Exchange Rate Disconnect and the Trade Balance*

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Abstract

We propose a model with costly international financial intermediation that links exchange rate movements to shifts in the demand for domestically produced goods relative to the demand for imported goods (trade rebalancing). Our model is consistent with stylized facts of exchange rate dynamics, including those related to the trade balance, which is typically overlooked in the literature on exchange rate determination. In a quantitative assessment, trade rebalancing explains nearly 50 percent of exchange rate fluctuations over the business cycle, whereas exogenous deviations from the uncovered interest rate parity—the primary source of exchange rate fluctuations in the literature—account for just above 20 percent. Using data on trade flows or the trade balance is key to properly identifying the determinants of the exchange rate. Thus, our model overcomes the sharp dichotomy between the real exchange rate and the macroeconomy embedded in other models of exchange rate determination.

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1 Introduction

The real exchange rate is central to international macroeconomics. Yet, the literature on exchange rate determination relies on mechanisms featuring a stark separation between the exchange rate and the remainder of the macroeconomy to resolve long-standing puzzles between economic theory and the data (Fama 1984; Backus and Smith 1993; Obstfeld and Rogoff 2000).[1] We propose a setup that does not impose such a stark separation. Using a model with costly international financial intermediation in the spirit of Gabaix and Maggiori (2015), we link exchange rate movements to shifts in the demand for domestically produced goods relative to the demand for imported goods (trade rebalancing). Our model is consistent with stylized facts of exchange rate dynamics over the business cycle, including those related to the trade balance, which is typically overlooked in the literature. In a quantitative assessment, trade rebalancing explains nearly 50 percent of exchange rate fluctuations, whereas exogenous deviations from the uncovered interest rate parity—the primary source of exchange rate fluctuations in the literature—account for just above 20 percent.

Both trade rebalancing shocks and costly international financial intermediation are needed to bring the model in line with the data. A trade rebalancing shock raises, at given prices, the demand for domestically-produced goods relative to the demand for imported goods, causing both a trade balance improvement and a real currency appreciation. The magnitude of the responses of the trade balance and the real exchange rate depend critically on the extent of consumption risk sharing across countries. Higher costs of financial intermediation reduce international borrowing and lending for a given-sized rebalancing shock and restrain the trade balance response while requiring larger real exchange rate adjustment to maintain goods market equilibrium. Under moderate financial intermediation costs, the movements of the real exchange rate relative to the trade balance in the model are consistent with the qualitative and quantitative patterns in the data.

[1] The three most prominent puzzles are (1) the exchange rate disconnect puzzle (Obstfeld and Rogoff 2000), i.e., the fact that exchange rates are an order of magnitude more volatile than aggregate consumption and output; (2) the Backus-Smith puzzle (Backus and Smith 1993), i.e., the fact that the correlation between the real exchange rate and relative consumption is low and often negative; and (3) the forward-premium puzzle (Fama 1984), i.e., the fact that contrary to the uncovered interest rate parity condition, high interest rate currencies are expected to appreciate.

Models that rely on exogenous departures from the uncovered interest rate parity (UIP) condition (Itskhoki and Mukhin 2021; Eichenbaum, Johannsen, and Rebelo 2021) as the main driver of the real exchange rate address major real exchange rate puzzles but imply excessive volatility of the trade balance relative to the real exchange rate and a counterfactually high positive correlation between the real exchange rate and the trade balance. The joint introduction of costly financial intermediation and trade rebalancing shocks in our model succeeds in resolving the mismatch between the data and other models of exchange rate determination with regard to the trade balance in part because costly financial intermediation restrains the financial net-flows associated with a UIP shock. The resulting smaller exchange rate response limits trade adjustment via expenditure switching.

We develop our argument in a stylized, real, two-country model with trade in goods and assets. Each country uses labor to produce a country-specific good. Goods are traded internationally but are imperfect substitutes for each other. In addition, household preferences are biased towards the domestically produced goods. To smooth economic shocks, households can trade one non-state-contingent bond in zero-net supply. International financial intermediation is costly, which limits the extent of international borrowing and lending and hence consumption risk sharing across countries originally introduced in Turnovsky (1985) and micro-founded in Gabaix and Maggiori (2015).[2] The model features three types of shocks. Trade rebalancing shocks shift household preferences over the two traded goods. Technology shocks alter total factor productivity. UIP shocks lead to exogenous deviations from the uncovered interest rate parity condition. In the presence of costly international financial intermediation, all three shocks induce endogenous departures from the UIP condition via their impact on international borrowing and lending.

