBC." The third set corresponds to the "Backus-Smith and Forward Premium" puzzles. In broad terms, the vast majority of the literature studying exchange rate dynamics focuses on these three groups of moments (Corsetti, Dedola, and Leduc 2008; Itskhoki and Mukhin 2021; Itskhoki and Mukhin 2025). We add a fourth set of statistics that captures the relationship between the real exchange rate and the trade balance. These moments have received hardly any attention in the context of the exchange rate disconnect analysis.[15] However, as our theoretical results show, these moments provide key identification restrictions to distinguish between UIP and trade rebalancing shocks.

Although we did not target all the moments in Table 2 as part of our calibration strategy, the model performs remarkably well compared to the data. Our model qualitatively captures the failure of the perfect risk-sharing benchmark, but the Backus-Smith correlation is substantially weaker than in the data which points to the need to introduce additional model features and shocks as discussed in Section 5.

[15] One exception is the paper of MacMullen and Woo (2025) which explores the exchange rate disconnect puzzle using a dynamic trade model calibrated to U.S. trade data.

Table 2: Empirical Moments

	Disconnect and PPP		IRBC	
	a. $\sigma(\Delta\hat{q})/\sigma(\Delta\hat{Y})$	**b.** $\rho(\hat{q})$	**c.** $\rho(\Delta\hat{Y}_1, \Delta\hat{Y}_2)$	**d.** $\rho(\Delta\hat{Y}, \Delta\hat{C})$
Data	4.14	0.96	0.40	0.67
Model	4.14	0.96	0.39	0.77
	Backus-Smith and Forward Premium		RER and Trade Balance	
	e. $\rho(\Delta\hat{q}, \Delta\hat{C}_1 - \Delta\hat{C}_2)$	**f.** Real Fama $\hat{\beta}$	**g.** $\rho(\Delta\tilde{T}, \Delta\hat{q})$	**h.** $\sigma(\Delta\tilde{T})/\sigma(\Delta\hat{q})$
Data	-0.02	-1.03	0.01	1.09
Model	-0.84	-4.35	0.01	1.09

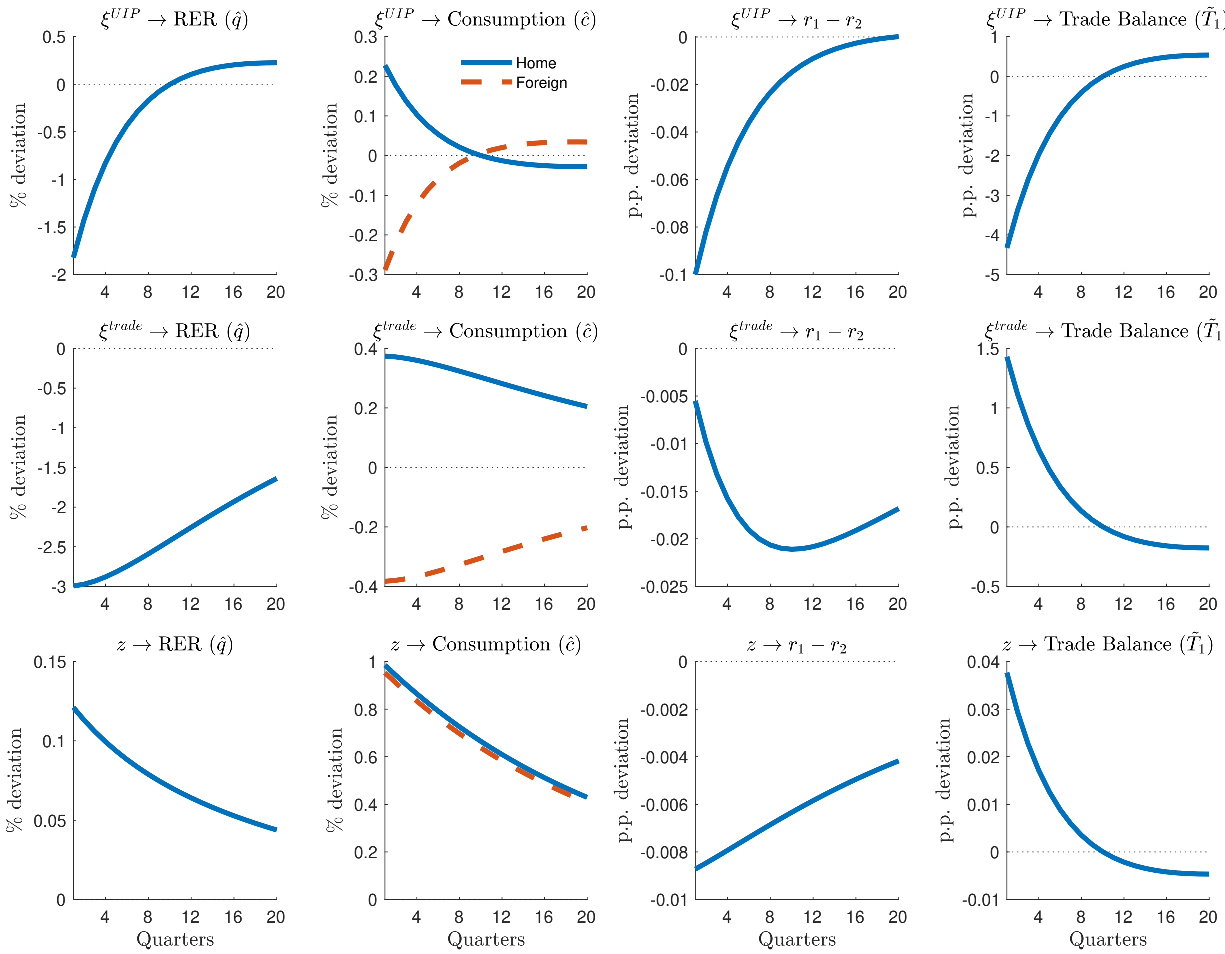
Notes: Empirical moments computed using quarterly data from 1985Q1 to 2019Q4.

3.3 Exchange Rate Puzzles

To understand the exchange rate dynamics associated with each group of shocks, Figure 1 plots the impulse responses to a UIP shock, ξ_1^{UIP}, a trade rebalancing shock towards the home good, ξ_1^{trade}, and a home country technology shock, z_1, under our preferred calibration.

The impulse responses confirm that the UIP shock helps address the major exchange rate puzzles. The UIP shock increases the demand of foreign households for the home country's bonds. The resulting drop in the home country's NFA position is reflected in the deterioration of the home country's trade balance. With aggregate foreign consumption, $\hat{C}_2$, being postponed into the future, foreign demand for the foreign good relative to the home good falls at given prices due to home bias in consumption. To clear the goods markets, the terms of trade (measured from the perspective of the home country) improve, and the real exchange rate appreciates on impact, so aggregate consumption in the home country, $\hat{C}_1$, can expand via increased demand for the foreign good relative to the home good. As a result, the real exchange rate and relative consumption, $\hat{C}_1 - \hat{C}_2$, are negatively correlated (Backus-Smith puzzle). Importantly, although the UIP shock alters the relative demand for the two traded goods in the model, the aggregate consumption response is much smaller than the real exchange rate response (the exchange rate disconnect puzzle). Finally, in addition to the expected depreciation of the real exchange rate following the initial appreciation, the foreign country's increased demand for the home country's bond causes a relative drop in the interest rate paid on the domestic bond so that $r_1 - r_2$ turns

Figure 1: Impulse Responses



Notes: Impulse responses to one standard deviation shocks. First row: UIP shock. Second row: trade rebalancing shock. Third row: TFP shock. The blue solid lines show the response of variables in the home country. The red dashed lines show the responses of variables in the foreign country.

negative (forward premium puzzle). Over time, as the direct effects of the shock dissipate, the NFA position determines the dynamics. As the foreign country sells off its accumulated assets, the home country's trade balance turns positive, and the real exchange rate depreciates relative to the steady state.

The trade rebalancing shock also addresses the three exchange rate puzzles; however, it moves the trade balance (and the NFA position) in the opposite direction from the UIP shock. The rebalancing shock raises the home country's appetite for its own good and shifts home demand away from the foreign good toward domestically produced goods at the given prices. Thus, the home country's terms of trade must improve for the goods markets to clear. Again, the real exchange rate appreciates, whereas relative consumption rises (Backus-Smith puzzle), and the increase in the real exchange rate is an order of magnitude larger than the movements in consumption (exchange rate disconnect puzzle). Contrary to the UIP shock, the exogenous boost to the demand for the home good dominates the terms of trade effects and pushes the trade balance into surplus. As the foreign country increases its borrowing, intermediation costs rise, increasing the overall cost of borrowing. In equilibrium, the interest rate paid on the domestic bond falls so that $r_1 - r_2$ turns negative, whereas the real exchange rate is expected to depreciate after its initial appreciation (forward premium puzzle). The responses of both the interest rate differential and the trade balance are significantly smaller for the trade rebalancing shock in comparison to the UIP shock.

Technology shocks impact the economy in a fundamentally different manner. While the UIP and trade rebalancing shock primarily reallocates goods between the two countries, the technology shock increases the amount of goods available to both countries. If a positive technology shock increases the production of the home good, their price has to fall, causing the real exchange rate to depreciate. With aggregate consumption in the home country rising by more than in the foreign country and the real exchange rate depreciating, the technology shock induces a positive comovement between $\hat{q}$ and $\hat{C}_1 - \hat{C}_2$. Similarly, the shock fails to generate the correct comovement between the real exchange rate and the interest rate differential. Finally, the technology shock fails to reproduce the exchange rate disconnect as the real exchange rate moves by less than aggregate consumption.

4 Exploring the Disconnect Mechanism

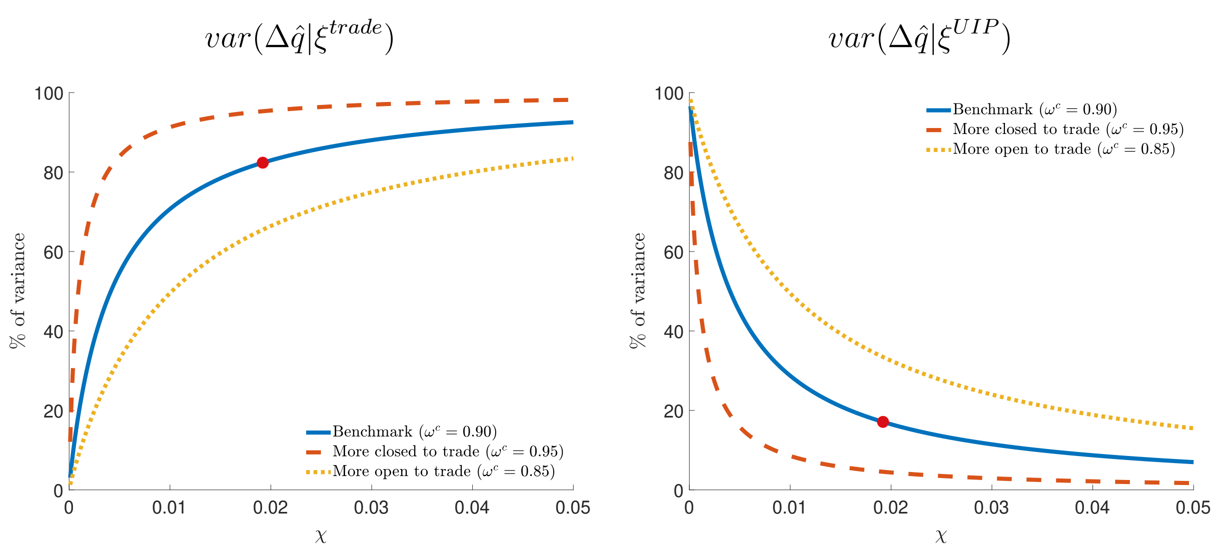
We use our model to examine the interplay between trade and financial integration in determining the drivers of the real exchange rate and the success of our model in matching the data. Our main finding is that trade balancing shocks are important drivers of exchange rate volatility through endogenous UIP deviations. This finding is important because it enables our model to reconnect exchange rate fluctuations to macroeconomic fundamentals, thereby addressing the exchange rate puzzles in the data.

4.1 What Drives the Exchange Rate?

As discussed in Theorem 1, international risk sharing and trade openness modulate endogenous UIP deviations in response to trade rebalancing and UIP shocks. Thus, we explore variance decompositions along these two dimensions. Figure 2 plots the contribution of trade rebalancing and UIP shocks to the variance of the real exchange rate. The left panel shows the share of variance of the real exchange rate accounted for by the rebalancing shocks in our calibrated model. The right panel shows the corresponding variance share due to the UIP shock. Solid lines depict the unconditional variance decomposition as a function of costly financial intermediation for a home bias parameter of $\omega^c = 0.9$. Our baseline calibration is marked with a red circle. The red dashed lines present the variance decomposition in an economy more closed to trade, and the yellow dotted line shows the variance decomposition in the case of an economy more open to trade.

In the benchmark calibration, trade rebalancing shocks account for around 80 percent of the variance in the real exchange rate. The UIP shock accounts for approximately 20 percent of the total variance, while the contribution of technology shocks is negligible. With greater international risk sharing, as measured by a lower value of the parameter χ, the variance contribution of trade rebalancing shocks declines, and the importance of UIP shocks increases. Intuitively,

Figure 2: Variance Decomposition of the Real Exchange Rate



Notes: Unconditional variance decomposition computed in the theoretical model. The blue line is our baseline model. The red dot marks our preferred calibration. Dashed and dashed-dotted lines correspond to alternative calibrations for different values of the home-bias parameter.

when intertemporal substitution is frictionless, as would be the case under $\chi \to 0$, any movement in the trade balance can be accommodated by a corresponding permanent change in the country's NFA position—the NFA position displays unit-root behavior. In other words, if countries can borrow freely, the variation of the trade balance decouples from real shocks. Whether the economy is open or closed to trade is irrelevant when trade in financial assets is frictionless.

The level of financial integration (χ) and trade openness (ω^c) significantly affect the contribution of shocks to the total variance of the real exchange rate. When trade integration is low, as illustrated by the calibration with $\omega^c = 0.95$, trade rebalancing shocks dominate the variance of the real exchange rate. By contrast, trade openness increases the role of the UIP shock.

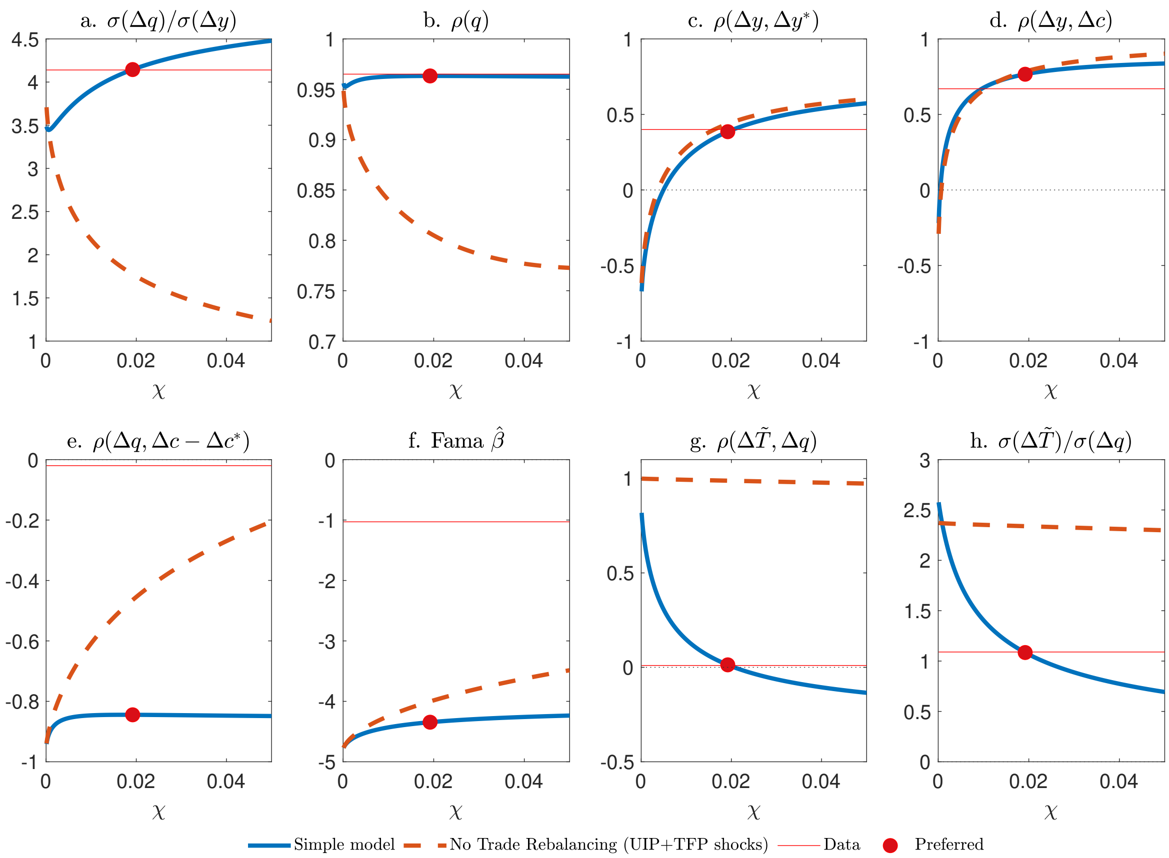
4.2 The Role of International Risk Sharing

We now illustrate how changes in the value of χ affect the exchange rate moments of interest. Figure 3 plots theoretical moments from our model, keeping all other structural parameters as in Table 1. The panels correspond to the moments in Table 2. The red line in each panel indicates the associated value computed in our dataset. The solid blue line depicts the theoretical moments from the model calculated for different values of χ with χ ranging from near-frictionless financial markets ($\chi = 0.001$) to configurations in which costly financial intermediation hampers the flow of financial assets ($\chi = 0.05$). Results for our preferred calibration of $\chi = 0.019$ are depicted with a red circle.

The top row in Figure 3 shows that reducing risk sharing through costlier financial intermediation amplifies the volatility of the real exchange rate, exacerbating the macroeconomic disconnect (panel a), and strengthens the international comovement of output and consumption (panels c and d). Costly financial intermediation reduces the flow of the non-state-contingent bond and thus the ability to smooth out shocks. For example, if the preference for the domestic good increases at home, this would require improving the trade balance and the NFA for the home country. However, when financial intermediation is more costly, the real exchange rate becomes more sensitive to changes in the NFA, resulting in a significantly larger appreciation of the exchange rate for a given-sized shock.

Turning to the panels in the bottom row of Figure 3, the Backus-Smith correlation (panel e) and the Fama coefficient (panel f) are nearly unaffected by changes in the parameter χ. This result is consistent with Theorems 2 and 3, indicating that technology (TFP) shocks play a

Figure 3: Financial Integration and Exchange Rate Moments



Notes: The blue line shows theoretical moments in the simple model with trade rebalancing shocks. The red dashed line shows theoretical moments in the model without trade rebalancing shocks. The solid red horizontal line indicates the corresponding empirical moment in each panel. The red dot is our preferred calibration.

limited role in our analysis here for these data moments.[16] One of our central results is that the model can account for the relationship between the exchange rate and trade flows. In particular, our model aligns with the weak correlation between the trade balance and the real exchange rate (panel g) and the volatility of trade flows relative to the real exchange rate (panel h).

How do trade rebalancing shocks interact with the degree of international risk sharing? The red dashed line in Figure 3 represents the exchange rate moments in a model without rebalancing shocks, while maintaining the calibration for all other parameters as specified in Table 1. The model becomes largely uninformative about the moments relating the exchange rate and the trade balance, complicating the inference of the parameter χ. At the same time, as we will show

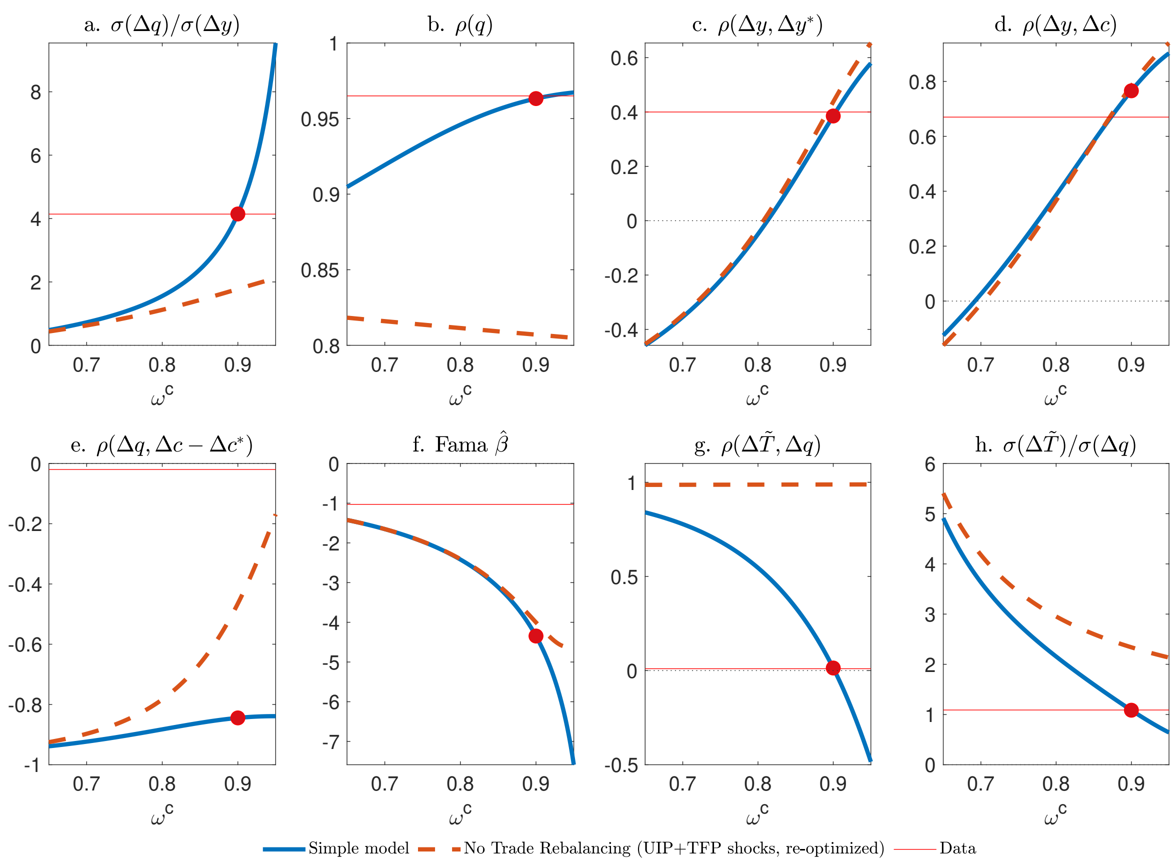
[16] As discussed in Appendix C.5, this result can be overturned under the assumptions in Corsetti, Dedola, and Leduc (2008).

momentarily, performing a similar moment matching exercise in the absence of trade shocks will result in a very low value of χ, as foreshadowed by panels a and b. In Section 5, we show that this identification issue also emerges in an estimated medium-scale model.

4.3 The Role of Trade Integration

How does trade integration affect our model's ability to address the exchange rate puzzles? Keeping χ fixed at 0.02, Figure 4 shows exchange rate moments in our model as a function of the home-bias parameter, ω^c. Lower values of ω^c correspond to a more open economy with greater trade integration. Higher values of ω^c correspond to an economy with less trade integration. The disconnect puzzles (panels a and b) become starker as the economy moves towards trade autarky. Similarly, the IRBC correlations also increase with the home-bias parameter.

Figure 4: Trade Integration and Exchange Rate Moments



Notes: The blue line shows theoretical moments in the simple model with rebalancing shocks. The red dashed line shows theoretical moments in the model without trade rebalancing shocks. The solid red horizontal line indicates the corresponding empirical moment in each panel. The red dot shows our preferred calibration.

These results follow the intertemporal approach of the current account pioneered in Obstfeld and Rogoff (1995). Recall Equation (18), reproduced here

$$\tilde{T}_{1,t} = \frac{\omega_1^c}{1-\omega_1^c}\xi_{1,t}^{trade} - \frac{\omega_1^c}{1-\omega_1^c}\xi_{2,t}^{trade} - z_{1,t} + z_{2,t} + \varpi\hat{\delta}_{1,t}, \tag{26}$$

and consider a trade rebalancing shock that improves the home country's trade balance. Holding the terms of trade constant, the trade balance response increases in the degree of home bias. However, as the economy becomes more closed, the aggregate resource constraint implies $\tilde{T}_{1,t}$ approaches 0. For the trade balance to remain unchanged, the terms of trade must improve by more, and the real exchange rate appreciates more strongly.

The intuition behind the response of the IRBC-related moments stems from the resource constraint of a closed economy. As home bias increases, the real exchange neutralizes movements in the trade balance, and productivity shocks become the only shocks affecting domestic output and consumption. In the trade-autarky limit, the correlation between domestic and foreign output will reflect our assumption about the cross-country correlation of technology shocks (panels c and d).

The Backus-Smith correlation (panel e) is negative and nearly equal to one, consistent with the predictions of Theorem 2. The degree of trade integration is irrelevant to this correlation. The forward premium puzzle (panel f), however, is heavily influenced by trade integration. The Fama coefficient becomes increasingly negative as the home-bias parameter approaches the closed-economy limit, as stated in Theorem 3.

The correlation between the trade balance and the exchange rate (panel g) and the volatility of the trade balance relative to the exchange rate (panel h) both decrease with the degree of home bias. As shown in Appendix B.1, the covariance between the trade balance and the exchange rate, absent technology shocks, depends on:

$$cov\left(\Delta\hat{\delta}_{1,t}, \Delta\tilde{T}_{1,t}\right) = \frac{\omega_1^c}{1-\omega_1^c} cov\left(\Delta\hat{\delta}_{1,t}, \Delta\xi_{1,t}^{trade}\right) + \left(1 + 2\frac{\omega_1^c}{\rho^c}\right) var\left(\Delta\hat{\delta}_{1,t}\right) \tag{27}$$

where the covariance between the terms of trade and the trade rebalancing shocks is negative, as previously discussed. With greater home bias, the first term on the right-hand side becomes more negative at a geometric rate, whereas the second term on the right-hand side becomes more positive at a linear rate. Even if the variance of the terms of trade grows without bound as we approach a closed-economy configuration, the correlation between the terms of trade and the trade balance will become increasingly negative with greater home bias.