Based on this framework, we make three distinct contributions relative to existing work. First, we obtain analytical solutions that characterize the exchange rate disconnect and the role of trade rebalancing in exchange rate determination. We start by showing how the UIP and the trade rebalancing shocks differ with regard

[2] See also Schmitt-Grohé and Uribe (2003), Bodenstein (2011), or more recently Fukui, Nakamura, and Steinsson (2023), Guo, Ottonello, and Perez (2023).

to their effects on the trade balance. A trade rebalancing shock that causes a real appreciation is associated with a trade balance surplus, whereas UIP shocks that cause a real appreciation are associated with a trade balance deficit. If international financial intermediation is more costly and financial markets provide less risk sharing, the real exchange rate appreciation is larger for the rebalancing shocks but smaller for the UIP shock. The trade balance response is smaller for both shocks in this case. These features of the model are key to bringing the model in line with the data evidence about exchange rates and the trade balance and the significantly reduced importance of the UIP shock in accounting for real exchange rate volatility. Information on the trade balance dynamics allows us to distinguish the UIP shock from trade rebalancing shocks. By contrast, the exchange rate puzzles usually cited in the literature are not helpful in obtaining the identification of these two types of shocks. We show analytically that both types of shocks satisfy the real exchange rate disconnect puzzle (Obstfeld and Rogoff 2000), the consumption-real-exchange-rate puzzle (Backus and Smith 1993), and the forward premium puzzle (Fama 1984).

Second, we show that our simple model captures the key features of the data not only qualitatively but also quantitatively. The model requires an intermediate level of intermediation costs to match selected data moments on the international business cycle, the real exchange rate, and the trade balance. For our preferred parameterization, trade rebalancing shocks account for a significantly larger share of exchange rate volatility than the UIP shock. In line with previous research, if intermediation costs are close to zero, the UIP shock is (almost) the sole driver of exchange rate volatility, but the trade balance turns out to be far too volatile and too tightly correlated with the real exchange rate.

Third, we extend our analytical insights to a New Keynesian open economy model of the U.S. and the Rest of the World that we estimate using Bayesian methods. We find that trade rebalancing shocks account for close to 50 percent of the variance of the real exchange rate at business cycle frequencies (8-32 quarters) whereas the UIP shocks explain just above 20 percent. Trade shocks also reduce the importance of exogenous UIP deviations in the medium term.

Relation to the literature. Our work relates to three strands in the literature.

First, we allow financial-type shocks to contribute to exchange rate fluctuations, in line with a large body of work building on Kollmann (2002) and Gabaix and Maggiori (2015). Relative to these papers, we demonstrate that data on trade flows are informative about the importance of different shocks in explaining the exchange rate and, through implications for international borrowing and lending, for UIP deviations. This sets us apart from, for example, Itskhoki and Mukhin (2021), Itskhoki and Mukhin (2023), and Eichenbaum, Johannsen, and Rebelo (2021), who assign a dominant role to exogenous deviations from UIP as the driver of the real exchange rate. In particular, the former two papers argue against trade rebalancing shocks as an important driver of the real exchange rate. Their conclusion rests on the behavior of the economy as it approaches trade autarky in the limit, i.e., complete home bias, which eliminates a role for trade rebalancing shocks by assumption. However, as shown by our results, trade rebalancing shocks can be consistent with the disconnect patterns once we step away from this limit. Trade data becomes informative for telling different shocks apart as drivers of the real exchange rate. Eichenbaum, Johannsen, and Rebelo (2021) estimate a medium-scale model and identify financial UIP shocks as the key driver of the exchange rate (around 75 percent of the variance). Their modeling strategy abstracts from both trade rebalancing shocks and limitations to risk sharing in the way that our model proposes.[3]