Equation (27) also captures the limitation of models that abstract from trade rebalancing shocks. In that case, $cov\left(\Delta\hat{\delta}_{1,t}, \Delta\xi_{1,t}^{trade}\right) = 0$, implying that the correlation between the trade balance and the terms of trade will always be positive and, for our parameterization, equal to one. The red dashed line in Figure 4 illustrates the effect of trade openness on the exchange rate moments in a model without trade rebalancing shocks. Models without trade rebalancing shocks are uninformative about the correlation between the real exchange rate and the trade balance (panel g). Although greater home bias reduces the volatility of the trade balance relative to the real exchange rate, a model without trade rebalancing shocks falls short of matching the empirical value of this moment (panel h). Again, these findings highlight the importance of using trade data, trade rebalancing shocks.

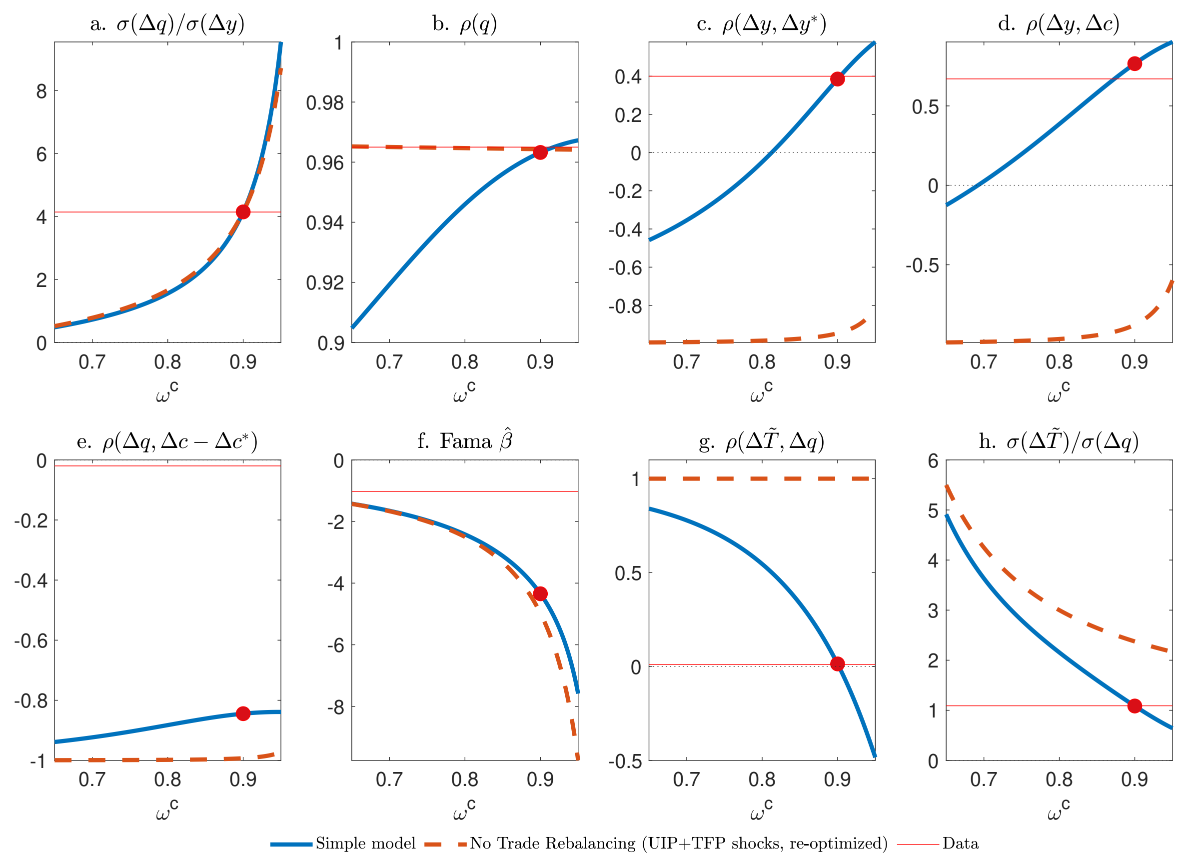
4.4 Relation to the Exchange Rate Disconnect Literature

Existing work that successfully addresses the exchange rate disconnect and related puzzles builds on models in which endogenous UIP deviations are negligible despite the presence of asset market segmentation (Itskhoki and Mukhin 2021), or due to nearly costless financial intermediation (Eichenbaum, Johannsen, and Rebelo 2021). We continue with a comparison between our benchmark model and a specification that, in addition to no trade rebalancing shocks, also features near frictionless international financial markets with $\chi = 0.0001$. To properly compare both models, we calibrate the parameters in the frictionless model to minimize the distance between the model and the relative volatility of the growth rate of the real exchange rate to gdp growth and the persistence of the real exchange rate as reported in Table 2.

Figure 5 plots the exchange rate moments of interest for the two models under varying degrees of home bias—blue solid lines for our baseline model and dashed red lines for the model without trade rebalancing shocks. Although, compared to Figure 4, lowering χ helps to bring the model without trade rebalancing shocks closer to some of the data moments when considering nearly closed economies, i.e., ω^c approaching 1, this version of the model still does not align with the observations regarding the relationship between the real exchange rate and the trade balance (panels g and h). Again, trade rebalancing shocks and costly financial intermediation are important to obtain models that are consistent with the data. Data on trade and the trade balance inform on the strength of the endogenous UIP deviation by identifying the value of χ.

The importance of endogenous UIP deviations—in our case via costly financial intermediation—is further strengthened by revisiting the evidence on UIP reversals. Engel (2016) and Valchev (2020) show that at longer time horizons the direction of UIP violations can reverse sign. Although their evidence pertains to an unconditional moment, it is worthwhile pointing out that exogenous UIP shocks can generate a sign reversal of the UIP violations in the longer run via the endogenous adjustment in the NFA position if financial intermediation is sufficiently costly. In the near term, the direct effect of the UIP shock, in our model, is to drive up excess returns when the shock improves the terms of trade; however, at longer horizons, the direct impact of the shock fades, and the endogenous effect of the persistently negative NFA position resulting from

Figure 5: Near Frictionless Risk Sharing and Home Bias



Notes: The blue line shows theoretical moments in the simple model with rebalancing shocks. The red dashed line shows theoretical moments in the model without trade rebalancing shocks and $\chi = 0.0001$. The solid red horizontal line indicates the corresponding empirical moment in each panel. The red dot shows our preferred calibration.

the appreciation of the real exchange rate and the deterioration of the trade balance becomes the dominant driver of excess returns. The switch in the sign of the expected excess return is more likely if the exogenous UIP shock is not too persistent and financial intermediation is sufficiently costly (i.e. χ is not too close to 0).[17]

4.5 External Validation

Our framework of exchange rate determination critically builds on endogenous UIP deviations that stem from the interaction of risk-bearing capacity and the home country's NFA position. We briefly discuss external evidence in support of this mechanism.

Equation (24) implies that currency excess returns are positively related to the home country's NFA position. We test this prediction using panel data for 28 currencies over the period from 1990 to 2019 and find a strong positive correlation between excess returns and the NFA position.[18] The correlation is robust across G10 and emerging economies, different time horizons, and alternative data measures of the NFA position. These results lend support to the role of external imbalances as the source of endogenous UIP deviations and complement similar findings with respect to deviations from covered interest parity in G10 economies (Liao and Zhang 2024). See Appendix E.1 for details.

4.6 Model Extensions

We close this section with a discussion of three of our modeling choices: the interpretation of the trade rebalancing shock, the role of discount factor shocks, and the labor supply elasticity. The analytical derivations supporting this discussion can be found in Appendix C.

[17] In our model, the horizon-k expected short-term excess return is:

$$E_t \iota_{t+k} = E_t\left(\Delta \hat{q}_{t+k+1} - (r_{1,t+k} - r_{2,t+k})\right) = E_t\left(\chi \tilde{B}_{1,t+k} + \xi_{t+k}^{UIP}\right).$$

A positive innovation of the UIP shock exerts exogenous upward pressure on the excess return via ξ_{t+k}^{UIP}, but endogenous downward pressure via the decline in the home country's NFA position, $\tilde{B}_{1,t+k}$.

[18] Our choice of countries and sample period is dictated by the availability of data on bond yield differentials, which we collected from Du, Im, and Schreger (2018).

4.6.1 Tariffs and Iceberg Trade Costs

The trade rebalancing shock directly alters households' preferences for domestic and foreign goods. We show in Appendix C.2 that the trade rebalancing shock in the original formulation of our model (ξ^{trade}) can be thought of as a linear combination of shocks to iceberg transportation costs (ξ^{ice}), shocks to import tariffs (ξ^{m}), shocks to export subsidies (ξ^{x}), and our original relative demand shock (now denoted as ξ^{c}). In detail, it is

$$\begin{aligned}\frac{\bar{\omega}^c}{1-\bar{\omega}^c}\xi^{trade}_{1,t} \quad = \quad & \frac{1}{1-(\tilde{\omega}^c-\omega^c)}\frac{\omega^c}{1-\omega^c}\xi^{c}_{1,t}+\left(1+\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\right)\frac{\tau^m}{1-\tau^m}\xi^{m}_{1,t} \\ & -\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tau^x}{1-\tau^x}\xi^{x}_{2,t}+\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tau^{ice}}{1-\tau^{ice}}\xi^{ice}_{1,t} \qquad (28)\end{aligned}$$

where $\bar{\omega}^c$ is the re-scaled home-bias parameter, and the parameter $\tilde{\omega}^c_1$ is a function of the steady state values of transportation costs (τ^{ice}), tariffs (τ^{m}), and subsidies (τ^{x}), and is equal to ω^c if those steady-state values are zero.

4.6.2 Discount Factor Shocks

Shocks that lead agents to discount future utility differently across countries can be an alternative source of exchange rate fluctuations. A relative increase in the patience of foreign households raises their demand for savings, which, in our framework, falls exclusively on the demand for the home country's bonds. The resulting rise in the foreign country's NFA position mirrors the drop in the home country's NFA position. With foreign demand for the foreign good relative to the home good falling more strongly, the home country's terms of trade improve, equilibrating the goods markets. This transmission of a (relative) discount factor shock follows the same logic as for a UIP shock. More formally, as shown in Appendix C.3, equally-sized (relative) discount factor and UIP shocks induce the same movements in the terms of trade, the trade balance, the NFA position, the real exchange rate, as well as, output and consumption in our framework. In the presence of discount factor shocks, $\xi^{disc}_{1,t}$ and $\xi^{disc}_{2,t}$, Equation (16) changes to

$$(z_{1,t}-E_t z_{1,t+1})-(z_{2,t}-E_t z_{2,t+1})-\left(\hat{\delta}_{1,t}-E_t\hat{\delta}_{1,t+1}\right)=\chi\tilde{B}_{1,t}+\xi^{UIP}_{1,t}+\left(\xi^{disc}_{2,t}-\xi^{disc}_{1,t}\right), \qquad (29)$$

while Equation (17) and Equation (18) remain unchanged.

A key dimension along which the UIP and the discount factor shocks differ is the induced comovement between the interest rate differential, $r_{1,t} - r_{2,t}$, and the exchange rate—used by Kekre and Lenel (2024) to distinguish the roles of UIP and discount factor shocks as drivers of exchange rate volatility. If the discount factor shocks make the foreign country relatively more patient, foreign interest rates fall relative to domestic rates, opening up a positive interest rate differential while the real exchange rate appreciates. By contrast, a UIP shock that appreciates the real exchange rate, the interest rate differential turns negative as shown in Figure 1.

However, regardless the role of UIP versus discount factor shocks, our analysis shows that trade rebalancing shocks and costly financial intermediation are still required for the model to be consistent with the stylized facts of the exchange rate and the trade balance in Table 2 and, as a direct consequence, trade rebalancing shocks must be a major driver of exchange rate volatility.

In quantitative DSGE models, the ability of discount factor shocks to impinge on the role of trade rebalancing shocks as key drivers of the exchange rate is further limited. While discount factor shocks have a negligible impact on economic activity and consumption in our setting or the endowment economy of Kekre and Lenel (2024), the effects can be sizable in models with nominal rigidities and capital accumulation. In the estimated medium-scale DSGE model of Section 5, the role of discount factor shocks as a driver of exchange rate fluctuations is constrained by the joint behavior of output, consumption, investment, and the exchange rate in the data and the presence of other shocks.

4.6.3 Generalized Labor Supply

It is central to our claim of trade rebalancing shocks being a key driver of the real exchange rate that trade rebalancing shocks satisfy the evidence in Backus and Smith (1993). While evident in our baseline model with a fully elastic labor supply (see Theorem 2), it is not immediately obvious that relative consumption increases after a trade rebalancing shock towards the domestic good when the labor supply is not fully elastic. Here we show that all our findings carry over to the case of a less elastic labor supply when financial intermediation is sufficiently costly. Appendix C.4 offers a more detailed treatment.

A trade rebalancing shock in favor of the domestic good primarily pushes up overall demand for this good despite the accompanying terms of trade improvement. In our baseline specification, expanding the supply of the domestic good is inexpensive as the labor supply is fully elastic. As, in addition, the terms of trade improvement reduces the cost of importing the foreign good, the home country's demand for imports in levels falls by less than its demand for the domestic good in levels expands. Thus, aggregate consumption in the home country rises. Ignoring technology shocks, Equations (21) and (22) imply

$$\hat{C}_{1,t} - \hat{C}_{2,t} = -2\left(1-\omega_1^c\right)\hat{\delta}_{1,t}, \tag{30}$$

revealing how the terms of trade effect raises aggregate consumption in the home country relative to the foreign country. Trade rebalancing shocks address the Backus-Smith puzzle by delivering a negative correlation between relative consumption and the real exchange rate (and the terms of trade) under a fully elastic labor supply.

When the labor supply is less elastic, expanding production is more costly which raises the question whether aggregate consumption in the home country can expand after a trade rebalancing shock that shifts demand towards the home country's good. Leaving details to Appendix C.4, relative consumption satisfies

$$\hat{C}_{1,t} - \hat{C}_{2,t} = -2\left(1-\omega_1^c\right)\hat{\delta}_{1,t} - \bar{\nu}\tilde{T}_{1,t} \tag{31}$$

with greater values of $\bar{\nu}$ indicating a lower labor supply elasticity. The extra term $-\bar{\nu}\tilde{T}_{1,t}$ reflects the increased utility cost from expanding production after a trade rebalancing shock. Since a trade rebalancing shock that induces a terms of trade improvement for the home country also raises its trade balance, relative consumption of the home country could decline when the labor supply is sufficiently inelastic. However, in our framework with costly financial intermediation, aggregate consumption of the home country will increase relative to the foreign country if the intermediation cost χ is sufficiently high. This finding results from the fact that Theorem 1 extends to the case of a less-than-fully elastic labor supply: higher intermediation costs amplify the terms of trade response and mute the trade balance response after a trade rebalancing shock all else equal. Hence, trade rebalancing shocks address the Backus-Smith puzzle also under more general preferences—a key ingredient of our finding that trade rebalancing shocks are a major driver of exchange rate movements.

To round off this discussion, Figure A.1 in Appendix C.4 repeats the exercise underlying Figure 3 for the case of a Frisch labor supply elasticity of 1. The fit of the model is largely unchanged, if not mildly improved. Interestingly, the preferred calibration of the extended model features $\chi \approx 0.02$, close to the value of χ in our baseline model.

5 A Medium-Scale DSGE Model

To assess the quantitative relevance of our mechanism using a broader range of macroeconomic data, we build and estimate an empirical medium-scale DSGE model that incorporates trade rebalancing shocks and costly international financial intermediation. Details on the model, data, estimation, and sensitivity analysis are provided in Appendix D.

The model is a two-country open economy extension of the seminal contributions by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), following the lines of Erceg, Guerrieri, and Gust (2005). The home country is assumed to be the U.S., and the foreign country is defined as the rest of the world. Households choose consumption subject to consumption habits, provide labor, and save in bonds and capital. International borrowing and lending are subject to costly financial intermediation, as in our analytical model. Nominal prices and wages are sticky, exports are subject to local currency pricing and pricing to market frictions.[19]

The final consumption good consists of a domestically produced good and an imported good. Trade rebalancing shocks affect the relative demand for the two goods. Trade adjustment costs introduce a difference between the short- and long-term trade elasticity. The production of the final investment good follows the same structure as for the final consumption good but assumes a lower home bias to reflect the higher import intensity of investment in the data.

Domestically produced and imported goods are bundles of a continuum of differentiated varieties. The production of these varieties utilizes a homogeneous input good, which itself is made from labor and capital services. Capital services are the product of the installed capital stock and an endogenously chosen utilization rate. The installation of new capital is subject to investment adjustment costs.

In each country, the government runs a balanced budget by financing its expenditures with lump-sum taxes on households. Monetary policy follows a Taylor-like instrument rule that responds to inflation and the output gap. In addition to trade rebalancing, UIP, and total factor productivity shocks, the model also features shocks to the marginal efficiency of investment, as well as shocks to monetary and fiscal policy, risk premium, and price and wage markups.

[19] We implemement pricing to market following Alessandria and Choi (2021).

We estimate the model using Bayesian methods with quarterly data on real GDP growth, consumption, investment, CPI inflation, and policy rates for the U.S. and the rest of the world. For the U.S., we also use data on the broad real dollar index, real wage growth, total hours worked, and the growth rate of the export-to-GDP and import-to-GDP ratios. All series run from 1985:Q1 to 2019:Q4 and are constructed as in Bodenstein, Cuba-Borda, Gornemann, Presno, Prestipino, Queraltó, and Raffo (2023). We estimate the parameters of all the shock processes and the parameter governing the financial intermediation cost, χ^{ms}. Unless stated otherwise, all other parameters are fixed at standard values.

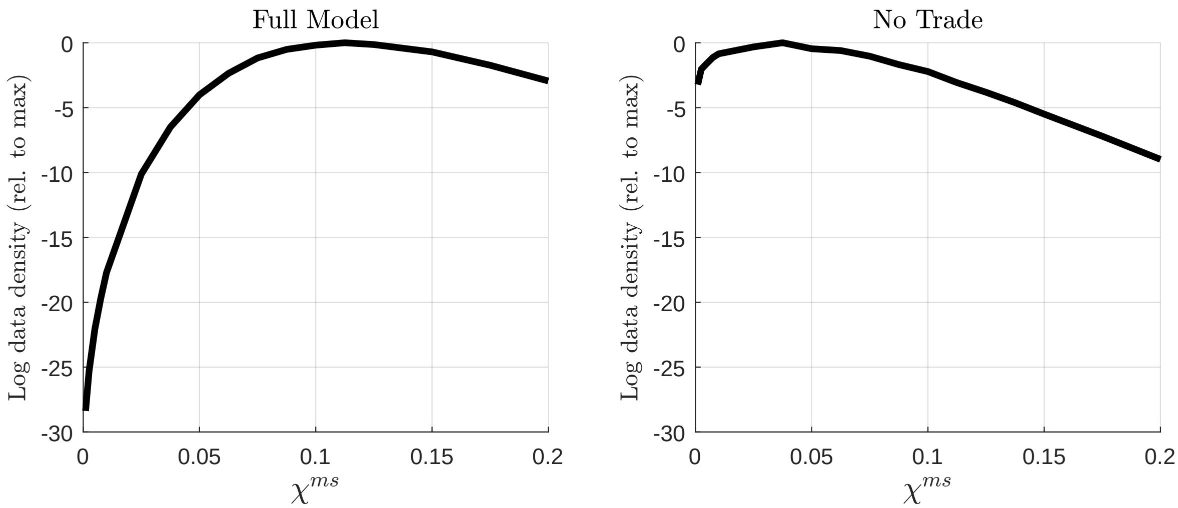
At its core, our model features a demand system close to the gravity framework (Anderson and van Wincoop 2003). The trade literature building on this framework has linked the wedge that we associate with trade rebalancing to various measures of trade and transportation costs, among others Fitzgerald (2012), Reyes-Heroles (2017), Dix-Carneiro, Pessoa, Reyes-Heroles, and Traiberman (2023). As shown in Appendix E.2, we find a strong correlation between our model's estimated trade rebalancing wedge and two empirical measures of trade costs, including one measure derived from a gravity framework. Relative to these contributions, our estimation of the rebalancing wedge leverages the full dynamic structure of the model by respecting general equilibrium channels and drawing inference from the comovements of all our data series.

5.1 Estimation Results

To let the data inform on the parameter χ^{ms} without prior judgement, we either use a wide uniform prior or re-estimate the model along a grid, fixing the value of χ^{ms} a priori. Because the NFA position is normalized by GDP in the medium-scale DSGE model, χ^{ms} must be multiplied by the export share in GDP of around 13.5 percent to obtain comparability with the values of χ used in the analytical model.

When using a uniform prior for χ^{ms}, the posterior distribution peaks at an estimated value of $\chi^{ms} = 0.12$. Translating this value of χ^{ms} to the NFA normalization applied in the analytical model, we obtain 0.016, which is close to the value of 0.019 obtained when calibrating the model in Section 3. When estimating the model along a grid for χ^{ms}, we vary the value of χ^{ms} between 0.001 and 0.2 and re-estimate the remaining model parameters. Figure 6 shows the resulting log-data density for each of these estimations of the full model in the left panel. The density peaks at $\chi^{ms} = 0.11$. The data clearly favor a model with both exogenous *and* endogenous UIP

Figure 6: Identifying χ^{ms}



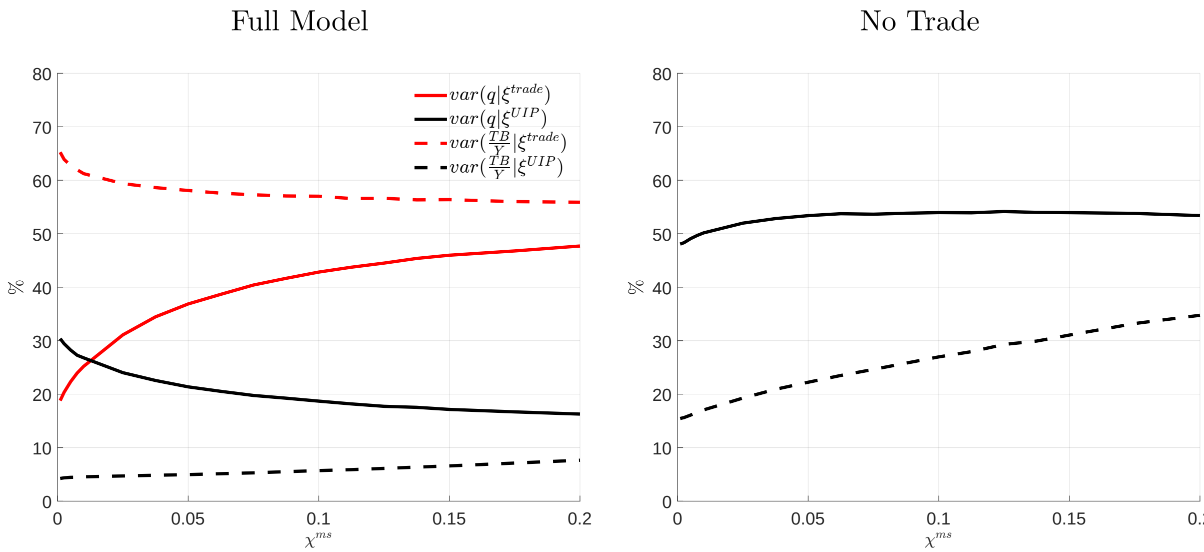
Note: The left panel shows the log-data density of the estimated model for different values of the parameter χ^{ms} when including U.S. export and import data in the estimation. The right panel shows the log-data density in an estimated model without trade data. In both panels, the density as a function of χ^{ms} is expressed relative to the maximum value attained on the grid for χ^{ms}.

deviations over a model that features mostly exogenous UIP deviations. As the right panel of Figure 6 reveals, including trade data into the estimation is key for this finding. If we exclude exports and imports from the estimation, the log-data density peaks at a value for χ^{ms} around 0.04, a much lower value than found in the model using trade data and trade rebalancing shocks.

Figure 7 shows the contributions of the UIP and the trade rebalancing shocks to the variances of the real exchange rate and the trade balance at business cycle frequencies for the different model estimations associated with each value of χ^{ms} on the grid. Each re-estimated model is evaluated at the posterior mean. The left panel of Figure 7 shows that, in line with our theoretical predictions, the share of variance of the real exchange rate explained by the trade rebalancing shocks grows with the magnitude of the financial intermediation costs, χ^{ms}, rising from as low as 20 percent to over 45 percent. By contrast, the contribution of the UIP shock falls as the parameter χ^{ms} increases. We also find that trade rebalancing shocks account for well over 50 percent of the variation in the trade balance. The right panel of Figure 7 shows that the model estimated without trade data and rebalancing shocks assigns a much higher share of the variance of both the real exchange rate and the trade balance to the UIP shock for any value of χ^{ms}. Together with our previous results, we conclude that accounting for trade data and trade rebalancing shocks is crucial for the determination of exchange rates in this class of models.