Second, we build on insights from quantitative models using trade data to inform on the drivers of the real exchange rate in other contexts. Here, our contribution is to show that trade rebalancing shocks can be consistent with the disconnect puzzles and inform about the degree of asset market frictions, which allows us to draw sharper inferences on their role. Alessandria and Choi (2021) find a sizable role for changes in trade costs for the path of the U.S. trade balance and real exchange rate after 1980. Gornemann, Guerrón-Quintana, and Saffie (2020), MacMullen and Woo (2023), Ayres, Hevia, and Nicolini (2021) point to the importance of trade data, trade frictions, and commodity prices in capturing the dynamics of the real exchange rate in quantitative international DSGE models. All these papers treat the degree of

[3] Although Eichenbaum, Johannsen, and Rebelo (2021) do not include trade data in the estimation, their model implies a low positive correlation between the (nominal) exchange rate and net exports because the trade elasticity is allowed to be significantly smaller in the short run.

friction in international financial markets as given. We show both analytically and in a quantitative model that trade data and its correlation with other series, like the real exchange rate, are, in fact, informative for these frictions as well. Our paper is, therefore, closer to work by Fitzgerald (2012), who uses bilateral trade data to learn about the degree of trade and asset market frictions between countries. While Fitzgerald (2012) derives a set of test statistics to probe for these frictions between countries using trade data, we utilize the co-movement of many macro-variables to draw inferences about the risk-sharing frictions and shocks, more in line with work in quantitative DSGE models. Our strategy also allows us to speak directly to the various puzzles in the international literature.

Third, we contribute to the literature by studying the drivers of the exchange rate more broadly, bringing new results to the discussion and complementing the analysis aimed at telling them apart. In this sense, we complement Itskhoki and Mukhin (2021) and Itskhoki and Mukhin (2023) in putting less emphasis on technology and technology-news based stories as in Corsetti, Dedola, and Leduc (2008), Colacito and Croce (2011), Heathcote and Perri (2014), Chahrour, Cormun, Leo, Guerron-Quintana, and Valchev (2021). Our emphasis is on the different drivers of the real exchange rate in concert with the UIP shock also aligns with Fukui, Nakamura, and Steinsson (2023) and Miyamoto, Nguyen, and Oh (2022).

Layout. The rest of the paper is organized as follows. Section 2 presents the analytical model and derives the main theoretical results. Section 3 revisits the exchange rate disconnect in the data and our simple model. Section 4 explores the role of trade and financial integration to account for the exchange rate disconnect (and related puzzles) observed in the data. Section 5 estimates a medium-scale model featuring real and nominal rigidities to quantify the main drivers of the real exchange rate in the data. We offer our conclusions in Section 6.

2 Analytical Model

We develop our argument in a simple model before turning to a quantitative business cycle model that encompasses the features of the analytical model in Section 5. A continuum of agents of mass one lives in each of two equally-sized countries. Each country produces one good using labor as the only input into production. Both prices and wages are fully flexible. Home and foreign goods are imperfect substitutes and traded across borders. International financial markets are incomplete, as captured by restricting financial flows to a single non-state-contingent bond. In addition, we assume that there are limits to the amount of debt intermediated internationally. This feature gives rise to *endogenous* departures from the uncovered interest rate parity (UIP) condition similar to the financial intermediation model of Gabaix and Maggiori (2015) or the convenience yield model of Valchev (2020).

The model features three sources of uncertainty: technology shocks that alter total factor productivity, rebalancing shocks that alter the relative demand for the domestically-produced and the imported goods, and a financial shock that gives rise to *exogenous* departures from the UIP condition as in Itskhoki and Mukhin (2023).[4]

2.1 Assumptions

The intertemporal preferences of the representative household in country 1, the home country, are

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln\left(C_{1,t+j}\right) - L_{1,t+j} \right\}. \tag{1}$$

The felicity function in period $t+j$ depends on consumption, $C_{1,t+j}$, as well as hours worked, $L_{1,t+j}$. The household chooses consumption, labor supply, and asset holdings to maximize intertemporal utility given the sequence of budget constraints

$$P^c_{1,t+j} C_{1,t+j} + \frac{P^b_{1,t+j} B_{1,t+j}}{\phi^b_{1,t+j}} = W_{1,t+j} L_{1,t+j} + B_{1,t-1+j} + U_{1,t+j}. \tag{2}$$

The difference between consumption expenditures, $P^c_{1,t+j} C_{1,t+j}$, on the left side of the budget constraint and income from wages, $W_{1,t+j} L_{1,t+j}$, and transfers, $U_{1,t+j}$, on the right side is accounted for by trade in and holdings of financial assets, $B_{1,t-1+j}$. If $B_{1,t-1+j} > 0$, then country 1 is a net lender in period $t+j$. These assets are acquired

[4] Itskhoki and Mukhin (2023) discusses several micro-foundations for exogenous UIP deviation shocks, including shocks to the utility from holding specific assets, noise traders, and time-varying risk premia.

at the cost $P^b_{1,t+j}$.