To complete the analysis, we compute the spectral decomposition of Δq_t, the change in the

Figure 7: Variance Share - Business Cycle Frequency



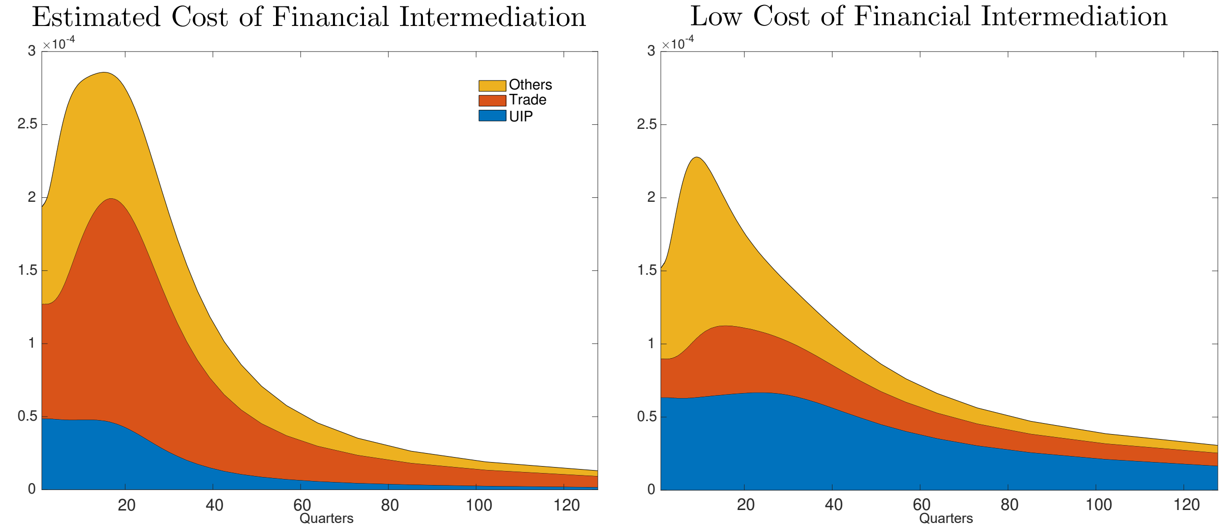
Notes: Solid lines depict the variance decomposition of the real exchange rate. Dashed lines correspond to the variance decomposition of the trade balance-to-output ratio. Red lines correspond to the share of the variance explained by trade rebalancing shocks. Black lines correspond to the share of the variance explained by UIP shocks. All model-implied series are bandpass filtered at 6-32 quarters.

real exchange rate, across cycles of different periods for two estimated versions of the model. In the first one, we set χ^{ms} equal to 0.11, the value maximizing the log-data density in Figure 6. In the second one, we fix χ^{ms} at 0.001, a level consistent with a low cost of financial intermediation. In both cases, the model is re-estimated given the value of χ^{ms} allowing for both UIP and trade rebalancing shocks. The results are shown in Figure 8. When χ^{ms} is set at the value that maximizes the log-data density, the UIP shock contributes less to the exchange rate volatility at every frequency than for the case of $\chi^{ms} = 0.001$—with the reverse being true for the trade rebalancing shock. The comparison across model specifications is most striking at lower frequencies, with the model featuring low costs of financial intermediation requiring UIP shocks to be the dominant driver of exchange rate volatility.

5.2 Exchange Rate Moments

We continue by demonstrating that the estimated model fits the key exchange rate moments on which the literature has focused. Contrary to the calibration exercise in Section 3, the estimation of the medium-scale model does not explicitly target these moments. Instead, the likelihood function trades off model fit along multiple dimensions, including the covariances and cross-correlations with the additional data series we include in the estimation beyond those

Figure 8: Spectral Decomposition - Real Exchange Rate Growth



Notes: The figure depicts the spectral decomposition of Δq_t across cycles of different periods at quarterly frequency. The blue area shows the contribution of the UIP shock. The orange area represents the contribution of trade rebalancing shocks, so that the combined blue and orange areas reflect the roles of UIP and rebalancing shocks. The yellow area represents the effect of all other shocks. The panel on the left shows the results for the main estimation. The panel on the right shows the decomposition for the model when fixing χ^{ms} at 0.001 and re-estimating the other parameters.

underlying the exchange rate moments of interest. Moreover, the medium-scale model imposes additional cross-equation restrictions that may penalize the moments we are interested in. Thus, we are not guaranteed that the estimated model fits the exchange rate moments well.

Table 3 shows that the medium-scale model with trade rebalancing shocks leans towards the moments related to the various exchange rate puzzles, despite the additional restrictions imposed to match a broader set of time series. The data moments lie, for the most part, within the confidence intervals for the moments obtained by simulating the estimated model.[20]

The columns labeled “No Trade” in Table 3 reports results obtained by re-estimating the medium-scale model without trade rebalancing shocks and trade data. The calibrated parameters assume the same values as in our benchmark medium-scale model. The posterior of parameter estimates is similar to that in our baseline model, but to compensate for the absence of trade shocks, the estimated volatility of the UIP and markup shocks is larger than for our benchmark estimates. The estimated model without trade rebalancing shocks is consistent with

[20] We also investigated the correlation between the real exchange rate and bond yields running a regressions of the logged real exchange rate on the ten-year bond rate. We find that a 1-percentage-point increase in U.S. bond yields leads to a 6-percent appreciation of the U.S. real exchange rate. This estimate is remarkably close to the empirical findings in Kekre and Lenel (2024) even without admitting discount factor shocks.

Table 3: Exchange Rate Moments - Medium-Scale Model

		Full Model		No Trade	
	Data	Median	Std. Error	Median	Std. Error
Disconnect and PPP puzzles					
$\sigma\Delta q/\sigma\Delta y$	4.14	3.19	(0.31)	3.52	(0.34)
$\rho(q)$	0.96	0.91	(0.03)	0.88	(0.04)
International Co-movement					
$\rho(\Delta y, \Delta y^*)$	0.39	0.13	(0.11)	0.28	(0.11)
$\rho(\Delta y, \Delta c)$	0.67	0.62	(0.07)	0.78	(0.04)
Backus-Smith and Forward Premium					
$\rho(\Delta q, \Delta c - \Delta c^*)$	-0.02	0.23	(0.08)	0.11	(0.08)
Fama (real) $\hat{\beta}$	-1.03	-0.20	(0.34)	-0.20	(0.29)
RER and NX					
$\rho(\Delta \frac{nx_t}{y_t}, \Delta q)$	-0.10	0.07	(0.09)	0.67	(0.05)
$\sigma(\Delta \frac{nx_t}{y_t})/\sigma(\Delta q)$	0.14	0.15	(0.01)	0.16	(0.01)
Percent of RER variance explained by					
UIP Shocks		15.3	(7.7)	53.6	(18.1)
Trade Rebalancing Shocks		50.3	(19.8)	0.00	(0.00)

Notes: Moments for the medium-scale models are computed using 1000 simulations drawn from the estimated innovations at the posterior mean. Each simulation includes 140 quarters to match the number of observations in our data sample from 1985Q1 to 2019Q4. The net-exports (nx) are normalized by output in this section instead of exports. To compare with the simple analytical model, we need to multiply the volatility of the nx-to-output ratio by the inverse of the trade share which yields 1.09 as in Table 2.

major exchange rate puzzles but is unable to reproduce the correlation between the trade balance and the exchange rate. Again, capturing the correlation between the trade balance and exchange rate has significant implications for assessing the main drivers of the exchange rate.[21]

5.3 Costly Financial Intermediation and the Mussa Puzzle

We explore the ability of our model to generate the celebrated Mussa Puzzle facts, which stress that a change in the monetary policy regime from fixed to flexible exchange rates leads to an increase in the volatility of the nominal and real exchange rates without affecting meaningfully the volatility of macroeconomic variables such as consumption and inflation (Itskhoki and Mukhin 2025). To test wether our model is consistent with the Mussa Puzzle, we augment the monetary policy rule such that it responds to changes in the nominal exchange rate in addition to inflation

[21] Estimating the model without trade shocks, but including trade data, results in a poor overall fit as the joint dynamics of trade and the exchange rate are hard to fit with domestic and UIP shocks only.

Table 4: Mussa Puzzle Moments

	RER $\sigma(\Delta(q))$	Relative Consumption $\sigma(\Delta(c) - \Delta(c^*))$	Relative Inflation $\sigma(\Delta(\pi) - \Delta(\pi^*))$	Risk-sharing wedge $\sigma(\Delta(c) - \Delta(c^*) - \Delta(q))$
Float	3.6	1.2	0.8	3.5
Peg	0.7	1.3	0.7	1.6
Float/Peg	5.0	1.0	1.1	2.2

Notes: The moments are calculated using 1,000 simulations drawn from the ergodic distribution of shocks using the posterior mean estimates. Standard deviations expressed in log points. Each simulation includes 116 quarters to match the number of observations used in the analysis of Itskhoki and Mukhin (2025).

and the output gap. By adjusting the parameters in the policy rule, we can accommodate both fixed and flexible exchange rate regimes—see Appendix D.

Table 4 shows that our estimated medium-scale model with costly financial intermediation and trade rebalancing delivers the following results when moving from a peg to a float: (i) the volatility of the real exchange rate increases; (ii) the volatility of relative consumption and relative inflation between the home and foreign economy are unchanged; (iii) the volatility of the risk sharing wedge, defined as the difference between relative consumptions and the real exchange rate, increases by a factor of two. Our model is therefore consistent with the Mussa facts but, compared to Itskhoki and Mukhin (2025), trade rebalancing shocks are now the main drivers of the exchange rate.

6 Conclusion

We develop a model of exchange rate determination that is consistent with key empirical features of exchange rates, such as the exchange rate volatility puzzle, the Backus-Smith puzzle, and the forward-premium puzzle, without overly relying on UIP shocks that are disconnected from the broader macroeconomy. In fact, we show analytically and numerically that trade rebalancing shocks—that is shifts in the relative demand for domestically produced versus imported goods—address these puzzles when risk-bearing capacity in international financial markets is limited. In contrast to UIP shocks which work primarily through exogenous deviations from UIP, trade rebalancing shocks lead to endogenous deviations from UIP in our setting.

In the data, the real exchange rate is weakly correlated with the trade balance. Theories of exchange rate determination that rely mainly on exogenous UIP shocks deliver too tight a connection between the two variables and imply too much volatility of the trade balance. By adding trade rebalancing shocks and costly financial intermediation to the model, the volatility of the trade balance drops and the correlation of the real exchange rate and the trade balance weakens bringing the model much closer to the data. Restricting the degree of consumption risk sharing through costly financial intermediation is key as higher costs shift the role of explaining exchange rate movements from the UIP to the trade rebalancing shocks. In an estimate medium-scale DSGE model of the U.S. and the rest of the world we find that trade rebalancing shocks account for about 50 percent of exchange rate fluctuations, whereas UIP shocks account for just around 15 percent.

Our results underscore the importance of trade data in correctly separating alternative drivers of exchange rate fluctuations and identifying the degree of impediments to international risk sharing. More broadly, our theory suggests that external imbalances are central to both macroeconomic and exchange rate dynamics.

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Online Appendix

Appendix "Exchange Rate Disconnect and the Trade Balance"

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A Appendix: Model Solution, Decision Rules, and Theoretical Moments of the Simple Model

A.1 First-order and equilibrium conditions

We present the first-order and equilibrium conditions of the model. Optimality of the household's decisions imply

$$C_{1,t} = \frac{P^d_{1,t}}{P^c_{1,t}} \frac{W_{1,t}}{P^d_{1,t}} \tag{A.1}$$

$$P^b_{1,t} = \phi^b_{1,t} \beta E_t \left\{ \frac{C_{1,t}}{C_{1,t+1}} \frac{P^c_{1,t}}{P^c_{1,t+1}} \right\}. \tag{A.2}$$

$$C^d_{1,t} = \omega^c_{1,t} \left(\frac{P^c_{1,t}}{P^d_{1,t}} \right)^{\frac{1+\rho^c}{\rho^c}} C_{1,t} \tag{A.3}$$

$$M_{1,t} = (1-\omega^c_{1,t}) \left(\frac{P^c_{1,t}}{P^m_{1,t}} \right)^{\frac{1+\rho^c}{\rho^c}} C_{1,t}. \tag{A.4}$$

These conditions and the consumption aggregator imply for the relative prices

$$\frac{P^c_{1,t}}{P^d_{1,t}} = \left[\omega^c_{1,t} + (1-\omega^c_{1,t}) \delta_{1,t}^{-\frac{1}{\rho^c}} \right]^{-\rho^c} = F_{1,t}^{-\rho^c} \tag{A.5}$$

where the terms of trade $\delta_{1,t}$ are the ratio of import prices expressed in common currency

$$\delta_{1,t} = \frac{e_{1,t} P^d_{2,t}}{P^d_{1,t}}. \tag{A.6}$$

Similar conditions are obtained for country 2, keeping in mind that the sole internationally traded bond pays in the currency of country 1

$$C_{2,t} = \frac{P^d_{2,t}}{P^c_{2,t}} \frac{W_{2,t}}{P^d_{2,t}} \tag{A.7}$$

$$P^b_{1,t} = \phi^b_{2,t} \beta E_t \left\{ \frac{C_{2,t}}{C_{2,t+1}} \frac{P^c_{2,t}}{P^c_{2,t+1}} \frac{e_{1,t}}{e_{1,t+1}} \right\} \tag{A.8}$$

$$C^d_{2,t} = \omega^c_{2,t} \left(\frac{P^c_{2,t}}{P^d_{2,t}} \right)^{\frac{1+\rho^c}{\rho^c}} C_{2,t} \tag{A.9}$$

$$M_{2,t} = (1-\omega^c_{2,t}) \left(\frac{P^c_{2,t}}{P^m_{2,t}} \right)^{\frac{1+\rho^c}{\rho^c}} C_{2,t} \tag{A.10}$$

$$\frac{P^c_{2,t}}{P^d_{2,t}} = \left[\omega^c_{2,t} + (1-\omega^c_{2,t})\delta_{1,t}^{\frac{1}{\rho^c}}\right]^{-\rho^c} = F_{2,t}^{-\rho^c}. \tag{A.11}$$

As we assume that prices and wages are flexible and that the production of each country's good is linear in the use of the country's labor,

$$Y_{1,t} = exp(z_{1,t})L_{1,t} \tag{A.12}$$

the production real wage equals the productivity level

$$\frac{W_{1,t}}{P^d_{1,t}} = exp(z_{1,t}). \tag{A.13}$$

Similarly, for country 2, we obtain

$$Y_{2,t} = exp(z_{2,t})L_{2,t} \tag{A.14}$$

$$\frac{W_{2,t}}{P^d_{2,t}} = exp(z_{2,t}). \tag{A.15}$$

Recall from the main text that market clearance in goods and financial markets requires

$$Y_{1,t} = C^d_{1,t} + M_{2,t} \tag{A.16}$$

$$Y_{2,t} = C^d_{2,t} + M_{1,t} \tag{A.17}$$

$$0 = B_{1,t} + B_{2,t}. \tag{A.18}$$

As defined in the main text, the trade balance (normalized by the value of exports) is

$$T_{1,t} \equiv e_t P^m_{2,t} M_{2,t} - P^m_{1,t} M_{1,t} \tag{A.19}$$

$$\tilde{T}_{1,t} = \frac{T_{1,t}}{e_t P^m_{2,t} M_{2,t}} \tag{A.20}$$

which implies that the consolidated budget constraint of households in country 1 follows

$$\frac{P^b_{1,t} B_{1,t}}{\phi^b_{1,t}} = T_{1,t} + B_{1,t-1}. \tag{A.21}$$

Last, the law of one price for the international bond implies the risk-sharing condition

$$\phi^b_{1,t} E_t \left\{ \frac{C_{1,t}}{C_{1,t+1}} \frac{P^c_{1,t}}{P^c_{1,t+1}} \right\} = \phi^b_{2,t} E_t \left\{ \frac{C_{2,t}}{C_{2,t+1}} \frac{P^c_{2,t}}{P^c_{2,t+1}} \frac{e_{1,t}}{e_{1,t+1}} \right\}. \tag{A.22}$$

A.2 Simplifying the Nonlinear Model

Noticing that consumption in the two countries can be expressed as

$$C_{1,t} = exp(z_{1,t})F_{1,t}^{\rho^c} \tag{A.23}$$

$$C_{2,t} = exp(z_{2,t})F_{2,t}^{\rho^c} \tag{A.24}$$

and, from the definition of the consumption real exchange rate, we obtain

$$q_{1,t} = \frac{e_{1,t}P_{2,t}^c}{P_{1,t}^c} = \left(\frac{F_{1,t}}{F_{2,t}}\right)^{\rho^c} \delta_{1,t}, \tag{A.25}$$

we can write the trade balance as

$$\begin{aligned} \tilde{T}_{1,t} &= 1 - \delta_{1,t}\frac{M_{1,t}}{M_{2,t}} \\ &= 1 - \frac{1-\omega_{1,t}^c}{1-\omega_{2,t}^c}\left(\frac{F_{1,t}}{F_{2,t}}\right)^{-1}\frac{exp(z_{1,t})}{exp(z_{2,t})}\delta_{1,t}^{1-2\frac{1+\rho^c}{\rho^c}}. \end{aligned} \tag{A.26}$$

Similarly, we write the risk sharing condition and the evolution of the NFA position as

$$E_t\left\{\frac{P_{1,t}^c}{F_{1,t+1}^{\rho^c}P_{1,t+1}^c}\left[\phi_{1,t}^b\frac{exp(z_{1,t})}{exp(z_{1,t+1})} - \phi_{2,t}^b\frac{exp(z_{2,t})}{exp(z_{2,t+1})}\frac{\delta_{1,t}}{\delta_{1,t+1}}\right]\right\} = 0 \tag{A.27}$$

$$\frac{P_{1,t}^b\tilde{B}_{1,t}}{\phi_{1,t}^b} = \tilde{T}_{1,t} + \frac{e_{1,t-1}P_{2,t-1}^m M_{2,t-1}}{e_{1,t}P_{2,t}^m M_{2,t}}\tilde{B}_{1,t-1} \tag{A.28}$$

where $\tilde{B}_{1,t} = \frac{B_{1,t}}{e_{1,t}P_{2,t}^m M_{2,t}}$.

We next express all variables entering in Equations (A.26), (A.27), and (A.28) in terms of $\tilde{T}_{1,t}$, $\delta_{1,t}$, $\tilde{B}_{1,t}$. The rebalancing shocks follow $\omega_{i,t}^c = \omega_i^c \exp\left(\xi_{i,t}^{trade}\right)$, for $i = 1,2$ and we assume that $\phi_{1,t}^b = \exp\left(-\frac{\chi}{2}\frac{B_{1,t}^*}{P_{1,t}^d M_{2,t}^*}\right)$ whereas $\phi_{2,t}^b = \exp\left(-\frac{\chi}{2}\frac{\frac{1}{e_{1,t}}B_{2,t}^*}{P_{2,t}^d M_{1,t}} + \xi_{1,t}^{UIP}\right)$. Then, linearization around the symmetric deterministic steady state with $\omega_1^c = \omega_2^c$ and balanced trade, i.e., $\tilde{T}_1 = 0$, $\delta_1 = 1$, $B_1 = 0$, yields the linear system

$$(z_{1,t} - E_t z_{1,t+1}) - (z_{2,t} - E_t z_{2,t+1}) - \left(\hat{\delta}_{1,t} - E_t\hat{\delta}_{1,t+1}\right) = \chi\tilde{B}_{1,t} + \xi_{1,t}^{UIP} \tag{A.29}$$

$$\beta\tilde{B}_{1,t} = \tilde{T}_{1,t} + \tilde{B}_{1,t-1} \tag{A.30}$$

$$\tilde{T}_{1,t} = \frac{\omega_1^c}{1-\omega_1^c}\xi_{1,t}^{trade} - \frac{\omega_1^c}{1-\omega_1^c}\xi_{2,t}^{trade} - z_{1,t} + z_{2,t} + \varpi\hat{\delta}_{1,t}, \tag{A.31}$$

where $\varpi = 1 + 2\frac{\omega_1^c}{\rho^c}$. The terms trade, $\hat{\delta}_{1,t}$, are measured in log-deviation from the steady state, $\tilde{T}_{1,t}$ and $\tilde{B}_{1,t}$ are in absolute deviations from 0. If the term $\chi > 0$, the NFA dynamics are stationary.

For later reference, the remaining model variables can be recovered from the following linear relationships

$$\hat{C}_{1,t} = z_{1,t} - (1 - \omega_1^c)\,\hat{\delta}_{1,t} \tag{A.32}$$

$$\hat{C}_{2,t} = z_{2,t} + (1 - \omega_1^c)\,\hat{\delta}_{1,t} \tag{A.33}$$

$$\hat{q}_{1,t} = (2\omega_1^c - 1)\,\hat{\delta}_{1,t}. \tag{A.34}$$

$$\hat{M}_{1,t} = -\frac{\omega_1^c}{1-\omega_1^c}\xi_{1,t}^{trade} + z_{1,t} - \frac{\varpi + 1}{2}\hat{\delta}_{1,t} \tag{A.35}$$

$$\hat{M}_{2,t} = -\frac{\omega_2^c}{1-\omega_2^c}\xi_{2,t}^{trade} + z_{2,t} + \frac{\varpi + 1}{2}\hat{\delta}_{1,t} \tag{A.36}$$

$$\hat{Y}_{1,t} = \omega_1^c\left(\xi_{1,t}^{trade} - \xi_{2,t}^{trade}\right) + \omega_1^c z_{1,t} + (1-\omega_1^c)\,z_{2,t} + \varpi\,(1-\omega_1^c)\,\hat{\delta}_{1,t} \tag{A.37}$$

$$\hat{Y}_{2,t} = -\omega_1^c\left(\xi_{1,t}^{trade} - \xi_{2,t}^{trade}\right) + \omega_1^c z_{2,t} + (1-\omega_1^c)\,z_{1,t} - \varpi\,(1-\omega_1^c)\,\hat{\delta}_{1,t} \tag{A.38}$$

Defining the real interest rate for country i as the real return on a bond that pays one unit of consumption in country i regardless of the state of the world, we obtain $r_{i,t} = \hat{C}_{i,t+1} - \hat{C}_{i,t}$ and can express Equation (A.29) in terms of the differential of real interest rates, $r_{1,t} - r_{2,t}$, between countries

$$r_{1,t} - r_{2,t} = E_t\left(\hat{q}_{1,t+1} - \hat{q}_{1,t}\right) - \chi\tilde{B}_{1,t} - \xi_{1,t}^{UIP}. \tag{A.39}$$

A.3 Applying the Method of Undetermined Coefficients

To compute the solution of the dynamic linear system, we employ the method of undetermined coefficients. Starting from the conjecture that in equilibrium the terms of trade evolve according to

$$\hat{\delta}_{1,t} = \gamma_1\xi_{1,t}^{trade} + \gamma_2\xi_{2,t}^{trade} + \gamma_3\xi_{1,t}^{UIP} + \gamma_4 z_{1,t} + \gamma_5 z_{2,t} + \gamma_b\tilde{B}_{1,t-1} \tag{A.40}$$

we compute the values of the unknown coefficients γ_1 through γ_5 and γ_b by substituting the conjectured solution into the dynamic system (A.29)-(A.31). Using Equation (A.31), the trade balance follows

$$\tilde{T}_{1,t} = \left(\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_1\right)\xi_{1,t}^{trade} + \left(-\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_2\right)\xi_{2,t}^{trade} + \varpi\gamma_3\xi_{1,t}^{UIP} + \left(-1+\varpi\gamma_4\right)z_{1,t} + \left(1+\varpi\gamma_5\right)z_{2,t} + \varpi\gamma_b\tilde{B}_{1,t-1}. \quad \text{(A.41)}$$

Turning to the risk sharing/UIP condition, Equation (A.29), we first evaluate the term $\left(\hat{\delta}_{1,t} - E_t\hat{\delta}_{1,t+1}\right)$ using the equation for the evolution of the NFA position, Equation (A.30), and Equation (A.41) to substitute out for $\tilde{B}_{1,t}$:

$$\begin{aligned}
\left(\hat{\delta}_{1,t} - E_t\hat{\delta}_{1,t+1}\right) &= \gamma_1\left(1-\rho_1^{trade}\right)\xi_{1,t}^{trade} + \gamma_2\left(1-\rho_2^{trade}\right)\xi_{2,t}^{trade} + \gamma_3\left(1-\rho_1^{UIP}\right)\xi_{1,t}^{UIP} \\
&\quad +\gamma_4\left(1-\rho_1^z\right)z_{1,t} + \gamma_5\left(1-\rho_2^z\right)z_{2,t} \\
&\quad +\gamma_b\left(\tilde{B}_{1,t-1} - \tilde{B}_{1,t}\right) \\
&= \gamma_1\left(1-\rho_1^{trade}\right)\xi_{1,t}^{trade} + \gamma_2\left(1-\rho_2^{trade}\right)\xi_{2,t}^{trade} + \gamma_3\left(1-\rho_1^{UIP}\right)\xi_{1,t}^{UIP} \\
&\quad +\gamma_4\left(1-\rho_1^z\right)z_{1,t} + \gamma_5\left(1-\rho_2^z\right)z_{2,t} \\
&\quad +\frac{\gamma_b}{\beta}\left(\left(\beta-1\right)\tilde{B}_{1,t-1} - \tilde{T}_{1,t}\right) \\
&= \left(\gamma_1\left(1-\rho_1^{trade}\right) - \frac{\gamma_b}{\beta}\left(\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_1\right)\right)\xi_{1,t}^{trade} \\
&\quad + \left(\gamma_2\left(1-\rho_2^{trade}\right) - \frac{\gamma_b}{\beta}\left(-\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_2\right)\right)\xi_{2,t}^{trade} \\
&\quad + \left(\gamma_3\left(1-\rho_1^{UIP}\right) - \frac{\gamma_b}{\beta}\varpi\gamma_3\right)\xi_{1,t}^{UIP} \\
&\quad + \left(\gamma_4\left(1-\rho_1^z\right) - \frac{\gamma_b}{\beta}\left(-1+\varpi\gamma_4\right)\right)z_{1,t} \\
&\quad + \left(\gamma_5\left(1-\rho_2^z\right) - \frac{\gamma_b}{\beta}\left(1+\varpi\gamma_5\right)\right)z_{2,t} \\
&\quad + \left(\frac{\gamma_b}{\beta}\left(\beta-1\right) - \frac{\gamma_b}{\beta}\varpi\gamma_b\right)\tilde{B}_{1,t-1}. \qquad \text{(A.42)}
\end{aligned}$$

Note that the UIP condition, Equation (A.29), can be written as

$$\begin{aligned}
\left(\hat{\delta}_{1,t} - E_t\hat{\delta}_{1,t+1}\right) &= -\frac{\chi}{\beta}\left(\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_1\right)\xi_{1,t}^{trade} \\
&\quad -\frac{\chi}{\beta}\left(-\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_2\right)\xi_{2,t}^{trade} \\
&\quad -\left(\frac{\chi}{\beta}\varpi\gamma_3 + 1\right)\xi_{1,t}^{UIP}
\end{aligned}$$

$$
\begin{aligned}
&-\left(\frac{\chi}{\beta}\left(-1+\varpi\gamma_4\right)-\left(1-\rho_1^z\right)\right)z_{1,t}\\
&-\left(\frac{\chi}{\beta}\left(1+\varpi\gamma_5\right)+\left(1-\rho_2^z\right)\right)z_{2,t}\\
&-\frac{\chi}{\beta}\left(\varpi\gamma_b+1\right)\tilde{B}_{1,t-1}.
\end{aligned}
\tag{A.43}
$$

Combining Equation (A.42) and Equation (A.43) yields the following condition that determines the coefficients of the decision rule for the terms of trade:

$$
\begin{aligned}
&\left(\gamma_1\left(1-\rho_1^{trade}\right)-\frac{\gamma_b}{\beta}\left(\frac{\omega_1^c}{1-\omega_1^c}+\varpi\gamma_1\right)\right)=-\frac{\chi}{\beta}\left(\frac{\omega_1^c}{1-\omega_1^c}+\varpi\gamma_1\right)\\
&\left(\gamma_2\left(1-\rho_2^{trade}\right)-\frac{\gamma_b}{\beta}\left(-\frac{\omega_1^c}{1-\omega_1^c}+\varpi\gamma_2\right)\right)=-\frac{\chi}{\beta}\left(-\frac{\omega_1^c}{1-\omega_1^c}+\varpi\gamma_2\right)\\
&\left(\gamma_3\left(1-\rho_1^{UIP}\right)-\frac{\gamma_b}{\beta}\varpi\gamma_3\right)=-\left(\frac{\chi}{\beta}\varpi\gamma_3+1\right)\\
&\left(\gamma_4\left(1-\rho_1^z\right)-\frac{\gamma_b}{\beta}\left(-1+\varpi\gamma_4\right)\right)=-\left(\frac{\chi}{\beta}\left(-1+\varpi\gamma_4\right)-\left(1-\rho_1^z\right)\right)\\
&\left(\gamma_5\left(1-\rho_2^z\right)-\frac{\gamma_b}{\beta}\left(1+\varpi\gamma_5\right)\right)=-\left(\frac{\chi}{\beta}\left(1+\varpi\gamma_5\right)+\left(1-\rho_2^z\right)\right)\\
&\left(\frac{\gamma_b}{\beta}\left(\beta-1\right)-\frac{\gamma_b}{\beta}\varpi\gamma_b\right)=-\frac{\chi}{\beta}\left(\varpi\gamma_b+1\right).
\end{aligned}
$$

We present the final coefficients next.