As in Turnovsky (1985) households face a cost of financial intermediation (measured in term of country 1's good) reflected by the term $\phi^b_{1,t+j}$. This feature of the model limits the extent of international financial intermediation and thus consumption risk sharing across countries. Gabaix and Maggiori (2015) offer a microfoundation for this approach in a model with international financiers who are constrained in their ability to bear risks from international imbalances. Appendix C.1 and C.4 discuss alternative approaches to costly financial intermediation that yield endogenous UIP deviations.

In the following, we assume intermediation costs to follow $\phi^b_{1,t} = \exp\left(-\frac{\chi}{2}\frac{B^*_{1,t}}{P^d_{1,t}M^*_{2,t}}\right)$, where $B^*_{1,t}$ denotes the aggregate amount of bonds issued, not the individual household's holdings normalized by the aggregate value of exports, $P^d_{1,t}M^*_{2,t}$. In other words, atomistic households do not internalize the effects of their asset choice on the intermediation cost. The parameter χ governs the intermediation cost: a given value of the net-foreign-asset (NFA) position is associated with higher overall bond prices from the perspective of the country if χ is larger, which in turn reduces households' inclination to borrow. This feature introduces endogenous departures from the UIP condition and gives rise to excess returns on foreign assets. If χ approaches infinity, international risk sharing is shut down, and countries exist in financial autarchy. Setting $\chi = 0$ shuts down endogenous deviations from the UIP condition.[5] We assume a similar intermediation function for the foreign country with $\phi^b_{2,t} = \exp\left(-\frac{\chi}{2}\frac{\frac{1}{e_{1,t}}B^*_{2,t}}{P^d_{2,t}M^*_{1,t}} + \xi^{UIP}_{1,t}\right)$.[6] Notice that in the foreign country's intermediation function we assume the presence of the stochastic component $\xi^{UIP}_{1,t}$ which introduces exogenous departures from the UIP condition as discussed in Itskhoki and Mukhin (2021).[7]

The final consumption good, $C_{1,t}$, is an aggregate of the home good, $C^d_{1,t}$, and

[5] The parameter χ not only controls the amount of borrowing and lending in response to shocks but also the speed with which the NFA position unwinds over time. If $\chi = 0$, the NFA position follows a unit-root process to a first-order approximation. See Bodenstein (2011) for an extensive discussion.

[6] Aggregate bond holdings abroad, $B^*_{2,t}$ are normalized by foreign exports, $P^d_{2,t}M^*_{1,t}$.

[7] In the linearized model, there is no difference whether the UIP shock enters only in one of the functions. We keep the subscript of the home country because we refer to a positive UIP shock as an increase in the demand for country 1's bond.

imports of the foreign good, $M_{1,t}$,

$$C_{1,t} = \left(\left(\omega^c_{1,t}\right)^{\frac{\rho^c}{1+\rho^c}} \left(C^d_{1,t}\right)^{\frac{1}{1+\rho^c}} + \left(1-\omega^c_{1,t}\right)^{\frac{\rho^c}{1+\rho^c}} \left(M_{1,t}\right)^{\frac{1}{1+\rho^c}} \right)^{1+\rho^c}. \tag{3}$$

The elasticity of substitution between the domestic and foreign goods is measured by $\frac{1+\rho^c}{\rho^c}$. The parameter $\omega^c_{1,t} = \omega^c_1 \exp\left(\xi^{trade}_{1,t}\right)$ is time-varying to allow for shifts in the relative demand for the home and foreign goods, that are not induced by a change in the relative price. Accordingly, we label shocks to $\omega^c_{1,t}$ trade rebalancing shocks.[8] As discussed in Section 4.5, the rebalancing shock can be obtained as a shock to import tariffs, export subsidies, transportation costs, or a combination thereof.

We denote the price of the home good by $P^d_{1,t}$ and the price of the imported foreign good by $P^m_{1,t}$. The price of the imported foreign good satisfies $P^m_{1,t} = e_{1,t}P^d_{2,t}$, where $e_{1,t}$ is the nominal exchange rate and $P^d_{2,t}$ is the price of the foreign good in the foreign country. The terms of trade, $\delta_{1,t}$, are the ratio of import prices of the two countries expressed in common currency

$$\delta_{1,t} = \frac{e_{1,t}P^d_{2,t}}{P^d_{1,t}}. \tag{4}$$

Relatedly, their real consumption exchange rate is defined as

$$q_{1,t} = \frac{e_{1,t}P^c_{2,t}}{P^c_{1,t}}. \tag{5}$$

We assume that prices and wages are flexible. Production of each country's goods is linear in the country's labor, which is prized at the wage $W_{1,t}$. Total output, $Y_{1,t}$, in country 1 is

$$Y_{1,t} = exp(z_{1,t})L_{1,t}. \tag{6}$$

The assumptions for preferences and productions in country 2 mirror those of

[8] Trade rebalancing shocks have a long tradition in international macroeconomic models dating back to the static trade model in Dornbusch, Fischer, and Samuelson (1977). More recently, Blanchard, Giavazzi, and Sa (2005) study relative demand shocks in a simple model of the U.S. exchange rate and the trade balance. Coeurdacier, Kollmann, and Martin (2007) introduces relative demand shocks for goods in a two-country open economy model to generate home bias in equity holdings and valuation effects consistent with the data. Pavlova and Rigobon (2008) use domestic demand shifters to study the comovement of exchange rates and stock prices.

country 1 in Equations 1-3 and 6. The rebalancing shock in country 2 follows $\omega_{2,t}^c = \omega_2^c \exp\left(\xi_{2,t}^{trade}\right)$. As already discussed, the financial intermediation costs in the foreign country are $\phi_{2,t}^b = \exp\left(-\frac{\chi}{2}\frac{\frac{1}{e_{1,t}}B_{2,t}^*}{P_{2,t}^d M_{1,t}} + \xi_{1,t}^{UIP}\right)$ and include the exogenous UIP shock, $\xi_{1,t}^{UIP}$.

Market clearance in goods and financial markets requires

$$Y_{1,t} = C_{1,t}^d + M_{2,t} \tag{7}$$

$$Y_{2,t} = C_{2,t}^d + M_{1,t} \tag{8}$$

$$0 = B_{1,t} + B_{2,t}. \tag{9}$$

We define the trade balance normalized by the value of exports, $P_{1,t}^d M_{2,t} = e_t P_{2,t}^m M_{2,t}$, as

$$\tilde{T}_{1,t} = \frac{T_{1,t}}{e_t P_{2,t}^m M_{2,t}} \equiv \frac{e_t P_{2,t}^m M_{2,t} - P_{1,t}^m M_{1,t}}{e_t P_{2,t}^m M_{2,t}}. \tag{10}$$

We choose the price of domestic goods to be the numeraire.

The exogenous technology, trade rebalancing, and UIP shocks follow autoregressive processes of order 1 with

$$\xi_{1,t}^{trade} = \rho_1^{trade}\xi_{1,t-1}^{trade} + \sigma_1^{trade}\epsilon_{1,t}^{trade} \tag{11}$$

$$\xi_{2,t}^{trade} = \rho_2^{trade}\xi_{2,t-1}^{trade} + \sigma_2^{trade}\epsilon_{2,t}^{trade} \tag{12}$$

$$\xi_{1,t}^{UIP} = \rho_1^{UIP}\xi_{1,t-1}^{UIP} + \sigma_1^{UIP}\epsilon_{1,t}^{UIP} \tag{13}$$

$$z_{1,t} = \rho_1^z z_{1,t-1} + \sigma_1^z \epsilon_{1,t}^z \tag{14}$$

$$z_{2,t} = \rho_2^z z_{2,t-1} + \sigma_2^z \epsilon_{2,t}^z. \tag{15}$$

2.2 Model Solution

We solve a linear approximation of the model around a symmetric deterministic steady state with $\omega_1^c = \omega_2^c$ and balanced trade, i.e., $\tilde{T}_1 = 0$, $\delta_1 = 1$, $B_1 = 0$. As shown in Appendix A, we can simplify the model to the following system of equations:

$$(z_{1,t} - E_t z_{1,t+1}) - (z_{2,t} - E_t z_{2,t+1}) - \left(\hat{\delta}_{1,t} - E_t\hat{\delta}_{1,t+1}\right) = \chi \tilde{B}_{1,t} + \xi_{1,t}^{UIP} \tag{16}$$

$$\beta \tilde{B}_{1,t} = \tilde{T}_{1,t} + \tilde{B}_{1,t-1} \tag{17}$$

$$\tilde{T}_{1,t} = \frac{\omega_1^c}{1-\omega_1^c}\xi_{1,t}^{trade} - \frac{\omega_1^c}{1-\omega_1^c}\xi_{2,t}^{trade} - z_{1,t} + z_{2,t} + \varpi\hat{\delta}_{1,t}, \tag{18}$$

where $\varpi = 1 + 2\frac{\omega_1^c}{\rho^c}$. Throughout the analysis, we assume that the trade elasticity, $\frac{1+\rho^c}{\rho^c}$, is large enough to ensure that $\varpi > 0$.[9] The terms trade, $\hat{\delta}_{1,t}$, are measured in log deviation from the steady state. The normalized trade balance, $\tilde{T}_{1,t}$, and the normalized NFA position, $\tilde{B}_{1,t} = \frac{B_{1,t}}{e_t P_{2,t}^m M_{2,t}}$, are measured in absolute deviation from their steady-state values of zero.

Equation 16 is the linearized risk-sharing or UIP condition under incomplete markets. The terms $\chi\tilde{B}_{1,t}$ and $\xi_{1,t}^{UIP}$ introduce time-varying wedges in the equation and play a key role in our analysis. Equation 17 is the linearized condition governing how the NFA position evolves. Finally, Equation 18 is the linearized definition of the trade balance. For completeness, notice that the (consumption) real exchange rate is proportional to the terms of trade, $\hat{q}_{1,t} = (2\omega_1^c - 1)\,\hat{\delta}_{1,t}$. An improvement in the terms of trade of country 1, $\hat{\delta}_{1,t} < 0$, goes along with an appreciation of the real exchange rate, $\hat{q}_{1,t} < 0$. We solve for the decision rules of the endogenous variables in Appendix A.

2.3 Importance of the Trade Balance

In the remainder of this section, we demonstrate how the trade balance interacts with the exchange rate. Theorem 1 establishes that the use of trade (balance) data allows to distinguish empirically between trade rebalancing and UIP shocks because the two shocks move the trade balance in opposite directions whenever they move the real exchange rate (or term of trade) in the same direction. Theorem 2 and Theorem 3 document that the major exchange rate puzzles in the literature do not contain sufficient information to distinguish empirically between rebalancing and UIP shocks. Both types of shocks have identical predictions about the exchange rate disconnect puzzle (Obstfeld and Rogoff 2000), the consumption-real-exchange-correlation puzzle (Backus and Smith 1993), and the forward premium puzzle (Fama 1984) and (Engel 2016).

[9] Under our parameterization, it is $\varpi > 0$ for values of the trade elasticity larger than 0.5. Most works on international business cycles assume values between 1 and 2.5.

2.3.1 Trade Balance and Shock Identification

Theorem 1 *A trade rebalancing shock that improves the home country's terms of trade (appreciates the real exchange rate), $\xi_{1,t}^{trade} > 0$ and/or $\xi_{2,t}^{trade} < 0$, is associated with an improvement of the trade balance. By contrast, a UIP shock that improves the terms of trade (appreciates the real exchange rate), $\xi_{1,t}^{UIP} > 0$, is associated with a deterioration of the trade balance. If financial markets provide less risk sharing due to higher intermediation costs, i.e., χ assumes a higher value, the terms of trade are more (less) sensitive to the trade rebalancing (UIP) shock, and the trade balance is less sensitive to both shocks.*

The proof of Theorem 1 is provided in Appendix B.1. We use the equilibrium decision rules for the endogenous variables associated with the linear system in Equations 16-18. In the decision rule for the terms of trade, the coefficients associated with the UIP and trade rebalancing shocks have the same sign. In the decision rule for the trade balance, the associated coefficients have opposite signs. The sensitivity of the effects to changes in the degree of risk sharing is measured by the derivatives of the coefficients with respect to χ.