A.4 Decision Rules

This subsection collects the decision roles of the main variables in our model, the terms of trade, the trade balance, and the NFA position. These three decision rules are used extensively when obtaining the proofs of our theorems. Recall that we assume $\varpi > 0$ throughout the analysis.

A.4.1 Terms of Trade, $\hat{\delta}_{1,t}$

The terms of trade are a linear function of the exogenous shocks and the inherited NFA position, $\tilde{B}_{1,t-1}$

$$
\hat{\delta}_{1,t}=\gamma_1\xi_{1,t}^{trade}+\gamma_2\xi_{2,t}^{trade}+\gamma_3\xi_{1,t}^{UIP}+\gamma_4 z_{1,t}+\gamma_5 z_{2,t}+\gamma_b\tilde{B}_{1,t-1}
\tag{A.44}
$$

with the coefficients

$$\gamma_1: \qquad \gamma_1 = \frac{\frac{\gamma_b-\chi}{\beta}\frac{\omega_1^c}{1-\omega_1^c}}{-\frac{\gamma_b-\chi}{\beta}\varpi + \left(1-\rho_1^{trade}\right)} = -\frac{\tilde{\gamma}_b\frac{1}{\varpi}\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_1^{trade}\right)} < 0 \tag{A.45}$$

$$\gamma_2: \qquad \gamma_2 = \frac{-\frac{\gamma_b-\chi}{\beta}\frac{\omega_1^c}{1-\omega_1^c}}{-\frac{\gamma_b-\chi}{\beta}\varpi + \left(1-\rho_2^{trade}\right)} = \frac{\tilde{\gamma}_b\frac{1}{\varpi}\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_2^{trade}\right)} > 0 \tag{A.46}$$

$$\gamma_3: \qquad \gamma_3 = \frac{-1}{-\frac{\gamma_b-\chi}{\beta}\varpi + (1-\rho_1^{UIP})} = -\frac{1}{\tilde{\gamma}_b + (1-\rho_1^{UIP})} < 0 \tag{A.47}$$

$$\gamma_4: \qquad \gamma_4 = \frac{-\frac{\gamma_b-\chi}{\beta} + (1-\rho_1^z)}{-\frac{\gamma_b-\chi}{\beta}\varpi + (1-\rho_1^z)} = \frac{\tilde{\gamma}_b\frac{1}{\varpi} + (1-\rho_1^z)}{\tilde{\gamma}_b + (1-\rho_1^z)} > 0 \tag{A.48}$$

$$\gamma_5: \qquad \gamma_5 = \frac{\frac{\gamma_b-\chi}{\beta} - (1-\rho_2^z)}{-\frac{\gamma_b-\chi}{\beta}\varpi + (1-\rho_2^z)} = -\frac{\tilde{\gamma}_b\frac{1}{\varpi} + (1-\rho_2^z)}{\tilde{\gamma}_b + (1-\rho_2^z)} < 0 \tag{A.49}$$

where the coefficient $\tilde{\gamma}_b = -\frac{\gamma_b-\chi}{\beta}\varpi$ and the coefficient γ_b is given by

$$\gamma_b \;\; = \;\; \frac{-(1-\beta-\chi\varpi) - \sqrt{(1-\beta-\chi\varpi)^2 + 4\chi\varpi}}{2\varpi}. \tag{A.50}$$

γ_b is the stable root associated with the quadratic equation

$$-\varpi\gamma_b^2 + (\beta - 1 + \chi\varpi)\,\gamma_b + \chi = 0. \tag{A.51}$$

The coefficient γ_b is negative and decreasing in χ, i.e., $\frac{\partial\gamma_b}{\partial\chi} < 0$ for $0 < \beta < 1$. For the coefficient $\tilde{\gamma}_b$ we therefore obtain $\frac{\partial\tilde{\gamma}_b}{\partial\chi} = \frac{\varpi}{\beta}\left(1 - \frac{\partial\gamma_b}{\partial\chi}\right) > 0$.

A.4.2 Trade Balance, $\tilde{T}_{1,t}$

Using Equation (A.41), the trade balance is a linear function of the exogenous shocks and the inherited NFA position, $\tilde{B}_{1,t-1}$

$$\tilde{T}_{1,t} \;\; = \;\; \alpha_1\xi_{1,t}^{trade} + \alpha_2\xi_{2,t}^{trade} + \alpha_3\xi_{1,t}^{UIP} + \alpha_4 z_{1,t} + \alpha_5 z_{2,t} + \alpha_b\tilde{B}_{1,t-1} \tag{A.52}$$

with the coefficients

$$\alpha_1: \qquad \alpha_1 = \frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_1 = \frac{\left(1-\rho_1^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_1^{trade}\right)} > 0 \tag{A.53}$$

$$\alpha_2: \qquad \alpha_2 = -\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_2 = -\frac{\left(1-\rho_2^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_2^{trade}\right)} < 0 \tag{A.54}$$

$$\alpha_3: \qquad \alpha_3 = \varpi\gamma_3 = -\frac{\varpi}{\tilde{\gamma}_b + (1-\rho_1^{UIP})} < 0 \tag{A.55}$$

$$\alpha_4: \qquad \alpha_4 = -1 + \varpi\gamma_4 = \frac{(1-\rho_1^z)(\varpi-1)}{\tilde{\gamma}_b + (1-\rho_1^z)} \tag{A.56}$$

$$\alpha_5: \qquad \alpha_5 = 1 + \varpi\gamma_5 = \frac{-(1-\rho_2^z)(\varpi-1)}{\tilde{\gamma}_b + (1-\rho_2^z)} \tag{A.57}$$

$$\alpha_b: \qquad \alpha_b = \varpi\gamma_b < 0. \tag{A.58}$$

A.4.3 Net Foreign Assets, $\tilde{B}_{1,t}$

The trade balance is a linear function of the exogenous shocks and the inherited NFA position, $\tilde{B}_{1,t-1}$

$$\tilde{B}_{1,t} = \beta_1\xi_{1,t}^{trade} + \beta_2\xi_{2,t}^{trade} + \beta_3\xi_{1,t}^{UIP} + \beta_b\tilde{B}_{1,t-1} + \beta_4 z_{1,t} + \beta_5 z_{2,t} \tag{A.59}$$

with the coefficients

$$\beta_1: \qquad \beta_1 = \frac{\alpha_1}{\beta} = \frac{1}{\beta}\frac{\left(1-\rho_1^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_1^{trade}\right)} > 0 \tag{A.60}$$

$$\beta_2: \qquad \beta_2 = \frac{\alpha_2}{\beta} = -\frac{1}{\beta}\frac{\left(1-\rho_2^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_2^{trade}\right)} < 0 \tag{A.61}$$

$$\beta_3: \qquad \beta_3 = \frac{\alpha_3}{\beta} = -\frac{1}{\beta}\frac{\varpi}{\tilde{\gamma}_b + (1-\rho_1^{UIP})} < 0 \tag{A.62}$$

$$\beta_4: \qquad \beta_4 = \frac{\alpha_4}{\beta} = \frac{1}{\beta}\frac{(1-\rho_1^z)(\varpi-1)}{\tilde{\gamma}_b + (1-\rho_1^z)} \tag{A.63}$$

$$\beta_5: \qquad \beta_5 = \frac{\alpha_5}{\beta} = -\frac{1}{\beta}\frac{(1-\rho_2^z)(\varpi-1)}{\tilde{\gamma}_b + (1-\rho_2^z)} \tag{A.64}$$

$$\beta_b: \qquad \beta_b = \frac{\alpha_b + 1}{\beta} = \frac{\gamma_b}{\gamma_b - \chi}. \tag{A.65}$$

A.5 Statistical Moments

Using the decision rules of the model, we compute analytical expressions for statistical moments displayed in the main text and the proofs. To ease notation just within this subsection we define

$$\rho_1 = \rho_1^{trade} \tag{A.66}$$

$$\rho_2 = \rho_2^{trade} \tag{A.67}$$

$$\rho_3 = \rho_1^{UIP} \tag{A.68}$$

$$\rho_4 = \rho_1^{z} \tag{A.69}$$

$$\rho_5 = \rho_2^{z} \tag{A.70}$$

and similarly for the variances σ_i. Although we generally assume shock processes to be uncorrelated, we do allow for this possibility in the following. The correlations coefficients between two shocks are denote by ρ_{ij}.

From the decision rule for $\tilde{B}_{1,t}$ in Equation (A.59), the unconditional variance of $\tilde{B}_{1,t}$ is

$$E_t\left(\tilde{B}_{1,t}^2\right) = \sum_i \sum_j \left\{ \frac{\beta_i \beta_j}{1-\beta_b^2} \frac{1+\rho_i\beta_b}{1-\rho_i\beta_b} \frac{\rho_{ij}\sigma_i\sigma_j}{1-\rho_i\rho_j} \right\} \tag{A.71}$$

where i and $j \in \{1,2,3,4,5\}$. The covariance between two exogenous shocks is given by

$$E_t\left(\xi_{i,t}\xi_{j,t}\right) = \frac{\rho_{ij}\sigma_i\sigma_j}{1-\rho_i\rho_j} \tag{A.72}$$

and the covariance between $\tilde{B}_{1,t}$ and shock $\xi_{i,t}$ is given by

$$E_t\left(\xi_{i,t}\tilde{B}_{1,t}\right) = \sum_j \left\{ \frac{\beta_j}{1-\rho_i\beta_b} \frac{\rho_{ij}\sigma_i\sigma_j}{1-\rho_i\rho_j} \right\}. \tag{A.73}$$

Let the coefficients in the decision rules for variables x_t and y_t be denoted by γ and α, respectively. The variance/covariance of these two variables (expressed in deviations from the steady state) is

$$E_t\left(x_t y_t\right) = \sum_i \sum_j \left\{ \left(\gamma_i\alpha_j + \Omega_i\beta_j\right) \frac{\rho_{ij}\sigma_i\sigma_j}{1-\rho_i\rho_j} \right\} \tag{A.74}$$

where

$$\Omega_i = \left(\gamma_i\alpha_b + \gamma_b\alpha_i\right)\frac{\rho_i}{1-\rho_i\beta_b} + \gamma_b\alpha_b\frac{\beta_i}{1-\beta_b^2}\frac{1+\rho_i\beta_b}{1-\rho_i\beta_b}. \tag{A.75}$$

Here the coefficients β_i or β_b are those in the decision rules of the NFA position.

Similarly, to compute the moments for $\Delta x_{t,t-1} = x_t - x_{t-1}$ and $\Delta y_{t,t-1} = y_t - y_{t-1}$ (variables expressed in growth rates), it is

$$E_t\left(\Delta x_{t,t-1}\Delta y_{t,t-1}\right) = \sum_i \sum_j \left\{\left(d\gamma_i d\alpha_j + \gamma_i \alpha_j \left(1-\rho_i\rho_j\right) + \Gamma_i \beta_j\right) \frac{\rho_{ij}\sigma_i\sigma_j}{1-\rho_i\rho_j}\right\} \tag{A.76}$$

where the coefficients for the decision rules $\Delta x_{t,t-1}$ and $\Delta y_{t,t-1}$ relate to those of the original rules associated with x_t and y_t as follows

$$\Gamma_i = \left(d\gamma_i d\alpha_b + d\gamma_b d\alpha_i\right)\frac{\rho_i}{1-\rho_i\beta_b} + d\gamma_b d\alpha_b \frac{\beta_i}{1-\beta_b^2}\frac{1+\rho_i\beta_b}{1-\rho_i\beta_b} \tag{A.77}$$

$$d\gamma_i = \gamma_i\left(\rho_i - 1\right) + \gamma_b\beta_i \tag{A.78}$$

$$d\gamma_b = \gamma_b\left(\beta_b - 1\right) = \chi\beta_b \tag{A.79}$$

$$d\alpha_i = \alpha_i\left(\rho_i - 1\right) + \alpha_b\beta_i \tag{A.80}$$

$$d\alpha_b = \alpha_b\left(\beta_b - 1\right). \tag{A.81}$$

B Appendix: Proofs of Theorems

B.1 Proof of Theorem 1 and Corollary 1

Theorem 1 *A trade rebalancing shock that improves the home country's terms of trade (appreciates the real exchange rate), $\xi_{1,t}^{trade} > 0$ and/or $\xi_{2,t}^{trade} < 0$, is associated with an improvement of the trade balance. By contrast, a UIP shock that improves the terms of trade (appreciates the real exchange rate), $\xi_{1,t}^{UIP} > 0$, is associated with a deterioration of the trade balance. If financial markets provide less risk sharing due to higher intermediation costs, i.e., χ assumes a higher value, the terms of trade are more (less) sensitive to the trade rebalancing (UIP) shock, and the trade balance is less sensitive to both shocks.*

Proof. We split the proof into two parts.

Claim 1: The home country's terms of trade improve (i.e., the real exchange rate appreciates) after a positive rebalancing shock towards the home country's good, $\xi_{1,t}^{trade} > 0$ and/or $\xi_{2,t}^{trade} < 0$, and a positive UIP shock $\xi_{1,t}^{UIP} > 0$. The magnitude of the terms of trade response to a given-sized shock is increasing in the value of χ for rebalancing shocks, but decreasing for the UIP shock.

Consider the decision rules for the terms of trade computed in Appendix A.4, where

$$\hat{\delta}_{1,t} = \gamma_1 \xi_{1,t}^{trade} + \gamma_2 \xi_{2,t}^{trade} + \gamma_3 \xi_{1,t}^{UIP} + \gamma_4 z_{1,t} + \gamma_5 z_{2,t} + \gamma_b \tilde{B}_{1,t-1}. \tag{B.1}$$

The coefficients of interest rate are γ_1 and γ_3, repeated here for convenience:

$$\gamma_1 = -\frac{\tilde{\gamma}_b \frac{1}{\varpi} \frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1 - \rho_1^{trade}\right)} < 0 \tag{B.2}$$

$$\gamma_3 = -\frac{1}{\tilde{\gamma}_b + (1 - \rho_1^{UIP})} < 0 \tag{B.3}$$

where $\tilde{\gamma}_b = -\frac{\gamma_b - \chi}{\beta}\varpi$.

Recall that γ_b is negative and decreasing in χ, i.e., $\gamma_b < 0$ and $\frac{\partial \gamma_b}{\partial \chi} < 0$ for $\beta > 0$. For the coefficient $\tilde{\gamma}_b$ we therefore obtain $\frac{\partial \tilde{\gamma}_b}{\partial \chi} = \frac{\varpi}{\beta}\left(1 - \frac{\partial \gamma_b}{\partial \chi}\right) > 0$. Hence, the derivatives of the coefficients γ_1 and γ_3 with respect to χ are

$$\frac{\partial \gamma_1}{\partial \chi} = -\frac{\frac{1}{\varpi} \frac{\omega_1^c}{1-\omega_1^c} \left(1 - \rho_1^{trade}\right)}{\left(\tilde{\gamma}_b + \left(1 - \rho_1^{trade}\right)\right)^2} \frac{\partial \tilde{\gamma}_b}{\partial \chi} < 0 \tag{B.4}$$

$$\frac{\partial \gamma_3}{\partial \chi} = \frac{1}{\left(\tilde{\gamma}_b + (1-\rho_1^{UIP})\right)^2} \frac{\partial \tilde{\gamma}_b}{\partial \chi} > 0. \tag{B.5}$$

Equations (B.2) - (B.5) establish Claim 1.

Claim 2: The trade and UIP shocks move the terms of trade (the real exchange rate) in the same direction, but move the trade balance in opposite directions. The higher the bond price elasticity to the NFA position, χ, the smaller are the effects on the trade balance of given-sized shocks. The effects on the terms of trade is larger (smaller) for the trade (UIP) shock, the larger the value of χ.

Consider the decision rules for the trade balance computed in Appendix A.4, where

$$\tilde{T}_{1,t} = \alpha_1 \xi_{1,t}^{trade} + \alpha_2 \xi_{2,t}^{trade} + \alpha_3 \xi_{1,t}^{UIP} + \alpha_4 z_{1,t} + \alpha_5 z_{2,t} + \alpha_b \tilde{B}_{1,t-1}. \tag{B.6}$$

The coefficients of interest rate are α_1 and α_3, repeated here for convenience:

$$\alpha_1 = \frac{\left(1-\rho_1^{trade}\right) \frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_1^{trade}\right)} > 0 \tag{B.7}$$

$$\alpha_3 = -\frac{\varpi}{\tilde{\gamma}_b + (1-\rho_1^{UIP})} < 0. \tag{B.8}$$

The derivatives of the coefficients α_1 and α_3 with respect to χ are

$$\frac{\partial \alpha_1}{\partial \chi} = -\frac{\left(1-\rho_1^{trade}\right) \frac{\omega_1^c}{1-\omega_1^c}}{\left(\tilde{\gamma}_b + \left(1-\rho_1^{trade}\right)\right)^2} \frac{\partial \tilde{\gamma}_b}{\partial \chi} < 0 \tag{B.9}$$

$$\frac{\partial \alpha_3}{\partial \chi} = \frac{\varpi}{\left(\tilde{\gamma}_b + (1-\rho_1^{UIP})\right)^2} \frac{\partial \tilde{\gamma}_b}{\partial \chi} > 0. \tag{B.10}$$

Equations (B.7) - (B.10) establish Claim 2.

The theorem follows directly from the two claims. ■

These features of the decision rules are also reflected in the unconditional moments of the trade balance and the terms of trade. Abstracting from technology shocks to simplify the exposition, the following corollary applies.

Corollary 1 *The UIP shock induces a positive covariance between the growth rate of the terms of trade and the growth rate of the trade balance. The trade rebalancing shock induces a negative covariance between the two growth rates. When both shocks are present in the model, the overall covariance is determined by the extent of costly financial intermediation as measured by* χ.

Proof. The proof assumes $\xi_{2,t}^{trade} = 0$ for ease of notation. Given the symmetry of our setting—the trade rebalancing shocks enter as the difference $\xi_{1,t}^{trade} - \xi_{2,t}^{trade}$ in the system (A.29)-(A.31)—this assumption does not impact the generality of the proof. The covariance between the growth rate of the terms of trade and the growth rate of the trade balance is

$$\begin{aligned} cov\left(\Delta\hat{\delta}_{1,t}, \Delta\tilde{T}_{1,t}\right) &= cov\left(\Delta\hat{\delta}_{1,t}, \frac{\omega_1^c}{1-\omega_1^c}\Delta\xi_{1,t}^{trade} + \varpi\Delta\hat{\delta}_{1,t}\right) \\ &= \frac{\omega_1^c}{1-\omega_1^c} cov\left(\Delta\hat{\delta}_{1,t}, \Delta\xi_{1,t}^{trade}\right) + \varpi var\left(\Delta\hat{\delta}_{1,t}\right) \end{aligned} \tag{B.11}$$

where, using Equation (A.76),

$$\begin{aligned} cov\left(\Delta\hat{\delta}_{1,t}, \Delta\xi_{1,t}^{trade}\right) &= -\left(1-\rho_1^{trade}\right)\left[d\gamma_1 - \left(1+\rho_1^{trade}\right)\gamma_1 + \gamma_b \frac{\rho_1^{trade}\beta_1}{1-\rho_1^{trade}\beta_b}\right] var\left(\xi_{1,t}^{trade}\right) \\ &= \left[\left(1-\left(\rho_1^{trade}\right)^2\right)\gamma_1 - \frac{\left(1-\rho_1^{trade}\right)\chi\beta_1}{1-\rho_1^{trade}\beta_b}\right] var\left(\xi_{1,t}^{trade}\right) \\ &= -\chi\beta_1\left[\frac{1+\rho_1^{trade}}{1-\beta_b} + \frac{1-\rho_1^{trade}}{1-\rho_1^{trade}\beta_b}\right] var\left(\xi_{1,t}^{trade}\right) < 0 \end{aligned} \tag{B.12}$$

and

$$\begin{aligned} var\left(\Delta\hat{\delta}_{1,t}\right) &= \left[d\gamma_1^2 + 2d\gamma_1 d\gamma_b \frac{\rho_1^{trade}\beta_1}{1-\rho_1^{trade}\beta_b} + d\gamma_b^2\beta_1^2 \frac{1}{1-\beta_b^2}\frac{1+\rho_1^{trade}\beta_b}{1-\rho_1^{trade}\beta_b}\right] var\left(\xi_{1,t}^{trade}\right) \\ &\quad + \left(1-\left(\rho_1^{trade}\right)^2\right)\gamma_1^2 var\left(\xi_{1,t}^{trade}\right) \\ &\quad + \left[d\gamma_3^2 + 2d\gamma_3 d\gamma_b \frac{\rho_1^{UIP}\beta_3}{1-\rho_1^{UIP}\beta_b} + d\gamma_b^2\beta_3^2 \frac{1}{1-\beta_b^2}\frac{1+\rho_1^{UIP}\beta_b}{1-\rho_1^{UIP}\beta_b}\right] var\left(\xi_{1,t}^{UIP}\right) \\ &\quad + \left(1-\left(\rho_1^{UIP}\right)^2\right)\gamma_3^2 var\left(\xi_{1,t}^{UIP}\right) \\ &= \frac{\chi^2\beta_1^2}{1-\beta_b^2}\frac{1+\rho_1^{trade}\beta_b}{1-\rho_1^{trade}\beta_b} var\left(\xi_{1,t}^{trade}\right) + \gamma_1^2\left(1-\left(\rho_1^{trade}\right)^2\right) var\left(\xi_{1,t}^{trade}\right) \\ &\quad + \frac{\chi^2\beta_3^2}{1-\beta_b^2}\frac{1+\rho_1^{UIP}\beta_b}{1-\rho_1^{UIP}\beta_b} var\left(\xi_{1,t}^{UIP}\right) + \left(1+2\frac{\chi\beta_3}{1-\rho_1^{UIP}\beta_b}\right) var\left(\xi_{1,t}^{UIP}\right) \\ &\quad + \gamma_3^2\left(1-\left(\rho_1^{UIP}\right)^2\right) var\left(\xi_{1,t}^{UIP}\right) \\ &= var\left(\Delta\hat{\delta}_{1,t}|\xi_{1,t}^{trade}\right) + var\left(\Delta\hat{\delta}_{1,t}|\xi_{1,t}^{UIP}\right). \end{aligned} \tag{B.13}$$

The overall variance of the terms of trade is the sum of the terms of trade variance that is due to the rebalancing shock and the terms of trade variance that is due to the UIP shock.

The variance of the change in the trade balance is given by

$$var\left(\Delta\tilde{T}_{1,t}\right) = \left(\frac{\omega_1^c}{1-\omega_1^c}\right)^2 var\left(\Delta\xi_{1,t}^{trade}\right) + \varpi^2 var\left(\Delta\hat{\delta}_{1,t}\right) +2\left(\frac{\omega_1^c}{1-\omega_1^c}\right)\varpi cov\left(\Delta\hat{\delta}_{1,t}, \Delta\xi_{1,t}^{trade}\right). \quad \text{(B.14)}$$

To understand the comovement between the changes in the terms of trade and the trade balance, we consider UIP and rebalancing shocks in turns. If the model admits UIP shocks only, it is $\frac{\omega_1^c}{1-\omega_1^c} cov\left(\Delta\hat{\delta}_{1,t}, \Delta\xi_{1,t}^{trade}\right) = 0$ and

$$cov\left(\Delta\hat{\delta}_{1,t}, \Delta\tilde{T}_{1,t}|\xi_{1,t}^{UIP}\right) = \varpi var\left(\Delta\hat{\delta}_{1,t}|\xi_{1,t}^{UIP}\right) > 0. \quad \text{(B.15)}$$

Because $var\left(\Delta\tilde{T}_{1,t}\right) = \varpi^2 var\left(\Delta\hat{\delta}_{1,t}|\xi_{1,t}^{UIP}\right)$, the associated correlation coefficient is equal to 1.