Intuitively, according to the risk sharing condition, Equation 16, a positive UIP shock, $\xi_{1,t}^{UIP} > 0$, induces an expected worsening of the terms of trade (depreciation of the real exchange rate) implying an initial improvement of the terms of trade (appreciation of the real exchange rate). Yet, this initial improvement of the terms of trade causes the trade balance to deteriorate, Equation 18, and reduces the home country's NFA position, Equation 17. In turn, the less favorable NFA position of the home country dampens the initial impact on the terms of trade in Equation 16. The larger χ, the stronger this dampening effect, and the smaller the terms of trade response for a given magnitude of the shock $\xi_{1,t}^{UIP}$.

The trade rebalancing shock works primarily on the trade balance, as seen in Equation 18. A shock that increases demand for country 1's goods, i.e., $\xi_{1,t}^{trade} > 0$ or $\xi_{2,t}^{trade} < 0$, causes the trade balance and the NFA position of country 1 to improve. From Equation 16, the improvement in the NFA position requires an initial improvement of the terms of trade (and an expected future worsening), dampening the trade balance's initial reaction. This dampening effect is stronger the larger the value of χ.

Appendix B.1 also shows that this intuition is consistent with the unconditional covariance between the growth rate of the terms of trade and the growth rate of the trade balance.

Corollary 1 *The UIP shock induces a positive covariance between the growth rate of the terms of trade and the growth rate of the trade balance. The trade rebalancing shock induces a negative covariance between the two growth rates. When both shocks are present in the model, the overall covariance is determined by the extent of costly financial intermediation as measured by* χ.

2.3.2 Exchange Rate Puzzles and Lack of Identification

In the data, the real exchange rate experiences large swings without being associated with swings of comparable magnitude in other macroeconomic variables (the real exchange rate disconnect). In addition, the correlation between the real exchange rate and relative consumption is low and often negative (Backus-Smith puzzle). Standard models of the international business cycle struggle to replicate exchange rate moments when they rely on shocks to technology and monetary policy. Under standard parameterization, a model with technology shocks only implies that the volatility of the real exchange rate is of similar magnitude as that of consumption and the correlation between relative consumption and the real exchange rate is very close to 1.[10] By contrast, the trade rebalancing and the UIP shocks bring the model better in line with the data.

Theorem 2 *Abstracting from technology shocks, the ratio of the standard deviation of the real exchange rate,* $\hat{q}_{1,t}$*, and consumption,* $\hat{C}_{1,t}$*, is independent of the relative*

[10] Exceptions are Benigno and Thoenissen (2008) and Corsetti, Dedola, and Leduc (2008). The latter shows that if the wealth effects from technology shocks dominate the substitution effects, their model can explain the two puzzles. As explained in Appendix C.4, using our model we show that these results rely on the use of endogenous discounting as in Uzawa (1968).

variances of the trade rebalancing and the UIP shocks,

$$\frac{std\left(\hat{q}_{1,t}\right)}{std\left(\hat{C}_{1,t}\right)} \;=\; \frac{std\left(\Delta\hat{q}_{1,t}\right)}{var\left(\Delta\hat{C}_{1,t}\right)} = \frac{2\omega_1^c - 1}{1-\omega_1^c}. \tag{19}$$

The correlation between relative consumption, $\hat{C}_{1,t} - \hat{C}_{2,t}$, and the real exchange rate is equal to minus one regardless of the relative variances of the trade rebalancing and the UIP shocks,

$$corr\left(\hat{C}_{1,t} - \hat{C}_{2,t}, \hat{q}_{1,t}\right) = -1. \tag{20}$$

Notice that for sufficient home bias, i.e., ω_1^c close to 1, the relative volatility of the real exchange rate can exceed the volatility of consumption multiple times. The proof in Appendix B.2 uses the first-order approximations (see Appendix A.2) to aggregate consumption and the real exchange rate (both in log-deviations)

$$\hat{C}_{1,t} \;=\; z_{1,t} - \left(1-\omega_1^c\right)\hat{\delta}_{1,t} \tag{21}$$

$$\hat{C}_{2,t} \;=\; z_{2,t} + \left(1-\omega_1^c\right)\hat{\delta}_{1,t} \tag{22}$$

$$\hat{q}_{1,t} \;=\; \left(2\omega_1^c - 1\right)\hat{\delta}_{1,t} \tag{23}$$

which relate consumption and the real exchange rate to the terms of trade and the technology shocks, $z_{1,t}$ and $z_{2,t}$. Neither the UIP nor the trade rebalancing shock enter directly into Equations 21-23 that determine the real exchange rate and consumption. Both shocks enter only indirectly through the terms of trade. Hence, the computed moments do not depend on the relative variances of the trade rebalancing and the UIP shocks.