If the model admits rebalancing shocks only,

$$\begin{aligned} cov\left(\Delta\hat{\delta}_{1,t}, \Delta\tilde{T}_{1,t}|\xi_{1,t}^{trade}\right) &= \Bigg[-\frac{\omega_1^c}{1-\omega_1^c}\frac{\left(1-\rho_1^{trade}\right)\chi\beta_1}{1-\rho_1^{trade}\beta_b} + \varpi\frac{\chi^2\beta_1^2}{1-\beta_b^2}\frac{1+\rho_1^{trade}\beta_b}{1-\rho_1^{trade}\beta_b} \\ &\quad + \frac{\omega_1^c}{1-\omega_1^c}\left(1-\rho_1^{trade}\right)\gamma_1 + \varpi\gamma_1^2\left(1-\left(\rho_1^{trade}\right)^2\right)\Bigg] var\left(\xi_{1,t}^{trade}\right) \\ &= -\frac{\omega_1^c}{1-\omega_1^c}\chi\beta_1\Bigg[\frac{\left(1-\rho_1^{trade}\right)^2}{1-\rho_1^{trade}\beta_b}\frac{\chi\beta_b + \frac{\beta}{\varpi}\left(1-\beta_b^2\right)}{\chi\frac{1+\beta_b}{1-\rho_1^{trade}} + \frac{\beta}{\varpi}\left(1-\beta_b^2\right)} \\ &\quad + \frac{1-\left(\rho_1^{trade}\right)^2}{1-\beta_b}\Bigg] var\left(\xi_{1,t}^{trade}\right) < 0. \end{aligned} \quad \text{(B.16)}$$

The negative covariance implies that the associated correlation coefficient is also negative (but larger than -1).

With the rebalancing shock inducing negative correlation between $\Delta\hat{\delta}_{1,t}$ and $\Delta\tilde{T}_{1,t}$ and the UIP shock inducing positive correlation, the covariance in a model with both shocks being active depends, among other parameters, on the extent of international risk sharing as governed by the value of χ. ■

B.2 Proof of Theorem 2 and Theorem 3

Theorem 2 *Abstracting from technology shocks, the ratio of the standard deviation of the real exchange rate, $\hat{q}_{1,t}$, and consumption, $\hat{C}_{1,t}$, is independent of the relative variances of the trade* *rebalancing and the UIP shocks,*

$$\frac{std\left(\hat{q}_{1,t}\right)}{std\left(\hat{C}_{1,t}\right)} = \frac{std\left(\Delta\hat{q}_{1,t}\right)}{var\left(\Delta\hat{C}_{1,t}\right)} = \frac{2\omega_1^c - 1}{1 - \omega_1^c}. \tag{B.17}$$

The correlation between relative consumption, $\hat{C}_{1,t} - \hat{C}_{2,t}$, and the real exchange rate is equal to minus one regardless of the relative variances of the trade rebalancing and the UIP shocks,

$$corr\left(\hat{C}_{1,t} - \hat{C}_{2,t}, \hat{q}_{1,t}\right) = -1. \tag{B.18}$$

Proof. Absent technology shocks the consumption-real-exchange-rate variance ratio is

$$\frac{std\left(\hat{q}_{1,t}\right)}{std\left(\hat{C}_{1,t}\right)} = \frac{\sqrt{\left(2\omega_1^c - 1\right)^2 var\left(\hat{\delta}_{1,t}\right)}}{\sqrt{\left(1 - \omega_1^c\right)^2 var\left(\hat{\delta}_{1,t}\right)}} = \frac{2\omega_1^c - 1}{1 - \omega_1^c}. \tag{B.19}$$

The same applies when expressing the variables in growth rates instead of levels.

For the correlation between relative consumption and the real exchange rate it is

$$corr\left(\hat{C}_{1,t} - \hat{C}_{2,t}, \hat{q}_{1,t}\right) = \frac{-2\left(1 - \omega_1^c\right)\left(2\omega_1^c - 1\right) E\left(\hat{\delta}_{1,t}^2\right)}{\sqrt{4(1 - \omega_1^c)^2 E\left(\hat{\delta}_{1,t}^2\right)}\sqrt{(2\omega_1^c - 1)^2 E\left(\hat{\delta}_{1,t}^2\right)}} = -1. \tag{B.20}$$

Neither the UIP shock nor the rebalancing shock enter directly into the equations determining the real exchange rate and consumption. Both shocks enter only indirectly through the terms of trade. The computed moments of interest do not depend on the relative variances of the rebalancing and the UIP shock. ■

Theorem 3 *Suppose the model admits only rebalancing and UIP shocks. In that case, the Fama coefficient is constant and negative independent of the degree of costly financial intermediation as measured by χ, as long as $\chi > 0$:*

$$\hat{\beta}^{Fama} = \frac{cov\left(E_t\Delta\hat{q}_{1,t+1}, r_{1,t} - r_{2,t}\right)}{var\left(r_{1,t} - r_{2,t}\right)} = -\frac{2\omega_1^c - 1}{2\left(1 - \omega_1^c\right)} = 1 - \frac{1}{2\left(1 - \omega_1^c\right)} < 0 \tag{B.21}$$

for $\omega_1^c > \frac{1}{2}$.

Online Appendix

Proof. The proof assumes $\xi_{2,t}^{trade}=0$ for ease of notation. Given the symmetry of our setting—the trade rebalancing shocks enter as the difference $\xi_{1,t}^{trade}-\xi_{2,t}^{trade}$ in the system (A.29)-(A.31)—this assumption does not impact the generality of the proof. We assume that rebalancing shocks and UIP shocks are the only shocks in the model. All shocks are uncorrelated with each other. First, note that using the UIP condition, Equation (A.39), it is

$$\begin{aligned} cov\left(\Delta\hat{q}_{1,t+1},r_{1,t}-r_{2,t}\right) &= var\left(r_{1,t}-r_{2,t}\right)+\chi cov\left(\Delta\hat{q}_{1,t+1},\tilde{B}_{1,t}\right) \\ &\quad +cov\left(\Delta\hat{q}_{1,t+1},\xi_{1,t}^{UIP}\right)-var\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right) \end{aligned}$$

which, after applying the relationship $\hat{q}_{1,t}=\left(2\omega_1^c-1\right)\hat{\delta}_{1,t}$, implies that the Fama coefficient can be stated as

$$\begin{aligned} \hat{\beta}^{Fama} &= 1+\frac{\left(2\omega_1^c-1\right)}{var\left(r_{1,t}-r_{2,t}\right)}\left[\chi cov\left(\Delta\hat{\delta}_{1,t+1},\tilde{B}_{1,t}\right)+cov\left(\Delta\hat{\delta}_{1,t+1},\xi_{1,t}^{UIP}\right)\right] \\ &\quad -\frac{1}{var\left(r_{1,t}-r_{2,t}\right)}\left[\chi^2 var\left(\tilde{B}_{1,t}\right)+2\chi cov\left(\tilde{B}_{1,t},\xi_{1,t}^{UIP}\right)+var\left(\xi_{1,t}^{UIP}\right)\right]. \end{aligned} \tag{B.22}$$

The decision rules presented in Appendix A.4, imply that the terms entering Equation (B.22) can be expressed solely in terms of the underlying exogenous shocks:

1. $\chi cov\left(\Delta\hat{\delta}_{1,t+1},\tilde{B}_{1,t}\right)$:

$$\begin{aligned} \chi cov\left(\Delta\hat{\delta}_{1,t+1},\tilde{B}_{1,t}\right) &= \chi E\left(\left[d\gamma_1\xi_{1,t}^{trade}+d\gamma_3\xi_{1,t}^{UIP}+d\gamma_b\tilde{B}_{1,t-1}\right]\tilde{B}_{1,t}\right) \\ &= \chi^2\beta_1\left(1+\rho_1^{trade}\beta_b\right)E\left(\xi_{1,t}^{trade}\tilde{B}_{1,t}\right)+\chi\left[1+\chi\beta_3\left(1+\rho_1^{UIP}\beta_b\right)\right]E\left(\xi_{1,t}^{UIP}\tilde{B}_{1,t}\right)+\chi^2\beta_b^2E\left(\tilde{B}_{1,t}^2\right) \\ &= \chi^2\frac{\beta_1^2}{1-\beta_b^2}\frac{1+\rho_1^{trade}\beta_b}{1-\rho_1^{trade}\beta_b}var\left(\xi_{1,t}^{trade}\right)+\chi\left[\chi\frac{\beta_3^2}{1-\beta_b^2}\frac{1+\rho_1^{UIP}\beta_b}{1-\rho_1^{UIP}\beta_b}+\frac{\beta_3}{1-\rho_1^{UIP}\beta_b}\right]var\left(\xi_{1,t}^{UIP}\right) \end{aligned}$$

2. $cov\left(\Delta\hat{\delta}_{1,t+1},\xi_{1,t}^{UIP}\right)$:

$$\begin{aligned} cov\left(\Delta\hat{\delta}_{1,t+1},\xi_{1,t}^{UIP}\right) &= E\left(\left[d\gamma_1\xi_{1,t}^{trade}+d\gamma_3\xi_{1,t}^{UIP}+d\gamma_b\tilde{B}_{1,t-1}\right]\xi_{1,t}^{UIP}\right) \\ &= \left(1+\frac{\chi\beta_3}{1-\rho_1^{UIP}\beta_b}\right)var\left(\xi_{1,t}^{UIP}\right) \end{aligned}$$

3. $\chi^2 var\left(\tilde{B}_{1,t}\right)+2\chi cov\left(\tilde{B}_{1,t},\xi_{1,t}^{UIP}\right)+var\left(\xi_{1,t}^{UIP}\right)$:

$$\chi^2 var\left(\tilde{B}_{1,t}\right)+2\chi cov\left(\tilde{B}_{1,t},\xi_{1,t}^{UIP}\right)+var\left(\xi_{1,t}^{UIP}\right)=\chi^2\frac{\beta_1^2}{1-\beta_b^2}\frac{1+\rho_1^{trade}\beta_b}{1-\rho_1^{trade}\beta_b}var\left(\xi_{1,t}^{trade}\right)$$

$$+\left[\chi^2\frac{\beta_3^2}{1-\beta_b^2}\frac{1+\rho_1^{UIP}\beta_b}{1-\rho_1^{UIP}\beta_b}+\left(1+\frac{2\chi\beta_3}{1-\rho_1^{UIP}\beta_b}\right)\right]var\left(\xi_{1,t}^{UIP}\right).$$

Note that the last term equals the sum of the first two terms

$$\Sigma_{\Delta\hat{\delta}_{1,t+1},\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)}=\chi cov\left(\Delta\hat{\delta}_{1,t+1},\tilde{B}_{1,t}\right)+cov\left(\Delta\hat{\delta}_{1,t+1},\xi_{1,t}^{UIP}\right). \tag{B.23}$$

Before turning to the variance of the interest rate differential, $r_{1,t}-r_{2,t}$, we first establish that the variance of the growth rate of the terms of trade, $\Delta\hat{\delta}_{1,t+1}$, is

$$\begin{aligned}
var\left(\Delta\hat{\delta}_{1,t+1}\right) &= E\left(\left[d\gamma_1\xi_{1,t}^{trade}+d\gamma_3\xi_{1,t}^{UIP}+d\gamma_b\tilde{B}_{1,t-1}\right]^2\right)\\
&= \frac{\chi^2\beta_1^2}{1-\beta_b^2}\frac{1+\rho_1^{trade}\beta_b}{1-\rho_1^{trade}\beta_b}var\left(\xi_{1,t}^{trade}\right)\\
&\quad+\frac{\chi^2\beta_3^2}{1-\beta_b^2}\frac{1+\rho_1^{UIP}\beta_b}{1-\rho_1^{UIP}\beta_b}var\left(\xi_{1,t}^{UIP}\right)\\
&\quad+\left(1+2\frac{\chi\beta_3}{1-\rho_1^{UIP}\beta_b}\right)var\left(\xi_{1,t}^{UIP}\right)\\
&= \Sigma_{\Delta\hat{\delta}_{1,t+1},\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)}.
\end{aligned} \tag{B.24}$$

Finally, we obtain for the variance of the interest rate differential, $r_{1,t}-r_{2,t}$, that

$$\begin{aligned}
var\left(r_{1,t}-r_{2,t}\right) &= var\left(\Delta\hat{q}_{1,t+1}\right)-2cov\left(\Delta\hat{q}_{1,t+1},\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)\\
&\quad+var\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)\\
&= \left(2\omega_1^c-1\right)^2 var\left(\Delta\hat{\delta}_{1,t+1}\right)-\left(4\omega_1^c-3\right)\Sigma_{\Delta\hat{\delta}_{1,t},\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)}\\
&= 4\left(1-\omega_1^c\right)^2\Sigma_{\Delta\hat{\delta}_{1,t},\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)}.
\end{aligned} \tag{B.25}$$

Applying these findings in Equation (B.22), the Fama coefficient reduces to

$$\hat{\beta}^{Fama} = -\frac{2\omega_1^c-1}{2\left(1-\omega_1^c\right)}=1-\frac{1}{2\left(1-\omega_1^c\right)}. \tag{B.26}$$

■

C Appendix: Model Extensions and Sensitivity

This section presents details on some model extensions:

1. portfolio adjustment costs and borrowing constraints,
2. tariffs and iceberg trade cost,
3. discount factor shocks,
4. generalized labour supply,
5. endogenous discounting.

C.1 Portfolio Adjustment Costs and Borrowing Constraints

Costly financial intermediation is one of many ways to restrict international borrowing and lending in international macro models and, in addition, obtain endogenous UIP deviations. In this section we lay out two alternative approaches and show how they relate to our modeling choices in the main text.

Consider the household budget constraint

$$P^c_{1,t+j}C_{1,t+j} + P^b_{1,t+j}B_{1,t+j} + J_{1,t+j} = W_{1,t+j}L_{1,t+j} + B_{1,t-1+j} \tag{C.1}$$

where the term $J_{1,t+j}$ stands for the mechanism that limits the ability of households to issue/hold debt. In the case of costly financial intermediation used in the main text it is

$$J_{1,t+j} = \left(\frac{1}{\phi^b_{1,t+j}} - 1\right) P^b_{1,t+j}B_{1,t+j} \tag{C.2}$$

where $\phi^b_{1,t+j}$ is a function of bonds outstanding.

Here we offer two alternative approaches:

1. a quadratic portfolio adjustment cost, as in Fukui, Nakamura, and Steinsson (2023) or Guo, Ottonello, and Perez (2023), with

$$J_{1,t+j} = \frac{1}{2}\tau\left(\frac{B_{1,t+j}}{P^d_{1,t+j}M^*_{2,t+j}}\right)^2 P^d_{1,t+j}M^*_{2,t+j} \tag{C.3}$$

where here the costs are measured by household bond holdings relative to the aggregate value of exports (not the individual household's choice), $P^d_{1,t+j}M^*_{2,t+j}$;

2. a borrowing constraint on debt, as in de Groot, Durdu, and Mendoza (2023), with

$$J_{1,t+j} = -\mu_{1,t+j}\left(B_{1,t+j} + \bar{B}\right) \tag{C.4}$$

where household assets cannot fall below the adhoc level $-\bar{B}$, i.e., $B_{1,t+j} > -\bar{B}$. The borrowing constraint is set to be sufficiently tight so that it binds occasionally. Denoting the Lagrange multiplier on the time-t budget constraint by $\beta^t\lambda_{1,t+j}$, the borrowing constraint can be absorbed into the budget constraint since $\beta^t\lambda_{1,t+j}\mu_{1,t+j}\left(B_{1,t+j} + \bar{B}\right) = 0$.

As for the case of costly financial intermediation, the portfolio adjustment cost affects the household's ability to smooth consumption because the level of asset holdings directly influences the price of debt. When asset holdings are high, higher portfolio costs discourage further accumulation of assets and limit risk sharing. In fact, absent the realization of additional shocks, households reduce their NFA position over time due to the cost which gives rise to endogenous deviations from UIP. To first order, portfolio adjustment costs and costly financial intermediation are observationally equivalent.

Borrowing constraints introduce a hard limit on the quantity of bonds in circulation. Once the constraint binds, no more bonds can be issued. The closer the quantity of outstanding debt is to the limit $\bar{B}$, the higher the shadow cost of borrowing. As a result, a wedge, that depends on the NFA position, emerges endogenously in the UIP condition similar to the case of costly finacial intermediation. However it is important to note, that the approach with borrowing constraints requires the use of global solution techniques to properly capture the emergence of this wedge.

C.2 Tariffs and Iceberg Trade Costs

In this section, we show that in the simple model the rebalancing shock is closely related to shocks to tariffs and trade costs. In detail, we distinguish between import tariffs, export subsidies, and iceberg trade costs.

We denote the import price of country 1 by $P^m_{1,t}$ and the producer price in country 2 by $P^d_{2,t}$. The different trading frictions affect international prices as follows:

- under iceberg trade costs a share $\tau_{1,t}^{ice}$ of the shipped good is lost in the shipping process, implying an import price $P_{1,t}^{m}$ to be

$$P_{1,t}^{m} = \frac{1}{1-\tau_{1,t}^{ice}} e_{1,t} P_{2,t}^{d}, \tag{C.5}$$

- under an import tariff $\tau_{1,t}^{m}$ the import price $P_{1,t}^{m}$ increases over the producer price according to

$$P_{1,t}^{m} = (1+\tau_{1,t}^{m}) e_{1,t} P_{2,t}^{d}, \tag{C.6}$$

- under an export subsidy $\tau_{2,t}^{x}$ the import price $P_{1,t}^{m}$ falls below the producer price according to

$$P_{1,t}^{m} = \left(1-\tau_{2,t}^{x}\right) e_{1,t} P_{2,t}^{d} = e_{1,t} \tilde{P}_{2,t}^{d}. \tag{C.7}$$

If all three elements are present, the following relationship applies between the import price of country 1 and the foreign production price of country 2

$$P_{1,t}^{m} = (1+\tau_{1,t}^{m}) \frac{1-\tau_{2,t}^{x}}{1-\tau_{1,t}^{ice}} e_{1,t} P_{2,t}^{d} = \frac{1+\tau_{1,t}^{m}}{1-\tau_{1,t}^{ice}} e_{1,t} \tilde{P}_{2,t}^{d}. \tag{C.8}$$

Similarly, import prices of country 2 and the foreign production price of country 1 are related via

$$P_{2,t}^{m} = (1+\tau_{2,t}^{m}) \frac{1-\tau_{1,t}^{x}}{1-\tau_{2,t}^{ice}} \frac{1}{e_{1,t}} P_{1,t}^{d} = \frac{1+\tau_{2,t}^{m}}{1-\tau_{2,t}^{ice}} \frac{1}{e_{1,t}} \tilde{P}_{1,t}^{d}. \tag{C.9}$$

Using Equations (A.3) and (A.4), the relative prices $\frac{P_{1,t}^{c}}{P_{1,t}^{d}}$ and $\frac{P_{2,t}^{c}}{P_{2,t}^{d}}$ are shown to be

$$\frac{P_{1,t}^{c}}{P_{1,t}^{d}} = \left[\omega_{1,t}^{c} + (1-\tilde{\omega}_{1,t}^{c}) \delta_{1,t}^{-\frac{1}{\rho^{c}}}\right]^{-\rho^{c}} = F_{1,t}^{-\rho^{c}} \tag{C.10}$$

$$\frac{P_{2,t}^{c}}{P_{2,t}^{d}} = \left[\omega_{2,t}^{c} + (1-\tilde{\omega}_{2,t}^{c}) \delta_{1,t}^{\frac{1}{\rho^{c}}}\right]^{-\rho^{c}} = F_{2,t}^{-\rho^{c}} \tag{C.11}$$

where

$$1-\tilde{\omega}_{1,t}^{c} = (1-\omega_{1,t}^{c}) \left((1+\tau_{1,t}^{m}) \frac{1-\tau_{2,t}^{x}}{1-\tau_{1,t}^{ice}}\right)^{-\frac{1}{\rho^{c}}} \tag{C.12}$$

$$1-\tilde{\omega}^c_{2,t} \quad = \quad \left(1-\omega^c_{2,t}\right)\left((1+\tau^m_{2,t})\frac{1-\tau^x_{1,t}}{1-\tau^{ice}_{2,t}}\frac{1}{\delta_{1,t}}\right)^{-\frac{1}{\rho^c}}. \tag{C.13}$$

In the presence of iceberg trade costs the market clearing condition is given by

$$Y_{1,t} = C^d_{1,t} + \frac{1}{1-\tau^{ice}_{2,t}}M_{2,t} \tag{C.14}$$

$$Y_{2,t} = C^d_{2,t} + \frac{1}{1-\tau^{ice}_{1,t}}M_{1,t} \tag{C.15}$$

where $M_{1,t}$ and $M_{2,t}$ denote the final consumption of imports (net of iceberg costs).

The government's net receipts from import tariffs and export subsidies amount to

$$U_{t,1} = \tau^m_{1,t}e_{1,t}\tilde{P}^d_{2,t}\frac{M_{1,t}}{1-\tau^{ice}_{1,t}} - \tau^x_{1,t}P^d_{1,t}\frac{M_{2,t}}{1-\tau^{ice}_{2,t}}. \tag{C.16}$$

From the consolidated budget constraint we obtain

$$P^d_{1,t}C^d_{1,t} + P^m_{1,t}M_{1,t} + \frac{P^b_{1,t}}{\phi^b_{1,t}}B_{1,t} \quad = \quad P^d_{1,t}C^d_{1,t} + P^d_{1,t}\frac{M_{2,t}}{1-\tau^{ice}_{2,t}} + B_{1,t-1} + U_{t,1} \tag{C.17}$$

or

$$\frac{P^b_{1,t}B_{1,t}}{\phi^b_{1,t}} = T_{1,t} + B_{1,t-1} \tag{C.18}$$

with

$$T_{1,t} \quad = \quad P^d_{1,t}\frac{M_{2,t}}{1-\tau^{ice}_{2,t}} - P^m_{1,t}M_{1,t} + Z_{t,1} \tag{C.19}$$

$$\quad = \quad \frac{1-\tau^x_{1,t}}{1-\tau^{ice}_{2,t}}P^d_{1,t}M_{2,t} - \frac{1-\tau^x_{2,t}}{1-\tau^{ice}_{1,t}}e_{1,t}P^d_{2,t}M_{1,t}. \tag{C.20}$$

Similar to the main text we define

$$\tilde{T}_{1,t} \quad = \quad \frac{T_{1,t}}{\frac{1-\tau^x_{1,t}}{1-\tau^{ice}_{2,t}}P^d_{1,t}M_{2,t}} = 1 - \frac{1-\tau^x_{2,t}}{1-\tau^x_{1,t}}\frac{1-\tau^{ice}_{2,t}}{1-\tau^{ice}_{1,t}}\delta_{1,t}\frac{M_{1,t}}{M_{2,t}}. \tag{C.21}$$

As the first-order conditions for consumption are unchanged, the model dynamics can be summarized by the same three equations as before:

$$E_t\left\{\frac{P^c_{1,t}}{F^{\rho^c}_{1,t+1}P^c_{1,t+1}}\left[\phi^b_{1,t}\frac{exp(z_{1,t})}{exp(z_{1,t+1})} - \phi^b_{2,t}\frac{exp(z_{2,t})}{exp(z_{2,t+1})}\frac{\delta_{1,t}}{\delta_{1,t+1}}\right]\right\} = 0 \tag{C.22}$$

$$\frac{P^b_{1,t}\tilde{B}_{1,t}}{\phi^b_{1,t}} = \tilde{T}_{1,t} + \frac{\frac{1-\tau^x_{1,t-1}}{1-\tau^{ice}_{2,t-1}} P^d_{1,t-1} M_{2,t-1}}{\frac{1-\tau^x_{1,t}}{1-\tau^{ice}_{2,t}} P^d_{1,t} M_{2,t}} \tilde{B}_{1,t-1} \tag{C.23}$$

with

$$\begin{aligned}
\tilde{T}_{1,t} &= 1 - \frac{1-\tilde{\omega}^c_{1,t}}{1-\tilde{\omega}^c_{2,t}} \frac{1+\tau^m_{2,t}}{1+\tau^m_{1,t}} \left(\frac{F_{1,t}}{F_{2,t}}\right)^{-1} \frac{exp(z_{1,t})}{exp(z_{2,t})} \delta_{1,t}^{1-2\frac{1+\rho^c}{\rho^c}} \\
&= 1 - \frac{1+\tau^m_{2,t}}{1+\tau^m_{1,t}} \left(\frac{\frac{\omega^c_{1,t}}{1-\tilde{\omega}^c_{1,t}} + \delta_{1,t}^{-\frac{1}{\rho^c}}}{\frac{\omega^c_{2,t}}{1-\tilde{\omega}^c_{2,t}} + \delta_{1,t}^{\frac{1}{\rho^c}}} \right)^{-1} \frac{exp(z_{1,t})}{exp(z_{2,t})} \delta_{1,t}^{1-2\frac{1+\rho^c}{\rho^c}}.
\end{aligned} \tag{C.24}$$

C.2.1 Linearized Model

We assume a symmetric steady state with $\omega^c_1 = \omega^c_2 = \omega^c$, $\tau^i_1 = \tau^i_2 = \tau^i$ with $i \in \{m, x, ice\}$ and $\delta_1 = 1$, $\tilde{B}_1 = 0$. The dynamics around the steady state are approximated by the equations

$$(z_{1,t} - E_t z_{1,t+1}) - (z_{2,t} - E_t z_{2,t+1}) - \left(\hat{\delta}_{1,t} - E_t\hat{\delta}_{1,t+1}\right) = \chi\tilde{B}_{1,t} + \xi^{UIP}_{1,t} \tag{C.25}$$

$$\beta\tilde{B}_{1,t} = \tilde{T}_{1,t} + \tilde{B}_{1,t-1} \tag{C.26}$$

$$\tilde{T}_{1,t} = \frac{\bar{\omega}^c}{1-\bar{\omega}^c}\left(\xi^{trade}_{1,t} - \xi^{trade}_{2,t}\right) - (z_{1,t} - z_{2,t}) + \bar{\varpi}\hat{\delta}_{1,t} \tag{C.27}$$

where we now define $\bar{\varpi} = 1 + 2\frac{\omega^c}{\rho^c}\frac{1}{1-(\tilde{\omega}^c-\omega^c)}$. The trade shock reflects movements in the four underlying shocks to preferences, import tariffs, export subsidies, and transportation costs:

$$\begin{aligned}
\frac{\bar{\omega}^c}{1-\bar{\omega}^c}\xi^{trade}_{1,t} = \; & \frac{1}{1-(\tilde{\omega}^c-\omega^c)}\frac{\omega^c}{1-\omega^c}\xi^c_{1,t} \\
& + \left(1 + \frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\right)\frac{\tau^m}{1-\tau^m}\xi^m_{1,t} \\
& - \frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tau^x}{1-\tau^x}\xi^x_{2,t} \\
& + \frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tau^{ice}}{1-\tau^{ice}}\xi^{ice}_{1,t}.
\end{aligned} \tag{C.28}$$

The solution for the terms of trade, the trade balance, and the NFA position in the model with differentiated trade shocks is isomorphic with the solution to the model with a single trade rebalancing shock.

C.3 Discount Factor Shocks

We show that the (relative) discount factor shock introduced in Kekre and Lenel (2024) is largely isomorph with the UIP shock in our settings, in the sense that the UIP and discount factor shocks induce identical movements in the terms of trade, the real exchange rate, the trade balance, and the NFA, despite inducing different real interest rate dynamics.

We denote the shock to the discount factor in country i in period t by $\xi_{i,t}^{disc}$. The price of the internationally traded bond for households in country 1 and 2 then satisfies

$$P_{1,t}^{b} = \phi_{1,t}^{b} \beta \exp\left(\xi_{1,t}^{disc}\right) E_t \left\{ \frac{C_{1,t}}{C_{1,t+1}} \frac{P_{1,t}^{c}}{P_{1,t+1}^{c}} \right\} \tag{C.29}$$

$$P_{1,t}^{b} = \phi_{2,t}^{b} \beta \exp\left(\xi_{2,t}^{disc}\right) E_t \left\{ \frac{C_{2,t}}{C_{2,t+1}} \frac{P_{2,t}^{c}}{P_{2,t+1}^{c}} \frac{e_{1,t}}{e_{1,t+1}} \right\}. \tag{C.30}$$

Following the same steps as in Appendix A.2, linearization of the dynamic system around its symmetric deterministic steady state with $\omega_1^c = \omega_2^c$ and balanced trade, i.e., $\tilde{T}_1 = 0$, $\delta_1 = 1$, $B_1 = 0$, yields the linear system

$$E_t \hat{\delta}_{1,t+1} - \hat{\delta}_{1,t} = \chi \tilde{B}_{1,t} + \xi_{1,t}^{UIP} + \left(\xi_{2,t}^{disc} - \xi_{1,t}^{disc}\right)$$

$$\beta \tilde{B}_{1,t} = \tilde{T}_{1,t} + \tilde{B}_{1,t-1} \tag{C.31}$$

$$\tilde{T}_{1,t} = \frac{\omega_1^c}{1-\omega_1^c} \left(\xi_{1,t}^{trade} - \xi_{2,t}^{trade}\right) + \varpi \hat{\delta}_{1,t}, \tag{C.32}$$

where $\varpi = 1 + 2\frac{\omega_1^c}{\rho^c}$. The terms trade, $\hat{\delta}_{1,t}$, are measured in log-deviation from the steady state, $\tilde{T}_{1,t}$ and $\tilde{B}_{1,t}$ are in absolute deviations from 0. If the term $\chi > 0$, the NFA dynamics are stationary. We abstract from technology shocks to simplify the exposition.

The relative discount factor shock, $\xi_{2,t}^{disc} - \xi_{1,t}^{disc}$, enters in Equations (C.31)-(C.32) in the same way as the UIP shock. Thus, conditional on the same realization, the two shocks induce the same paths of the terms of trade, the trade balance, and the NFA. The real exchange rate will also follow the same path given its proportionality to the terms of trade. Note that if the steady state value of the NFA is not zero, the presence of valuation effects may lead to small differences via Equation (C.31).

However, the discount factor shocks lead to very different real interest rate paths than the UIP shock. Defining the real interest rates for country i as the real return on a bond that pays one unit of consumption in country i regardless of the state of the world, it is $r_{i,t} = -\xi_{i,t}^{disc} + \hat{C}_{i,t+1} - \hat{C}_{i,t}$.

Applying this information and the linearized equations for consumption (A.32) and (A.33), and the real exchange rate (A.34), we can rewrite the UIP condition as

$$r_{1,t} - r_{2,t} = E_t\left(\hat{q}_{1,t+1} - \hat{q}_{1,t}\right) - \chi \tilde{B}_{1,t} - \xi_{1,t}^{UIP}. \tag{C.33}$$

In contrast to the UIP shock, the relative discount factor shock is absorbed into the relative interest rate term. If the (relative) discount factor shock induces an appreciation of the home country's real exchange rate, relative consumption of the home country rises relative to the foreign country, but the trade balance deteriorates. As a result, Equation (C.33) implies that the (relative) discount factor shocks induce a positive interest rate gap, i.e., $r_{1,t} - r_{2,t} > 0$. This is in sharp contrast to the UIP shock, which induces a negative interest rate differential on impact because of the term $-\xi_{1,t}^{UIP}$ on the right hand side of Equation (C.33).

Based on data for G10 currencies, Kekre and Lenel (2024) argue that the *expected change* in the real exchange rate comoves negatively with the interest rate differential—the forward premium puzzle—whereas the *level* of the real exchange comoves positively with the interest rate differential—interpreted as UIP holding in the long-run. The former evidence favors the UIP shock, the latter discount factor shocks. When the model is required to match both features of the data, Kekre and Lenel (2024) conclude that discount factor shocks account for 75 perccent in the variance of the exchange rate between the dollar and a basket of G10 currencies whereas the UIP shock accounts for 25 percent. Because the UIP and the (relative) discount factor shocks have identical implications for the behavior of the trade balance vis-à-vis the exchange rate, as just shown, bringing the model in line with the data still requires trade rebalancing shocks and costly financial intermediation as in our baseline specification.

C.4 Generalized Labor Supply

We show that our finding regarding the importance of trade rebalancing shocks as the main driver of exchange rate movements applies under more general household preferences that allow for a less-than-fully elastic labor supply. In addition to deriving equilibrium conditions of the generalized model, we sketch the extension of Theorem 1 and Theorem 2 in this case.

Online Appendix

Let the intertemporal preferences of the household be

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln\left(C_{1,t+j}\right) - \frac{\nu_0}{1+\nu} L_{1,t+j}^{1+\nu} \right\}, \tag{C.34}$$

with $\nu \geq 0$. Focusing on the home country, this generalization only affects the first-order condition with respect to labor, yielding the consumption-labor trade-off

$$\nu_0 L_{1,t+j}^{\nu} C_{1,t} = \frac{P_{1,t}^d}{P_{1,t}^c} \frac{W_{1,t}}{P_{1,t}^d} = exp\left(z_{1,t}\right) F_{1,t}^{\rho_c}, \tag{C.35}$$

where

$$F_{1,t} \quad = \quad \omega_{1,t}^c + (1-\omega_{1,t}^c)\delta_{1,t}^{-\frac{1}{\rho^c}}. \tag{C.36}$$

Using the demand functions for goods 1 and 2, the market clearing conditions imply

$$Y_{1,t}^d \quad = \quad C_{1,t}^d + M_{2,t} = A_{1,t} C_{1,t} \tag{C.37}$$

$$Y_{2,t}^d \quad = \quad C_{2,t}^d + M_{1,t} = A_{2,t} C_{2,t} \tag{C.38}$$

where

$$A_{1,t} \quad = \quad \left[\omega_{1,t}^c + (1-\omega_{1,t}^c)\delta_{1,t}^{-\frac{1}{\rho^c}} \frac{1}{1-\tilde{T}_{1,t}} \right] F_{1,t}^{-(1+\rho^c)} \tag{C.39}$$

$$A_{2,t} \quad = \quad \left[\omega_{2,t}^c + (1-\omega_{2,t}^c)\delta_{1,t}^{\frac{1}{\rho^c}} \frac{1-\tilde{T}_{1,t}}{\delta_{1,t}} \right] F_{2,t}^{-(1+\rho^c)} \tag{C.40}$$

$$1-\tilde{T}_{1,t} \quad = \quad \frac{1-\omega_{1,t}^c}{1-\omega_{2,t}^c} \left(\frac{F_{1,t}}{F_{2,t}} \right)^{-(1+\rho^c)} \delta_{1,t}^{1-2\frac{1+\rho^c}{\rho^c}} \frac{C_{1,t}}{C_{2,t}}. \tag{C.41}$$

Using Equation (C.35) and the fact that $L_{1,t} = \frac{1}{\exp(z_{1,t})} Y_{1,t}^d$, we can express consumption as

$$C_{1,t} \quad = \quad \exp\left(z_{1,t}\right) \left(\frac{1}{\nu_0} F_{1,t}^{\rho^c} \left(A_{1,t}\right)^{-\nu} \right)^{\frac{1}{1+\nu}}. \tag{C.42}$$

Notice, that for $\nu > 0$ aggregate consumption depends on of the terms of trade (via $F_{1,t}$ and $A_{1,t}$) and on the trade balance (via $A_{1,t}$).

C.4.1 Linearized Model

From here on, we abstract from technology shocks, $z_{1,t}$ and $z_{2,t}$. Approximating Equation (C.42) around the deterministic steady state of the model yields

$$\hat{C}_{1,t} \quad = \quad -\left(1-\omega_1^c\right)\hat{\delta}_{1,t} - \frac{1}{2}\bar{\nu}\tilde{T}_{1,t}, \tag{C.43}$$

where $\bar{\nu} = 2\frac{\nu}{1+\nu}\left(1-\omega_1^c\right) > 0$ and $1-\bar{\nu} > 0$ for $\omega_1^c > 0.5$. With this expression in hand, linearizing the definition of the trade balance, the risk sharing condition, and the NFA accumulation condition delivers the equilibrium system

$$E_t\hat{\delta}_{1,t+1} - \hat{\delta}_{1,t} = \check{\chi}\tilde{B}_{1,t} + \frac{1}{1+\bar{\nu}\check{\varpi}}\xi_{1,t}^{UIP} + \frac{\bar{\nu}\left(1-\rho_1^{trade}\right)}{1+\bar{\nu}\check{\varpi}}\frac{\check{\omega}_1^c}{1-\check{\omega}_1^c}\left(\xi_{1,t}^{trade}-\xi_{2,t}^{trade}\right) \tag{C.44}$$

$$\beta\tilde{B}_{1,t} = \tilde{T}_{1,t} + \tilde{B}_{1,t-1} \tag{C.45}$$

$$\tilde{T}_{1,t} = \frac{\check{\omega}_1^c}{1-\check{\omega}_1^c}\left(\xi_{1,t}^{trade}-\xi_{2,t}^{trade}\right) + \check{\varpi}\hat{\delta}_{1,t} \tag{C.46}$$

with the parameter definitions

$$\check{\varpi} = \frac{\varpi}{1-\bar{\nu}} \qquad \check{\chi} = \frac{\chi}{1+\frac{\bar{\nu}}{1-\bar{\nu}}\varpi} \qquad \frac{\check{\omega}_1^c}{1-\check{\omega}_1^c} = \frac{\omega_1^c}{1-\omega_1^c}\frac{1}{1-\bar{\nu}}. \tag{C.47}$$

Other than redefinitions of the model parameters, the trade rebalancing shocks now enter directly into the risk sharing condition.

For completeness note, that production follows

$$\hat{Y}_{1,t} \quad = \quad \hat{C}_{1,t} + \left(1-\omega_1^c\right)\left(\tilde{T}_{1,t}+\hat{\delta}_{1,t}\right) = \frac{1}{1+\nu}(1-\omega_1^c)\tilde{T}_{1,t}. \tag{C.48}$$

Absent technology shocks, output is proportional to the trade balance. For given-sized trade surplus, the expansion in output is smaller for less elastic labor supply. Obviously, if the labor supply is fully inelastic, i.e., $\nu \approx \infty$, output does not adjust endogenously.

C.4.2 Decision Rules and Extension of Theorem 1

We obtain the decision rules in the model with less-than-fully elastic labor supply using the method of undetermined coefficients. The equilibrium terms of trade follow

$$\hat{\delta}_{1,t} = \gamma_1\xi_{1,t}^{trade} + \gamma_2\xi_{2,t}^{trade} + \gamma_3\xi_{1,t}^{UIP} + \gamma_b\tilde{B}_{1,t-1} \tag{C.49}$$

with the coefficients

$$\gamma_1: \qquad \gamma_1 = -\frac{\tilde{\gamma}_b + \frac{\bar{\nu}\breve{\varpi}}{1+\bar{\nu}\breve{\varpi}}\left(1-\rho_1^{trade}\right)}{\tilde{\gamma}_b + \left(1-\rho_1^{trade}\right)} \frac{1}{\breve{\varpi}} \frac{\breve{\omega}_1^c}{1-\breve{\omega}_1^c} < 0 \tag{C.50}$$

$$\gamma_2: \qquad \gamma_2 = \frac{\tilde{\gamma}_b + \frac{\bar{\nu}\breve{\varpi}}{1+\bar{\nu}\breve{\varpi}}\left(1-\rho_2^{trade}\right)}{\tilde{\gamma}_b + \left(1-\rho_2^{trade}\right)} \frac{1}{\breve{\varpi}} \frac{\breve{\omega}_1^c}{1-\breve{\omega}_1^c} > 0 \tag{C.51}$$

$$\gamma_3: \qquad \gamma_3 = -\frac{\frac{1}{1+\bar{\nu}\breve{\varpi}}}{\tilde{\gamma}_b + (1-\rho_1^{UIP})} < 0 \tag{C.52}$$

where $\tilde{\gamma}_b = -\frac{\gamma_b - \check{\chi}}{\beta}\breve{\varpi}$ and γ_b is given by

$$\gamma_b \quad = \quad \frac{-(1-\beta-\check{\chi}\breve{\varpi}) - \sqrt{(1-\beta-\check{\chi}\breve{\varpi})^2 + 4\check{\chi}\breve{\varpi}}}{2\breve{\varpi}}. \tag{C.53}$$

γ_b is the stable root associated with the quadratic equation

$$-\breve{\varpi}\gamma_b^2 + (\beta - 1 + \check{\chi}\breve{\varpi})\,\gamma_b + \check{\chi} = 0. \tag{C.54}$$

Similarly, the equilibrium trade balance follows

$$\tilde{T}_{1,t} \quad = \quad \alpha_1 \xi_{1,t}^{trade} + \alpha_2 \xi_{2,t}^{trade} + \alpha_3 \xi_{1,t}^{UIP} + \alpha_b \tilde{B}_{1,t-1} \tag{C.55}$$

with the coefficients

$$\alpha_1: \qquad \alpha_1 = \frac{\frac{1}{1+\bar{\nu}\breve{\varpi}}\left(1-\rho_1^{trade}\right)}{\tilde{\gamma}_b + \left(1-\rho_1^{trade}\right)} \frac{\breve{\omega}_1^c}{1-\breve{\omega}_1^c} > 0 \tag{C.56}$$

$$\alpha_2: \qquad \alpha_2 = -\frac{\frac{1}{1+\bar{\nu}\breve{\varpi}}\left(1-\rho_2^{trade}\right)}{\tilde{\gamma}_b + \left(1-\rho_2^{trade}\right)} \frac{\breve{\omega}_1^c}{1-\breve{\omega}_1^c} < 0 \tag{C.57}$$

$$\alpha_3: \qquad \alpha_3 = -\frac{\frac{1}{1+\bar{\nu}\breve{\varpi}}\breve{\varpi}}{\tilde{\gamma}_b + (1-\rho_1^{UIP})} < 0 \tag{C.58}$$

$$\alpha_b: \qquad \alpha_b = \varpi\gamma_b < 0. \tag{C.59}$$

The coefficients in the decision rules have the same signs for $\nu \neq 0$ as under our baseline specification with $\nu = 0$. In addition, the derivatives of the coefficients with regard to χ continue to have the same signs, with $\frac{\partial \tilde{\gamma}_b}{\partial \chi} > 0$, $\frac{\partial \gamma_1}{\partial \chi} < 0$, $\frac{\partial \gamma_3}{\partial \chi} > 0$, $\frac{\partial \alpha_1}{\partial \chi} < 0$, $\frac{\partial \alpha_3}{\partial \chi} > 0$. Thus, the theoretical predictions of Theorem 1 continue to apply.

C.4.3 Backus-Smith Puzzle and Theorem 2

It is central to our claim of trade rebalancing shocks being a key driver of the real exchange rate that trade rebalancing shocks satisfy the evidence in Backus and Smith (1993). While evident in our baseline model, it is not immediately obvious that relative consumption increases after a trade rebalancing shock towards the domestic good when the labor supply is less elastic.

Equation (C.43) and its counterpart for the foreign country imply that relative consumption relates to the terms of trade and the trade balance as follows

$$\hat{C}_{1,t} - \hat{C}_{2,t} \;=\; -2\left(1-\omega_1^c\right)\hat{\delta}_{1,t} - \bar{\nu}\tilde{T}_{1,t}. \tag{C.60}$$

Given that a trade rebalancing shock that improves the home country's terms of trade also improves the country's trade balance according to the extension of Theorem 1, relative consumption could fall if $\bar{\nu}$ assumes a sufficiently high value. Less mechanically, the trade rebalancing shock raises the home country's demand for its own good. Ideally, production of the home good expands. However, if the labor supply is inelastic, such an expansion can be quite costly. The additional term $-\bar{\nu}\tilde{T}_{1,t}$ captures this effect.

However, given the fact that Theorem 1 continues to apply, higher costs of financial intermediation increase the sensitivity of the terms of trade to the rebalancing shock (γ_1) and lower the sensitivity of the trade balance (α_1) to this shock. Thus, for a sufficiently high value of χ relative consumption of the home country increases regardless of the value of the labor supply elasticity. More formally, substituting out for the trade balance, we can express relative consumption as

$$\hat{C}_{1,t} - \hat{C}_{2,t} \;=\; \left\{-\left[2\left(1-\omega_1^c\right)+\bar{\nu}\breve{\varpi}\right]\gamma_1 - \bar{\nu}\frac{\breve{\omega}_1^c}{1-\breve{\omega}_1^c}\right\}\left(\xi_{1,t}^{trade}-\xi_{2,t}^{trade}\right) + \text{other terms.} \tag{C.61}$$

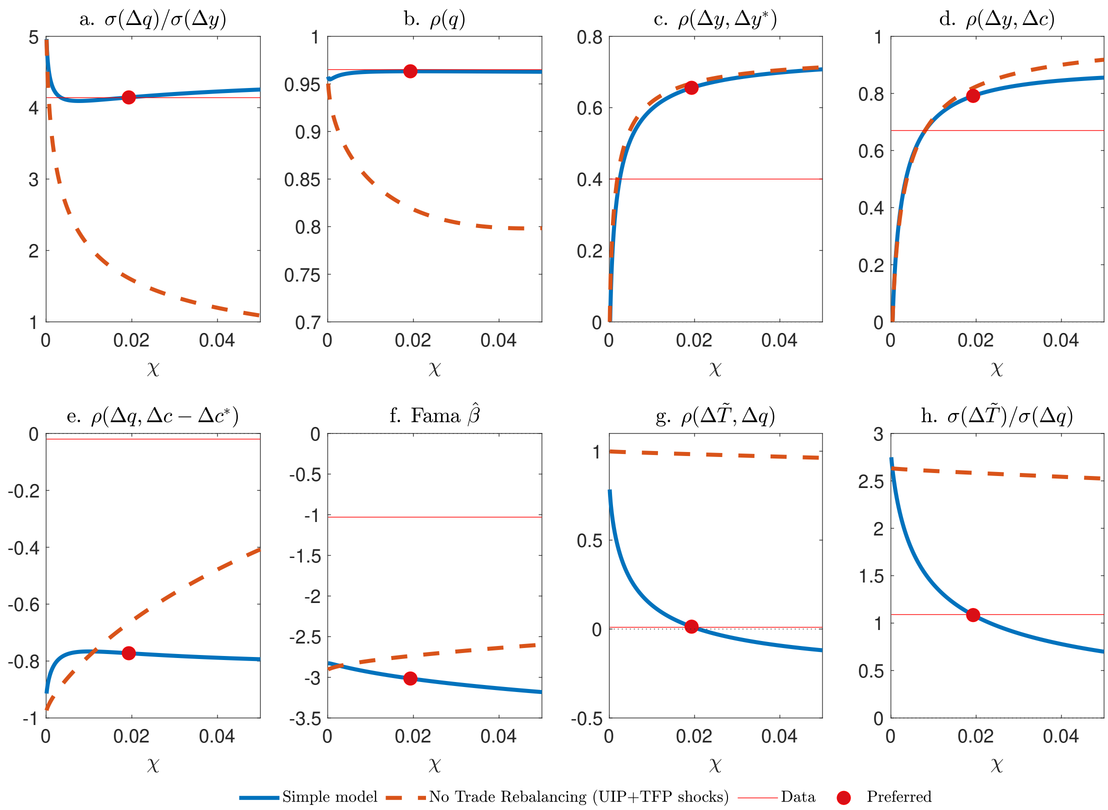
The term $-\left[2\left(1-\omega_1^c\right)+\bar{\nu}\breve{\varpi}\right]\gamma_1$ is positive and increases in χ, whereas the term $-\bar{\nu}\frac{\breve{\omega}_1^c}{1-\breve{\omega}_1^c}$ is negative and independent of χ. For the structural parameters in Table 1, a value of $\chi > 0.00125$ implies an increase in the home country's relative consumption after a trade rebalancing shock that appreciates the home country's real exchange rate for all $\nu \geq 0$.

C.4.4 Financial Integration, Exchange Rate Moments, and the Labor Supply

Finally, we repeat the exercise from Figure 3 for $\nu = 1$, shown in Figure A.1. The fit of the model to the data continues to be surprisingly good given the simplicity of the model. With

$\chi \approx 0.02$, the degree of costly financial intermediation is unchanged in our preferred calibration compared to the baseline model.

Figure A.1: Standard Labor Supply and Exchange Rate Moments



Notes: The blue line shows theoretical moments for different values of χ. The red horizontal line indicates the corresponding empirical moment in each panel. The red dot shows our preferred calibration. The parameters used for this exercise are: $\omega^c = 0.9$, $\rho_{\xi^{trade}} = \rho_{\xi^{UIP}} = \rho_z = 0.96$, $100\sigma_{\xi^{trade}} = 0.85$, $100\sigma_{\xi^{UIP}} = 0.57$, $100\sigma_z = 1$, $\rho(z_1, z_2) = 0.75$, $\nu = 1$.

C.5 Endogenous Discounting

Under endogenous discounting as in Uzawa (1968), the intertemporal preferences of the household in country 1 are given by

$$E_t \sum_{j=0}^{\infty} \Theta_{1,t+j+1} \left\{ \ln\left(C_{1,t+j}\right) - \frac{\nu_0}{1+\nu} L_{1,t+j}^{1+\nu} \right\}, \tag{C.62}$$

where

$$\Theta_{1,t+j+1} = \Psi_0 \left(1 + exp\left(\ln\left(C_{1,t+j}\right) - \frac{\nu_0}{1+\nu} L_{1,t+j}^{1+\nu}\right)\right)^{-\Psi} \Theta_{1,t+j}. \tag{C.63}$$

Ψ_0 and Ψ are chosen such that $\Psi_0\left(1+exp\left(\ln\left(C_1\right)-\frac{\nu_0}{1+\nu}L_1^{1+\nu}\right)\right)^{-\Psi}=\beta$ in the deterministic steady state. With this change in preferences and an analogous change for country 2, the risk sharing condition changes from (A.27) to

$$E_t\left\{\frac{P^c_{1,t}}{F^{\rho^c}_{1,t+1}P^c_{1,t+1}}\left[\frac{\Theta_{1,t+1}}{\Theta_{1,t}}\frac{exp(z_{1,t})}{exp(z_{1,t+1})}-\frac{\Theta_{2,t+1}}{\Theta_{2,t}}\frac{exp(z_{2,t})}{exp(z_{2,t+1})}\frac{\delta_{1,t}}{\delta_{1,t+1}}\right]\right\}=0. \tag{C.64}$$

With the equations for the trade balance and the evolution of the NFA position unchanged, the linear system that governs the equilibrium dynamics of $\hat{\delta}_{1,t}$, $\tilde{T}_{1,t}$, and $\tilde{B}_{1,t}$ is given by

$$\Delta\widehat{\Theta}_t-E_t\Delta z_{1,t+1}+E_t\Delta z_{2,t+1}+E_t\Delta\hat{\delta}_{1,t+1}+\bar{\nu}E_t\Delta\tilde{T}_{1,t+1}=\xi^{UIP}_{1,t} \tag{C.65}$$

$$\beta\tilde{B}_{1,t}=\tilde{T}_{1,t}+\tilde{B}_{1,t-1} \tag{C.66}$$

$$\left(1-\bar{\nu}\right)\tilde{T}_{1,t}=\frac{\omega^c_1}{1-\omega^c_1}\xi^{trade}_{1,t}-\frac{\omega^c_1}{1-\omega^c_1}\xi^{trade}_{2,t}-z_{1,t}+z_{2,t}+\varpi\hat{\delta}_{1,t} \tag{C.67}$$

where differences in endogenous discounting across countries result in the wedge

$$\begin{aligned}\Delta\widehat{\Theta}_t &= -\tilde{\Psi}\left(\hat{C}_{1,t}-\hat{L}_{1,t}\right)+\tilde{\Psi}\left(\hat{C}_{2,t}-\hat{L}_{2,t}\right)\\ &= -\tilde{\Psi}\left(z_{1,t}-z_{2,t}-2\left(1-\omega^c_1\right)\left(\hat{\delta}_{1,t}+\tilde{T}_{1,t}\right)\right),\end{aligned} \tag{C.68}$$

with $\tilde{\Psi}=\frac{exp\left(\ln(C)-\frac{\nu_0}{1+\nu}L^{1+\nu}\right)}{1+exp\left(\ln(C)-\frac{\nu_0}{1+\nu}L^{1+\nu}\right)}\Psi$.

We solve for the coefficients in the policy rule for the terms of trade using the method of undetermined coefficients

$$\hat{\delta}_{1,t}=\gamma^{end}_1\xi^{trade}_{1,t}+\gamma^{end}_2\xi^{trade}_{2,t}+\gamma^{end}_3\xi^{UIP}_{1,t}+\gamma^{end}_4 z_{1,t}+\gamma^{end}_5 z_{2,t}+\gamma^{end}_b\tilde{B}_{1,t-1} \tag{C.69}$$

with the parameters

$$\gamma^{end}_1:\qquad \gamma^{end}_1=-\frac{\frac{1-\beta}{\beta}+2\tilde{\Psi}\left(1-\omega^c_1\right)}{\varpi\left(\frac{1}{\beta}-\rho^{trade}_1\right)}\frac{\omega^c_1}{1-\omega^c_1}<0 \tag{C.70}$$

$$\gamma^{end}_2:\qquad \gamma^{end}_2=\frac{\frac{1-\beta}{\beta}+2\tilde{\Psi}\left(1-\omega^c_1\right)}{\varpi\left(\frac{1}{\beta}-\rho^{trade}_2\right)}\frac{\omega^c_1}{1-\omega^c_1}>0 \tag{C.71}$$

$$\gamma^{end}_3:\qquad \gamma^{end}_3=-\frac{1}{\frac{1}{\beta}-\rho^{UIP}_1}<0 \tag{C.72}$$

$$\gamma_4^{end}: \qquad \gamma_4^{end} = \frac{(1-\rho_1^z)-\tilde{\Psi}}{\left(\frac{1}{\beta}-\rho_1^z\right)} + \frac{\frac{1-\beta}{\beta}+2\tilde{\Psi}\left(1-\omega_1^c\right)}{\varpi\left(\frac{1}{\beta}-\rho_1^z\right)} \tag{C.73}$$

$$\gamma_5^{end}: \qquad \gamma_5^{end} = -\frac{(1-\rho_2^z)-\tilde{\Psi}}{\left(\frac{1}{\beta}-\rho_2^z\right)} - \frac{\frac{1-\beta}{\beta}+2\tilde{\Psi}\left(1-\omega_1^c\right)}{\varpi\left(\frac{1}{\beta}-\rho_2^z\right)} \tag{C.74}$$

$$\gamma_b^{end}: \qquad \gamma_b^{end} = -\frac{\beta}{\varpi}\left(\frac{1-\beta}{\beta}+2\tilde{\Psi}\left(1-\omega_1^c\right)(1+\varpi)\right) < 0. \tag{C.75}$$

The parameters γ_1^{end}, γ_2^{end}, γ_3^{end} have the same sign as under costly financial intermediation.

For completeness, we report the coefficients in the decision rule for the trade balance

$$\tilde{T}_{1,t} = \alpha_1^{end}\xi_{1,t}^{trade} + \alpha_2^{end}\xi_{2,t}^{trade} + \alpha_3^{end}\xi_{1,t}^{UIP} + \alpha_4^{end}z_{1,t} + \alpha_5^{end}z_{2,t} + \alpha_b^{end}\tilde{B}_{1,t-1} \tag{C.76}$$

which are given by

$$\alpha_1^{end}: \qquad \alpha_1^{end} = \left[1-\frac{\frac{1-\beta}{\beta}+2\tilde{\Psi}\left(1-\omega_1^c\right)}{\left(\frac{1}{\beta}-\rho_1^{trade}\right)}\right]\frac{\omega_1^c}{1-\omega_1^c} \tag{C.77}$$

$$\alpha_2^{end}: \qquad \alpha_2^{end} = -\left[1-\frac{\frac{1-\beta}{\beta}+2\tilde{\Psi}\left(1-\omega_1^c\right)}{\left(\frac{1}{\beta}-\rho_2^{trade}\right)}\right]\frac{\omega_1^c}{1-\omega_1^c} \tag{C.78}$$

$$\alpha_3^{end}: \qquad \alpha_3^{end} = -\frac{\varpi}{\frac{1}{\beta}-\rho_1^{UIP}} \tag{C.79}$$

$$\alpha_4^{end}: \qquad \alpha_4^{end} = \frac{(1-\rho_1^z)-\tilde{\Psi}}{\left(\frac{1}{\beta}-\rho_1^z\right)}\varpi - \left[1-\frac{\frac{1-\beta}{\beta}+2\tilde{\Psi}\left(1-\omega_1^c\right)}{\left(\frac{1}{\beta}-\rho_1^z\right)}\right] \tag{C.80}$$

$$\alpha_5^{end}: \qquad \alpha_5^{end} = -\frac{(1-\rho_2^z)-\tilde{\Psi}}{\left(\frac{1}{\beta}-\rho_2^z\right)}\varpi + \left[1-\frac{\frac{1-\beta}{\beta}+2\tilde{\Psi}\left(1-\omega_1^c\right)}{\left(\frac{1}{\beta}-\rho_2^z\right)}\right] \tag{C.81}$$

$$\alpha_b^{end}: \qquad \alpha_b^{end} = -\beta\left(\frac{1-\beta}{\beta}+2\tilde{\Psi}\left(1-\omega_1^c\right)(1+\varpi)\right). \tag{C.82}$$

For appropriate choices of Ψ_0 and Ψ, we obtain $\alpha_1^{end} > 0$ and $\alpha_2^{end} < 0$. The effect of the UIP shock on the trade balance is always negative regardless of the choices for Ψ_0 and Ψ.

Finally, the coefficients in the decision rules for the NFA position

$$\tilde{B}_{1,t} = \beta_1^{end}\xi_{1,t}^{trade} + \beta_2^{end}\xi_{2,t}^{trade} + \beta_3^{end}\xi_{1,t}^{UIP} + \beta_4^{end}z_{1,t} + \beta_5^{end}z_{2,t} + \beta_b^{end}\tilde{B}_{1,t-1} \tag{C.83}$$

satisfy $\beta_i = \frac{\alpha_i}{\beta}$ for $i = \{1,2,3,4,5\}$ and $\beta_b^{end} = 1-2\tilde{\Psi}\left(1-\omega_1^c\right)(1+\varpi)$. Endogenous discounting does not change the fundamental feature of the baseline model that the trade rebalancing shock can be a serious contender to the UIP shock as the driving force of exchange rate fluctuations.

As an aside, the model with endogenous discounting offers an interesting window into Corsetti, Dedola, and Leduc (2008). These authors show that a positive technology shock in the home country can induce an appreciation of the real exchange rate under specific assumptions on the value of the trade elasticity of substitution. They argue that for this to be the case the trade elasticity must be either very low or quite high (if paired with fairly persistent technology shocks). The assumption of Uzawa-type preferences is key to this finding. As shown in Bodenstein (2011), the dynamics around the symmetric deterministic steady state are explosive for very low trade elasticities under the assumptions of Section 2, but stable under endogenous discounting.

Using the solution of the model shown earlier, we can explore the conditions under which a positive technology shock causes an appreciation on impact. As the terms of trade and the real exchange rate are proportional to each other, a positive home (foreign) technology shock is followed by an appreciation if $\gamma_4^{end} < 0$ (or $\gamma_5^{end} > 0$). This can be achieved in two ways:

1. High trade elasticity and persistent technology shocks: Assuming that the home technology shock follows (close to) a unit root process, γ_4^{end} reduces to

$$\gamma_4^{end} = \frac{-\tilde{\Psi}}{\frac{1}{\beta} - 1} + \frac{\frac{1-\beta}{\beta} + 2\tilde{\Psi}\left(1 - \omega_1^c\right)}{\varpi\left(\frac{1}{\beta} - 1\right)}. \tag{C.84}$$

   Recalling that the trade elasticity is given by $\frac{1+\rho^c}{\rho^c}$ and $\varpi = 1 + \frac{\omega^c}{\rho^c}$, the second term in Equation (C.84) decreases as the trade elasticity rises and vanishes as the trade elasticity approaches infinity. By contrast, the first term is independent of the trade elasticity. Thus, there is a threshold value of the trade elasticity above which $\gamma_4^{end} < 0$.

2. Low trade elasticity: As for the low-elasticity case discussed in Corsetti, Dedola, and Leduc (2008), notice that ϖ is negative for low values of the trade elasticity, as is γ_4^{end}.

D Appendix: Medium-scale Model

D.1 Model Description

The model consists of two country blocs, which we index with subscript $j \in \{1, 2\}$. The home country, which we associate with the U.S., is indexed as $j = 1$, and a foreign block, which we associate with the rest of the world, is indexed as $j = 2$. Let parameter $n_j \in (0, 1)$ denote the country size, with the restriction that $n_1 + n_2 = 1$. Each country is populated by households, wholesale retailers, and intermediate goods producers, and a government who conducts fiscal and monetary policy. The good produced by each country is made up from a continuum of varieties; the domestic and foreign good are imperfect substitutes for each other. The only asset trading internationally is one non-state-contingent bond that is denominated in the currency of the domestic country. Trade in the financial asset is constrained by the fact that international financial intermediation is costly.

Households. Each country has a continuum of risk-averse households of measure one. Each household consists of a continuum of workers who supply differentiated labor to firms through an employment agency. We assume perfect risk-sharing within the household. Households derive utility from consumption and disutility from labor.

Let $C_{j,t}$ denote households' consumption of the final good in country j, $B_{j,t}$ their holdings of the bonds denominated in the currency of country 1 with price $P^{b,1}_{j,t}$. International asset markets are incomplete, and the only asset traded internationally is the non-state-contingent bond denominated in the currency of country 1. Thus, international borrowing and lending occur only in U.S. dollars. $\Pi_{j,t}$ are the profits received from firms, $T_{j,t}$ are lump-sum taxes collected by the government, $n_{j,t}(i)$ denotes the labor supply of differentiated labor variety $i \in (0, 1)$, and $w_{j,t}(i)$ is the associated nominal wage. Households in country j choose $C_{j,t}, B_{j,t}$, and $\{n_{j,t}(i), w_{j,t}(i)\}$ to maximize expected lifetime utility given by

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \log\left(C_{j,t} - b\tilde{C}_{j,t-1}\right) - \frac{\psi_N}{1+\eta} \int n_{j,t}(i)^{1+\eta} di \right\}.$$

$\eta > 0$ is the Frisch elasticity of labor supply, ψ_N controls the disutility from labor, and $\tilde{C}_{j,t}$ is aggregate consumption in country j, which implies that households exhibit external habits on their consumption decisions.[22] The households' budget constraint is

$$P_{j,t}C_{j,t} + \exp(\zeta_{j,t}^{RP})\frac{P_{j,t}^{b,1}B_{j,t}}{e_{j,t}^{1}\phi_{j,t}} = \int w_{2,t}(i)n_{2,t}(i)di + \frac{B_{j,t-1}}{e_{j,t}^{1}} + \Pi_{j,t} + T_{j,t},$$

where $e_{j,t}^{k}$ is the price of currency from country k in units of currency of country j, the nominal exchange rate, and obviously $e_{j,t}^{k} = 1$ for $k = j$. As in our analytical model, $\phi_{j,t}^{k}$ captures the financial intermediation cost of trading bonds for country j. The budget constraint states that purchasing final consumption and new bonds must equal total labor income, proceeds from existing bond holdings, and firms' profits, net of lump-sum taxes. $\xi_{j,t}^{RP}$ is a risk-premium shock that shifts the demand for bonds, whose shifts in income are rebated back lump sum to the household.[23]

Asset Markets. We assume that financial intermediation costs affect bond trade across borders but not within borders, thus $\phi_{1,t} = 1$. With this restrictions the only non-trivial intermediation cost is $\phi_{2,t}$, which we assume the functional form $\phi_{2,t} = \exp\left(-\chi^{ms}\frac{B_{2,t}}{e_{2,t}^{1}P_{2,t}Y_{2,t}} + \xi_{1,t}^{UIP}\right)$, with $\chi^{ms} > 0$. Notice that we normalize the NFA position by aggregate output in this section following most of the literature and not by exports as we did in the analytical model for convenience. The term $\xi_{1,t}^{UIP}$ is the UIP shock that scales the return of the dollar bonds for foreign households.

Final Consumption and Investment Goods. The production of the final consumption and investment goods is conducted by perfectly competitive firms and is symmetric between the two countries. We describe a generic country $j \in \{1, 2\}$. The final consumption good $\mathcal{C}_{j,t}$ is produced by combining the final intermediate good $C_{j,t}^{d}$ and imports $M_{j,t}^{c}$ of the final intermediate according to

$$\mathcal{C}_{j,t} = \left[\omega_t^{1/\theta}C_{j,t}^{d\,\frac{\theta-1}{\theta}} + (1-\omega_t)^{1/\theta}\left((1-\psi_{j,t}^{i})M_{j,t}^{c}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

where $\theta > 1$ is the elasticity of substitution between home and foreign intermediate goods, with $\omega_t \equiv \omega\exp\left(\xi_{j,t}^{\omega}\right)$, where $\omega \in (0.5, 1]$ is the home-bias parameter, and $\xi_{j,t}^{\omega}$ is a shock to the home

[22] For notation, we use X to denote the allocation by an individual agent, whereas $\tilde{X}$ denotes the allocation by the representative agent.

[23] As a result in equilibrium the household receives a transfer $\left(1 - \frac{\exp(\zeta_{j,t}^{RP})}{\phi_{j,t}^{k}}\right)\frac{P_{j,t}^{b,1}B_{j,t}}{e_{j,t}^{1}}$, whose size he takes as given.

bias in domestic consumption.

Similarly, the final investment good $I_{j,t}$ is produced as a composite of the final intermediate good $I^d_{j,t}$ and imports $M^i_{j,t}$ of the final intermediate from the other economy according to

$$I_{j,t} = \left[{\omega^i_t}^{1/\theta} {I^d_{j,t}}^{\frac{\theta-1}{\theta}} + \left(1-\omega^i_t\right)^{1/\theta} \left((1-\psi^i_{j,t})M^i_{j,t}\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

with $\omega^i_t \equiv \omega^i \exp\left(\xi^{\omega^i}_{j,t}\right)$ where $\omega^i \in (0.5, 1]$ is the home-bias parameter for investment goods and $\xi^{\omega^i}_{j,t}$ is a shock to the home bias in domestic investment.

The import adjustment costs for consumption and for investment, parameterized by $\psi^i_{j,t}$, attenuate the response of imports to changes in relative prices in the short run, allowing for differences between the short-term and the long-term trade elasticities. We assume these adjustment costs take the quadratic form proposed in Erceg, Guerrieri, and Gust (2005).

Domestic and imported intermediate goods are bundles of a continuum of intermediate varieties aggregated according to the following technologies

$$Y^d_{j,t} = \left(\int_0^1 (Y^d_{j,t}(h))^{\frac{\mu^d_{j,t}-1}{\mu^d_{j,t}}} dh \right)^{\frac{\mu^d_{j,t}}{\mu^d_{j,t}-1}} \qquad M_{j,t} = \left(\int_0^1 (M_{j,t}(h))^{\frac{\mu^M_{j,t}-1}{\mu^M_{j,t}}} dh \right)^{\frac{\mu^M_{j,t}}{\mu^M_{j,t}-1}}$$

where $\mu^k_{j,t}$ are time-varying elasticities of substitution, defined as $\mu^d_{j,t} = \mu^d \exp(\xi^{\mu^d_j}_{j,t})$ and $\mu^M_{j,t} = \mu^M \exp(\xi^{\mu^M_j}_{j,t} - \zeta q_t)$ with shocks $\xi^{\mu^k_j}_{j,t}$ for $k \in \{d, M\}$ following AR(1) processes. ζq_t induces time varying markups for exporters capturing pricing-to-market-type behavior in a reduced form manner following Alessandria and Choi (2021).

In equilibrium, the supply of final consumption goods has to equal the consumption demand by households and the government, $\mathcal{C}_{j,t} = C_{j,t} + G_{j,t}$. The market clearing conditions for the demand for domestic and imported goods are $Y^d_{j,t} = C^d_{j,t} + I^d_{j,t}$, and $M_{j,t} = M^c_{j,t} + M^i_{j,t}$.

Intermediate Goods Producers. A continuum of perfectly competitive firms produces a homogeneous intermediate good sold to intermediate retailers. Intermediate producers rent labor from the employment agency and rent capital from capital goods producers to operate a Cobb-Douglas production technology $Y_{j,t} = \exp(\xi^A_{j,t})\bar{K}^{\alpha}_{j,t}N^{1-\alpha}_{j,t}$, where $\alpha \in (0, 1)$, $\xi^A_{j,t}$ is an aggregate technology shock, and $\bar{K}_{j,t}$ is effective units of capital. We allow variable capital utilization, with $u_{j,t}$ the utilization rate. Therefore, effective capital is related to installed capital as follows: $\bar{K}_{j,t} = K_{j,t-1}u_{j,t}$. We assume that adjusting the utilization rate is costly and proportional to the level of capital, $\mathcal{A}(u_{j,t})K_{j,t-1}$, where $\mathcal{A}(u_{j,t})$ has the following functional form $\mathcal{A}(u_{j,t}) = r^K \frac{\exp(\xi(u_{j,t}-1)-1)}{\xi}$, where $\xi > 0$ and r^K is the steady-state rental rate of capital.

Capital Goods Producers. Every period, perfectly competitive firms, investment goods to augment the undepreciated capital stock $K_{j,t} = (1-\delta)K_{j,t-1} + \exp(\xi^I_{j,t})F(I_{j,t}, I_{j,t-1})$, where $\xi^I_{j,t}$ is a shock to the marginal efficiency of investment as in Justiniano, Primiceri, and Tambalotti (2010), and $F(I_{j,t}, I_{j,t-1}) = I_{j,t}\left[1 - S\left(\frac{I_{j,t}}{I_{j,t-1}}\right)\right]$ represents flow adjustments costs, and $S(.)$ is a convex adjustment cost function, as in Christiano, Eichenbaum, and Evans (2005).

Intermediate Retailers. Intermediate retailers purchase the homogeneous intermediate goods and produce a differentiated variety at no cost. Each country has two types of retailers: domestic retailers who sell in local markets and exporters. Retailers operate in monopolistically competitive product markets and sell the intermediate varieties to the wholesale retailers described above. They purchase their inputs at price $MC_{j,t}$ from intermediate good producers and turn them one for one into units of their variety. We assume that intermediate retailers set prices subject to a Rotemberg-type quadratic adjustment cost, where we map the scale of the adjustment costs to a Calvo price adjustment parameter θ_p and θ_p^x for the domestic good and exports, respectively, following the mapping in Keen and Wang (2007). We assume that prices are sticky in the currency of the country in which the sale takes place, giving us local currency pricing behavior.

In more detail, domestic retailer h in country j solve the following optimization problem:

$$J^d_{j,t}(p^d_{j,t-1}(h)) = \max_{p^d_{j,t}(h)} \quad \left[\frac{p^d_{j,t}(h)}{P_{j,t}}Y^d_{j,t}(h) - MC_{j,t}Y^d_{j,t}(h) - \frac{\Phi_P}{2}\left(\frac{p^d_{j,t}(h)}{p^d_{j,t-1}(h)} - \bar{\pi}_j\right)^2 + \beta\mathbb{E}_t\frac{\lambda_{j,t+1}}{\lambda_{j,t}}J^d_{j,t+1}(p^d_{j,t}(h))\right]$$

$$\text{subject to} \qquad Y^d_{j,t}(h) = \left(\frac{p^d_{j,t}(h)}{P^d_{j,t}}\right)^{-\mu^d_{j,t}} Y^d_{j,t}.$$

Here $\lambda_{j,t}$ denotes the marginal utility of consumption of the representative consumer in country j in period t. For ease of interpretation we map the scale of these adjustment costs to a Calvo probability θ_p: $\Phi_P = (\mu^d - 1)\frac{\theta_p}{1-\theta_p}\frac{1}{1-\beta\theta_p}$.

Meanwhile exporters solve:

$$J^x_{j,t}(p^x_{j,t-1}(i)) = \max_{p^x_{j,t}(i)} \quad \left[\frac{p^x_{j,t}(i)}{P_{j,t}e^{-j}_{j,t}}M_{j,t}(i) - MC_{j,t}M_{j,t}(i) - \frac{\Phi^x_P}{2}\left(\frac{p^x_{j,t}(i)}{p^x_{j,t-1}(i)} - \bar{\pi}_{-j}\right)^2 + \beta\mathbb{E}_t\frac{\lambda_{j,t+1}}{\lambda_{j,t}}J^x_{j,t+1}(p^x_{j,t}(i))\right]$$

$$\text{subject to} \qquad M_{j,t}(i) = \left(\frac{p^x_{j,t}(i)}{P^x_{j,t}}\right)^{-\mu^M_{j,t}} M_{j,t}.$$

An index of $-j$ refers to the index of the trading partner, so $-j = 1$ if $j = 2$ and vice versa. For ease of interpretation, we map the scale of these adjustment costs to a Calvo probability θ_p: $\Phi^x_P = (\mu^M - 1)\frac{\theta^x_p}{1-\theta^x_p}\frac{1}{1-\beta\theta^x_p}$.

Labor markets. Workers in country j supply differentiated labor through employment agencies, which bundle the differentiated varieties into a homogeneous labor input $N_{j,t}$ and sell it to intermediate producers at a nominal wage W_t. Therefore, the demand for the labor varieties is given by $N_{j,t}(i) = \left(\frac{W_{j,t}(i)}{W_{j,t}}\right)^{-\mu^w} N_{j,t}$. The employment agency sets the wages for each labor variety subject to Rotemberg-type quadratic adjustment frictions. To be more specific, employment agency i solves the following maximization problem in period t expressed in recursive form:

$$\begin{aligned} J^W_{j,t}(W_{j,t-1}(i)) = \max_{W_{j,t}(i)} \quad & \Bigg[\frac{W_{j,t}(i)}{P_{j,t}} N_{j,t}(i) - \frac{\psi_N}{1+\eta}\frac{(N_{j,t}(i))^{1+\eta}}{\lambda_{j,t}} - \frac{\Phi_W}{2}\left(\frac{W_{j,t}(i)}{W_{j,t-1}(i)} - \bar{\pi}_j\right)^2 \\ & + \beta \mathbb{E}_t \frac{\lambda_{j,t+1}}{\lambda_{j,t}} J^W_{j,t+1}(W_{j,t}(i))\Bigg] \\ \text{subject to} \quad & N_{j,t}(i) = \left(\frac{W_{j,t}(i)}{W_{j,t}}\right)^{-\mu^w} N_{j,t}. \end{aligned}$$

The solution to the maximization problem results in a standard wage Phillips curve for the real wage ($w_{j,t} = \frac{W_{j,t}}{P_{j,t}}$). For ease of interpretation we map the scale of these adjustment costs to a Calvo probability θ_w using the mapping in Born and Pfeifer (2020):[24]

$$\Phi_W = (\mu^W - 1)\frac{\mu^d - 1}{\mu^d}\frac{\theta_w}{1-\theta_w}\frac{1-\alpha}{1-\beta*\theta_w}\bar{Y}.$$

Monetary and Fiscal Policy. The government issues no debt and runs a balanced budget by financing its expenditures with lump-sum taxes levied on the households, that is, $T_{j,t} = G_{j,t}$. Government expenditures are given by $G_{j,t} = \exp(\xi^G_{j,t})G$, where G is the steady-state level and $\xi^G_{j,t}$ is a government expenditure shock.

The monetary authority in each country sets nominal interest rates following a Taylor-like

[24] Here, we implicitly impose already that in our calibration $\mu^d = \mu^M$. $\bar{Y}$ denotes steady state output.

rule which reacts to inflation and the output gap:

$$\frac{R_{j,t}}{R} = \left(\frac{R_{j,t-1}}{R}\right)^{\varphi_R} \left[\left(\frac{\bar{\pi}_{j,t}}{\bar{\pi}_j}\right)^{\varphi_\pi} \left(\frac{Y_{j,t}}{Y_{j,t}^{flex}}\right)^{\varphi_Y}\right]^{1-\varphi_R} \exp(\xi_{j,t}^R)$$

where $\bar{\pi}_{j,t} = \left(\prod_{s=0}^{3} \pi_{j,t-s}\right)^{1/4}$ is the 4-quarter average inflation with $\pi_{j,t} \equiv \frac{P_{j,t}^c}{P_{j,t-1}^c}$, and $Y_{j,t}^{flex}$ is aggregate output in the flexible-price version of the economy and $\xi_{j,t}^R$ is a monetary policy shock following an AR(1) process.

For the analysis of the Mussa puzzle in Section 5.3 we modify the policy rule as follows:

$$\frac{R_{j,t}}{R} = \left(\frac{R_{j,t-1}}{R}\right)^{\varphi_R} \left[\left(\frac{\bar{\pi}_{j,t}}{\bar{\pi}_j}\right)^{\varphi_\pi} \left(\frac{Y_{j,t}}{Y_{j,t}^{flex}}\right)^{\varphi_Y} \left(\frac{e_{2,t}^1}{e_{2,t-1}^1}\right)^{\varphi_e}\right]^{1-\varphi_R} \exp(\xi_{j,t}^R).$$

Where φ_e controls the response of the policy instrument to the growth rate of the nominal exchange rate. A flexible exchange rate regime corresponds to $\varphi_e = 0$. We model a fixed exchange rate regime with $\varphi_e = 1000$.

D.2 Calibrated Parameters

We calibrate most model parameters to standard values in the literature. Table A.1 reports the calibrated parameters. On the household side, we set the (inverse) Frisch elasticity to 1 and target total hours of 0.33 in steady state. External habit is set at 0.8, and we target an interest rate of 4 percent annually.[25] The wage stickiness parameter, θ_w, is set to 0.85 and the elasticity of substitution between labor varieties, μ^w to 4. Regarding production-related parameters, we set $\alpha = 0.29$, the depreciation rate to 2.5 percent per quarter and the curvature parameters of the cost of capital utilization and investment adjustment to 1 and 5, respectively. The "Calvo" parameters controlling the frequency of price changes are set to 0.85. We target an elasticity of substituion between goods varieties of 4 and set the slope of the export elasticity to the exchange rate to 0.75 following Alessandria and Choi (2021).

We set $\theta = 1.5$ and $\psi^C = \psi^i = 10$ for the parameters affecting trade flows, implying a long-run and a short-run trade elasticity of 1.5 and around 0.3, respectively. Regarding country

[25] Our measurement equation for the interest rate adjusts the intercept to be consistent with average short-term interest rates.

size, we assume that the U.S. is 25 percent of the world economy. We set the share of imported goods in U.S. consumption to 7 percent and 50 percent for U.S. investment. For the rest of the world, we re-scale these two moments proportionally to obtain balanced trade in a steady state.

Concerning policy parameters, we set the government expenditure-to-GDP ratio to 22 percent and assume that the Taylor rule responds to the lagged interest rate with a weight of 0.75, to 4-quarter average inflation with a weight of $1.5 \times (1-0.75)$, and to the output gap with a weight of $0.1 \times (1-0.75)$.

Table A.1: Calibrated Parameters

Parameter	**Description**	**Value**
η	Inverse Frisch Elasticity	1
b	Consumption Habit	0.8
β	Household Discount Factor	0.99
θ	Elasticity of Substitution Home Foreign Good	1.5
ψ^C, ψ^i	Trade Adjustment Costs	10
ω^C	Consumption Home Bias U.S.	0.93
ω^i	Investment Home Bias U.S.	0.5
n_1	Size U.S.	0.25
μ^w, μ^p	Elasticity of Substitutions	4
θ_w	Calvo Wage	0.85
θ_p	Calvo Domestic	0.85
θ_p^x	Calvo Export	0.85
ζ	Export Markup elasticity	0.75
α	Capital Elasticity Production	0.29
δ_k	Depreciation Capital	0.025
S''	Investment Adjustment Costs	5
ξ	Slope Utilization	1
$\bar{g}$	Government Expenditures Share GDP	0.22
φ_R	Inertia Taylor Rule	0.75
φ_π	Taylor Rule Inflation Response	1.5
φ_Y	Taylor Rule Output Gap Response	0.1

Notes: This table lists the parameters that are calibrated to values shown here. See the text for the details on the calibration targets. We omit scale parameters like $\bar{\psi}_1$, as they depend on estimated parameters and vary across different exercises.

D.3 Data and Inference

We use Bayesian methods to estimate the parameters governing the shock processes and the parameter χ^{ms} that governs the costly international financial intermediation process using the data described in the main text. In matching the data to the model, we allow for intercept terms in the measurement equations and measurement error in all the observable time series of the rest of the world.[26] Table A.2 gives details of the prior specification for the persistence and standard deviations of the exogenous processes. For the parameter χ^{ms}, we either use a wide uniform prior or re-estimate the model along a grid, fixing the values of χ^{ms} a-priori.

D.4 Estimation Results

Table A.2 shows the posterior distribution of parameter estimates. The estimated persistence of UIP shocks is close to unity, while the home bias shocks have significantly lower persistence. As previously discussed, the transmission of endogenous UIP deviations hinges on the degree of financial integration. The top row in Table A.2 shows the prior and posterior distribution of the parameter χ^{ms}. Although there is substantial uncertainty in the estimation, the posterior distribution peaks at an estimated value of $\chi^{ms} = 0.12$. We note that this estimated value of χ^{ms} is crucial to recovering the endogenous deviations of UIP from the data. Using χ^{ms} to induce first-order stationary dynamics results in a substantial deterioration of model fit.

D.5 Sensitivity Analysis

This section collects more details on the robustness experiments for our estimated quantitative DSGE model discussed in the main text.

D.5.1 Variations of the Medium-Scale Model

To compare different estimation exercises, we focus on the same set of moments presented in the main text, but expanded to include the relative volatility of investment and consumption growth. Table A.4 reproduces the moments in our dataset and the simulated counterparts from the Baseline model on the left. We show results for four alternatives. First, while we assume Local Currency Pricing (LCP) in the baseline model, a common alternative in the literature is Dollar Currency Pricing (DCP). We re-estimated the model under DCP. Overall results are similar to our baseline findings, as shown in the table. While the DCP model produces a real

[26] Measurement error is set to equal five percent of the in-sample variance of the underlying series. We use DYNARE to implement a standard RWMH algorithm for our estimation. See Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto, and Villemot (2011) for details.

Table A.2: Estimated Parameters - Medium-Scale Model

Parameter	Description	Prior Distribution		Posterior Distribution	
		Family	[P(1), P(2)]	U.S.	Rest of World
Financial Integration					
χ^{ms}	Fin. Intermediation Cost	$\mathcal{U}$	$[0, 0.2]$	–	0.12 [0.07,0.17]
Standard Deviations					
$100 \times \sigma_R$	Monetary Policy Shock	$\mathcal{IG}$	$[0.1, 1]$	0.12 [0.10,0.13]	0.11 [0.08,0.13]
$100 \times \sigma_G$	Government Policy Shock	$\mathcal{IG}$	$[1, 5]$	2.29 [2.06,2.52]	1.76 [1.54,1.98]
$100 \times \sigma_I$	MEI Shock	$\mathcal{IG}$	$[1, 5]$	3.89 [3.11,4.68]	7.64 [6.16,9.08]
$100 \times \sigma_{RP}$	Risk Premium Shock	$\mathcal{IG}$	$[0.5, 1]$	0.76 [0.57,0.95]	0.95 [0.73,1.16]
$100 \times \sigma_A$	TFP Shock	$\mathcal{IG}$	$[0.5, 1]$	0.43 [0.39,0.47]	0.39 [0.14,0.67]
$100 \times \sigma_\omega$	Home Bias Shock	$\mathcal{IG}$	$[1, 5]$	3.86 [3.29,4.41]	0.99 [0.84,1.14]
$100 \times \sigma_\mu$	Markup Shock	$\mathcal{IG}$	$[1, 5]$	26.33 [16.86,35.62]	9.60 [6.18,12.99]
$100 \times \sigma_{UIP}$	UIP Shock	$\mathcal{IG}$	$[1, 1]$	–	0.19 [0.15,0.23]
Persistence Parameters					
ρ_R	Monetary Policy Shock	$\mathcal{B}$	$[0.5, 0.25]$	0.71 [0.66,0.76]	0.77 [0.70,0.84]
ρ_G	Government Policy Shock	$\mathcal{B}$	$[0.5, 0.25]$	0.97 [0.94,0.99]	0.99 [0.98,1.00]
ρ_I	MEI Shock	$\mathcal{B}$	$[0.5, 0.25]$	0.65 [0.54,0.76]	0.21 [0.04,0.37]
ρ_{RP}	Risk Premium Shock	$\mathcal{B}$	$[0.5, 0.25]$	0.81 [0.75,0.86]	0.77 [0.71,0.83]
ρ_A	TFP Shock	$\mathcal{B}$	$[0.5, 0.25]$	0.99 [0.98,1.00]	0.56 [0.16,0.99]
ρ_ω	Home Bias Shock	$\mathcal{B}$	$[0.5, 0.25]$	0.73 [0.68,0.78]	0.68 [0.61,0.75]
ρ_μ	Markup Shock	$\mathcal{B}$	$[0.5, 0.25]$	0.63 [0.44,0.82]	0.88 [0.82,0.95]
ρ_{UIP}	UIP Shock	$\mathcal{B}$	$[0.5, 0.25]$	–	0.99 [0.98,1.00]

Notes: $\mathcal{U}$ is Uniform distribution; $\mathcal{IG}$ is Inverse Gamma distribution; $\mathcal{B}$ is Beta distribution. P(1) and P(2) are the mean and standard deviations for Beta and Inverse Gamma distributions. For Uniform distributions, P(1) and P(2) represent the hyper-parameters determining the lower and upper bound of the support of the distribution. The table reports the posterior mean and the 90% credible set in square brackets. We omit the level shifters in the measurement equations. We rounded to the second decimal. As a result, for example, ρ_G^* lies between 0.98 and 1.00. Using greater decimal precision, the interval is $[0.9772, 0.9997]$.

Table A.3: Estimated Parameters - Medium-Scale Model without Trade Shocks/Data

Parameter	Description	Prior Distribution		Posterior Distribution	
		Family	[P(1), P(2)]	U.S.	Rest of World
Financial Integration					
χ^{ms}	Fin. Intermediation Cost	$\mathcal{U}$	$[0, 0.2]$	–	0.05 [0.01,0.08]
Standard Deviations					
$100 \times \sigma_R$	Monetary Policy Shock	$\mathcal{IG}$	$[0.1, 1]$	0.12 [0.10,0.13]	0.10 [0.08,0.13]
$100 \times \sigma_G$	Government Policy Shock	$\mathcal{IG}$	$[1, 5]$	2.01 [1.81,2.22]	1.50 [1.29,1.70]
$100 \times \sigma_I$	MEI Shock	$\mathcal{IG}$	$[1, 5]$	3.49 [2.66,4.29]	7.95 [6.52,9.39]
$100 \times \sigma_{RP}$	Risk Premium Shock	$\mathcal{IG}$	$[0.5, 1]$	0.91 [0.71,1.11]	1.02 [0.80,1.25]
$100 \times \sigma_A$	TFP Shock	$\mathcal{IG}$	$[0.5, 1]$	0.43 [0.39,0.47]	0.38 [0.14,0.63]
$100 \times \sigma_\mu$	Markup Shock	$\mathcal{IG}$	$[1, 5]$	32.46 [24.71,39.85]	9.85 [6.82,13.04]
$100 \times \sigma_{UIP}$	UIP Shock	$\mathcal{IG}$	$[1, 1]$	–	0.30 [0.21,0.37]
Persistence Parameters					
ρ_R	Monetary Policy Shock	$\mathcal{B}$	$[0.5, 0.25]$	0.72 [0.67,0.77]	0.78 [0.71,0.85]
ρ_G	Government Policy Shock	$\mathcal{B}$	$[0.5, 0.25]$	0.96 [0.93,0.99]	0.98 [0.97,1.00]
ρ_I	MEI Shock	$\mathcal{B}$	$[0.5, 0.25]$	0.70 [0.59,0.82]	0.18 [0.02,0.32]
ρ_{RP}	Risk Premium Shock	$\mathcal{B}$	$[0.5, 0.25]$	0.77 [0.72,0.82]	0.76 [0.71,0.82]
ρ_A	TFP Shock	$\mathcal{B}$	$[0.5, 0.25]$	0.98 [0.97,1.00]	0.58 [0.19,0.99]
ρ_μ	Markup Shock	$\mathcal{B}$	$[0.5, 0.25]$	0.49 [0.35,0.63]	0.88 [0.82,0.93]
ρ_{UIP}	UIP Shock	$\mathcal{B}$	$[0.5, 0.25]$	–	0.95 [0.91,0.98]

Notes: $\mathcal{U}$ is Uniform distribution; $\mathcal{IG}$ is Inverse Gamma distribution; $\mathcal{B}$ is Beta distribution. P(1) and P(2) are the mean and standard deviations for Beta and Inverse Gamma distributions. For Uniform distributions, P(1) and P(2) represent the hyper-parameters determining the lower and upper bound of the support of the distribution. The table reports the posterior mean and the 90% credible set in square brackets. We omit the level shifters in the measurement equations. We rounded to the second decimal. As a result, for example, ρ_G^* lies between 0.97 and 1.00. Using greater decimal precision, the interval is $[0.9659, 0.9996]$.

exchange rate volatility closer to the data, it performs somewhat worse in terms of the correlation between the growth rates of the RER and net exports. Similarly, removing the reduced form Pricing-to-Market (PTM) assumption from the international markup to the real exchange rate has limited effects once the entire model is re-estimated—see column "No PTM". Next, one concern with our exercises is that the country-specific home bias shocks may partially capture changes in trade openness that are common across countries. To assess this possibility, we add a third home bias shock that is perfectly correlated across countries and whose relative impact on the two countries leaves the real exchange rate unchanged.[27] The results are shown under the column "Global ω". While the share of the real exchange rate explained by trade rebalancing shocks declines, they remain the main contributor to the variance of the exchange rate. Finally, we replace the risk premium shock in each country with a preference shock in the form of a shock to β similar to (Kekre and Lenel 2024)–see column "β" Shock". While this shock helps generate a more volatile real exchange rate, it leads to a deterioration of the correlation between the growth rates of net exports and the real exchange rate and the Backus-Smith correlation.[28]

D.5.2 Results for Advanced Economies countries

The reader might also be concerned that our estimation sample includes emerging market economies as part of the rest of the world, whereas a large part of the literature has focused on developed economies, such as the G-10, as a counterpart to the US. To address this concern, we re-estimated our baseline model using a dataset that includes only advanced economies to represent the rest of the world, as well as trade flows and real exchange rates. Results are shown in table A.5. While the exact moments change, we can see that our main arguments and results are robust to this change (see "Main"). In addition, a model estimated without the trade rebalancing shocks and trade data still struggles to match the correlation of the real exchange rate and net export growth ("No Trade"). Finally, although we lacked sufficient data coverage to construct a series for total hours for the rest of the world at a quarterly frequency, we can do so for the advanced economy sample using the data from Ohanian and Raffo (2012). This

[27] If the two countries were of the same size, this would mean that the direct impact would be the same for both.

[28] We have also experimented with calibrating the persistence of the preference shock to a very value in line with (Kekre and Lenel 2024) and found that, while such calibration improves some of the international moments, the overall fit of the model declines noticeably.

Table A.4: Exchange Rate Moments - Medium-Scale Model - Robustness

	Data	Baseline	DCP	No PTM	Global ω	β Shock
Disconnect and PPP puzzles						
$\sigma\Delta q/\sigma\Delta y$	4.14	3.19	3.73	3.22	3.05	5.43
$\rho(q)$	0.96	0.91	0.91	0.90	0.90	0.88
International Co-movement						
$\rho(\Delta y, \Delta y^*)$	0.39	0.13	0.13	0.13	0.16	0.10
$\rho(\Delta y, \Delta c)$	0.67	0.62	0.73	0.63	0.65	0.47
Backus-Smith and Forward Premium						
$\rho(\Delta q, \Delta c - \Delta c^*)$	-0.02	0.23	0.17	0.23	0.26	0.39
Fama (real) $\hat{\beta}$	-1.03	-0.20	-0.18	-0.20	-0.21	-0.11
RER and NX						
$\rho(\Delta \frac{nx_t}{y_t}, \Delta q)$	-0.10	0.07	-0.38	0.08	0.12	0.53
$\sigma(\Delta \frac{nx_t}{y_t})/\sigma(\Delta q)$	0.14	0.15	0.14	0.15	0.15	0.13
Percent of RER variance explained by						
UIP Shocks		15.38	17.04	14.74	18.70	19.14
Trade Rebalancing Shocks		50.31	37.31	47.87	36.53	27.91
Consumption/Investment Volatility						
$\sigma\Delta i/\sigma\Delta y$	3.16	2.59	2.82	2.68	2.60	3.15
$\sigma\Delta c/\sigma\Delta y$	1.03	0.75	0.81	0.74	0.76	0.81

Notes: Moments for the medium-scale models are computed using 1000 simulations drawn from the estimated innovations at the posterior mean. Each simulation includes 140 quarters to match the number of observations in our data sample from 1985Q1 to 2019Q4.

addition has little impact on the displayed moments or other results ("Foreign Hours").

Table A.5: Exchange Rate Moments - Medium-Scale Model - Advanced Economies

	Data	Main	No Trade	Foreign Hours
Disconnect and PPP puzzles				
$\sigma\Delta q/\sigma\Delta y$	8.62	5.27	5.17	5.18
$\rho(q)$	0.97	0.89	0.89	0.88
International Co-movement				
$\rho(\Delta y, \Delta y^*)$	0.57	0.02	0.08	0.03
$\rho(\Delta y, \Delta c)$	0.67	0.77	0.84	0.77
Backus-Smith and Forward Premium				
$\rho(\Delta q, \Delta c - \Delta c^*)$	-0.06	0.25	0.10	0.24
Fama (real) $\hat{\beta}$	-1.96	-0.52	-0.32	-0.47
RER and NX				
$\rho(\Delta \frac{nx_t}{y_t}, \Delta q)$	-0.03	0.01	0.89	-0.01
$\sigma(\Delta \frac{nx_t}{y_t})/\sigma(\Delta q)$	0.04	0.04	0.05	0.04
Percent of RER variance explained by				
UIP Shocks		10.60	78.38	11.45
Trade Rebalancing Shocks		64.97	0.00	66.35
Consumption/Investment Volatility				
$\sigma\Delta i/\sigma\Delta y$	3.16	2.28	2.71	2.28
$\sigma\Delta c/\sigma\Delta y$	1.03	0.35	0.82	0.76

Notes: Moments for the medium-scale models are computed using 1000 simulations drawn from the estimated innovations at the posterior mean. Each simulation includes 140 quarters to match the number of observations in our data sample from 1985Q1 to 2019Q4.

E Appendix: External Validation

E.1 Cross Country Evidence

Our model predicts that expected excess returns on dollar denominated bonds are positively related to the home country's net holdings of foreign assets (NFA position).[29] To test this prediction without relying on the full structure of the model we run the following panel regression of monthly ex-post excess returns of country-i ($\iota_{i,t}$) on country fixed effects (δ_i^c), month fixed effects (δ_t^m) and its net foreign asset position as a share of GDP ($NFA_{i,t}$):

$$\iota_{i,t} = \delta_i^c + \delta_t^m + \beta \times NFA_{i,t} + \epsilon_{i,t}. \tag{E.1}$$

To be consistent with our model's prediction, the regression coefficient β has to be positive. We test this condition using monthly data from 28 currencies that includes G10 and emerging economies over the period 1990:m1-2019:m12. Our choice of countries and sample periods is dictated by the availability of comparable data on bond yield differentials. We compute realized three-month and 1-year ahead excess currency returns using spot exchange rates and interest rate differentials based on government bond yields at the three-month and 1-year tenors from Du, Im, and Schreger (2018). To approximate the NFA position, we rely on estimates of the net international investment position to GDP ratio from Milesi-Ferretti (2024).

Table A.6 summarizes our findings. Columns (1) and (2) report results for G10 countries, while columns (3) and (4) report results for all countries in our sample. In line with our theory, the coefficient on the NFA position is positive and statistically significant across all specifications.[30]

[29] The expected excess return on holding the internationally traded bond, ι_t, can be obtained either using real or nominal variables since $\iota_t = E_t(\hat{q}_{t+1} - \hat{q}_t + \hat{r}_{2,t} - \hat{r}_{1,t}) = E_t(\hat{e}_{t+1} - \hat{e}_t + \hat{i}_{2,t} - \hat{i}_{1,t})$ with the nominal exchange rate e_t and the nominal interest rates $i_{j,t}$, $j \in \{1, 2\}$. Using nominal variables to construct the return series is more direct because it does not require the use of inflation data. Following Equation (24), the expected excess return is increasing in the NFA position.

[30] Our results regarding the correlation between excess returns and the NFA position are robust to controlling for interest rate differentials or to alternative measures of the NFA position such as the GDP share of dollar-denominated net debt estimated in Benetrix, Gautam, Juvenal, and Schmitz (2019).

Table A.6: USD Nominal Excess Returns and NFA

	G10 currencies		All currencies	
	3 months	1 year	3 months	1 year
	(1)	(2)	(3)	(4)
NFA	0.02***	0.05***	0.02***	0.03***
	(0.005)	(0.011)	(0.005)	(0.009)
Currencies	9	9	28	28
Observations	2,188	2,188	5,696	5,696
R^2	0.06	0.03	0.15	0.02

Notes: Standard errors in parentheses. * $p < 0.1$, ** $p < 0.01$, *** $p < 0.001$

E.2 Trade rebalancing, trade costs, and tariffs

To provide external validity of the trade rebalancing shocks we compare the model implied import wedge ω_t^c extracted from our estimation, with two measures measuring the relative cost of imports in the data. For comparability, we report the resulting time series at the annual frequency for comparability and in standard deviation units.

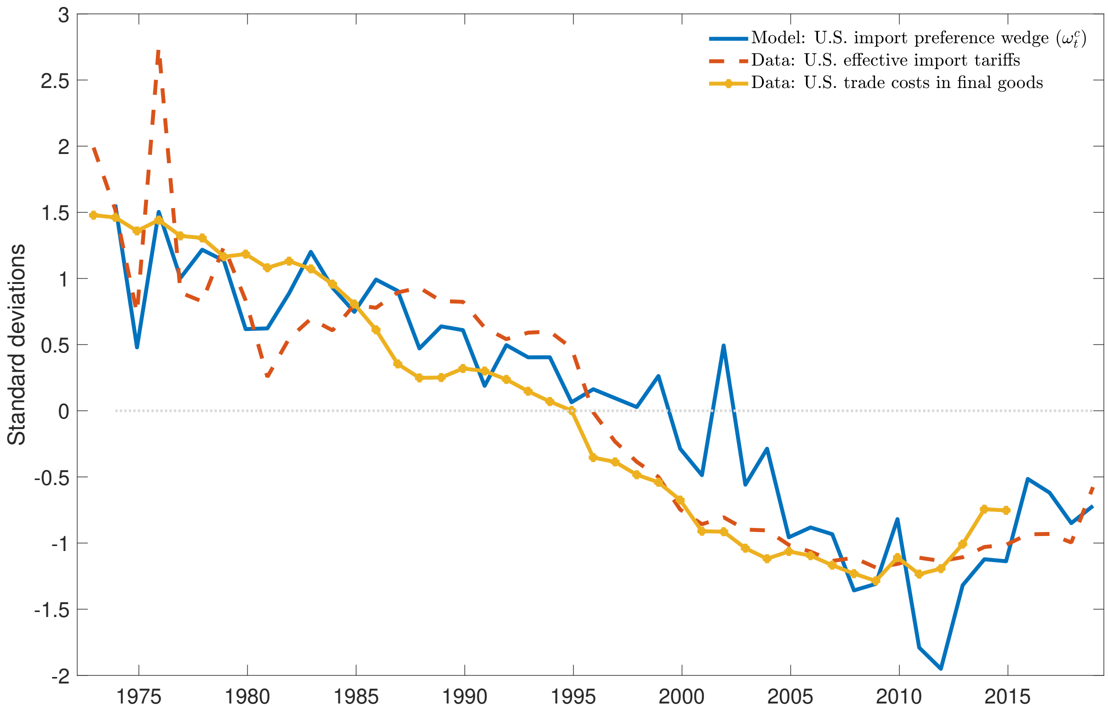
First, we measure the effective import tariff rate using data from the U.S. Federal Government's Current Receipts and Expenditures from the National Income and Products Accounts (Table 3.2). We collect total custom duties, CD_t, and construct import tariffs as $\tau_t = \frac{CD_t}{CD_t + M_t}$, where M_t is the dollar value of total imports, net of tariffs. Second, we construct a broad measure of trade costs using data from the World Input-Output Database. We follow the methodology from Cuba-Borda, Reyes-Heroles, Queralto, and Scaramucci (2025) to recover bilateral trade costs in final consumption goods. This measure of trade costs encompasses tariffs, non-tariff barriers such as trade quotas, and any other barrier to trade, such as shipping costs or trade uncertainty. We compute these bilateral trade costs for the U.S. and 25 trading partners from 1972 to 2014.

Figure A.2 shows that the model implied trade wage, ω_t^c, shown in the blue solid line, tracks the evolution of the two empirical counterparts in the data. Our model's trade wedge picks up the secular decline in trade costs and tariffs and some important trade shocks, such as President Ford's tariff on oil imports in 1975 or the increase in trade costs in 2009 associated with the Great Trade Collapse.

Our model implied rebalancing wedge has a correlation of 0.86 with the data on effective U.S.

import tariffs over the sample period, while the correlation with the second measure is about 0.92. Our interpretation of these correlations is that fluctuations in our rebalancing wedge are consistent with the dynamics of trade costs, thus offering support to the link between exchange rate and the trade balance dynamics we explore in this paper.

Figure A.2: U.S. Tariffs and Model Implied Home Bias



F Appendix: Data

We estimate the exchange rate moments presented in Section 3 and the medium-scale model in Section 5 using quarterly macroeconomic data of a trade-weighted sum of foreign economies and the United States.

We collect quarterly time series for the following 35 country/blocs: Argentina, Australia, Brazil, Bulgaria, Canada, Colombia, Chile, China, Croatia, Czech Republic, Denmark, Euro Area, Hong Kong, Hungary, India, Indonesia, Israel, Japan, Malaysia, Mexico, New Zealand, Philippines, Poland, Romania, Russian Federation, Saudi Arabia, Singapore, South Africa, South Korea, Sweden, Taiwan, Thailand, Turkey, United Kingdom and the United States. Our sample of countries represents about 85 % of PPP-adjusted world GDP in 2019. Unless otherwise note, all data is seasonally adjusted. Our data covers the period 1985Q1-2019Q4.

Below we list the variables used in the analysis together with the required transformations. For a detailed list of data sources, see (Bodenstein, Cuba-Borda, Gornemann, Presno, Prestipino, Queraltó, and Raffo 2023).

F.1 Interest rates and Inflation

For consistency across countries, we measure the policy rate using the money market interest rate where available, otherwise we use the deposit rate. We use the short rate time series constructed by Krippner (2020) for Canada, the Euro Area, Japan, UK, and the U.S. in periods of a binding effective lower bound instead of the policy rate itself to proxy for other policy measures being taken. Our measure of inflation is the quarterly change in the CPI expressed in annual rate terms. The real interest rate is the difference between the nominal interest rate and ex-post one-period ahead GDP deflator inflation.

F.2 Real GDP, consumption and investment

We source nominal GDP, nominal personal consumption expenditures, nominal gross private investment, from quarterly national accounts data obtained through Haver Analytics. We convert GDP and its components to per capita terms using the "Resident Working Age Population: 15-64 years" or an equivalent concept from the United Nations World Population Prospects database. We linearly interpolate annual population estimates to quarterly frequency. We use the implicit GDP price deflator to express all variables in real terms.

F.3 Real exchange rate

Our measure of the real exchange rate is the "Real Broad Effective Exchange Rate" for United States obtained from FRED and extended back in time using trade-weighted averages of real effective exchange rate of major trading partners.

F.4 Trade flows

We collect data of U.S. nominal imports of goods and services and U.S. nominal exports of goods and services from the Bureau of Economic Analysis. For the exchange rate moments reported in Section 3 we construct exports minus imports relative to total exports. For the estimation of the medium-scale model in Section 5 we use the ratios of nominal exports and imports relative to nominal GDP.