Turning to the forward premium puzzle, Fama (1984) finds that the hypothesis of uncovered interest rate parity is violated in the data. While this theory predicts that the coefficient in the regression of the exchange rate on the interest rate differential is equal to one, empirical work estimates a negative value for the coefficient. Put differently, in the data the high interest rate currency is expected to appreciate. Engel (2016) shows that the forward premium puzzle documented in Fama (1984) also holds in real terms.

Using the conditions for pricing a non-state-contingent bond in each country, Equation 16 can be written in terms of the real interest rate differential, $r_{1,t} - r_{2,t}$,

$$E_t \left(\hat{q}_{1,t+1} - \hat{q}_{1,t}\right) = r_{1,t} - r_{2,t} + \chi \tilde{B}_{1,t} + \xi_{1,t}^{UIP}. \tag{24}$$

The expected one-period-ahead excess return of the foreign bond over the home bond, ι_t, is equal to $\chi \tilde{B}_{1,t} + \xi_{1,t}^{UIP}$. The UIP condition holds if $\iota_t = 0$. The UIP shock introduces exogenous departures from UIP. If households are limited in their ability to share consumption risk, i.e. $\chi > 0$, any shock causes endogenous UIP departures through the NFA position, $\tilde{B}_{1,t}$. Whether the direction of the endogenous departures conforms with the data evidence is not guaranteed by assumption.

Theorem 3 *Suppose the model admits only rebalancing and UIP shocks. In that case, the Fama coefficient is constant and negative independent of the degree of costly financial intermediation as measured by* χ*, as long as* $\chi \neq 0$*:*

$$\hat{\beta}^{Fama} = \frac{cov\left(E_t \Delta \hat{q}_{1,t+1}, r_{1,t} - r_{2,t}\right)}{var\left(r_{1,t} - r_{2,t}\right)} = -\frac{2\omega_1^c - 1}{2\left(1 - \omega_1^c\right)} = 1 - \frac{1}{2\left(1 - \omega_1^c\right)} < 0 \tag{25}$$

for $\omega_1^c > \frac{1}{2}$.

See Appendix B.2 for the proof. The presence of the wedge $\chi \tilde{B}_{1,t} + \xi_{1,t}^{UIP}$ allows for the expected depreciation of the home country's currency to be associated with a negative interest rate differential, i.e., $cov\left(E_t \Delta \hat{q}_{1,t+1}, r_{1,t} - r_{2,t}\right) < 0$.[11]

Engel (2016) and Valchev (2020) show that in the data, the direction of the departures from UIP switches signs at longer horizons. Without exploring this aspect in greater detail, it is noteworthy that the overall UIP departures induced by the UIP shock switch sign at longer horizons if the UIP shock is not too persistent and there are significant limits to households' ability to share risk, i.e., χ is not too close

[11] In alternative formulations of this puzzle, researchers regresses the excess return, $\iota_t = E_t \Delta \hat{q}_{1,t+1} - (r_{1,t} - r_{2,t})$, on the interest rate differential. In this case, the regression coefficient is

$$\hat{\beta}^{Fama,\iota} = \frac{cov\left(\iota_t, r_{1,t} - r_{2,t}\right)}{var\left(r_{1,t} - r_{2,t}\right)} = \hat{\beta}^{Fama} - 1 = -\frac{1}{2\left(1 - \omega_1^c\right)}.$$

to 0.[12]

2.4 Discussion

Both UIP and rebalancing shocks can help align the observations from international macro models with those in the data, particularly about exchange rate dynamics. However, as shown by Theorems 2 and 3, celebrated exchange rate puzzles do not provide relevant information to distinguish the driving forces behind exchange rate determination in macroeconomic models.

Our main result is that the trade balance dynamics interact with exchange rate dynamics. Theorem 1 shows that the trade balance response helps distinguish UIP shocks from other potential drivers of exchange rate movements, which in our case are the trade rebalancing shocks. Moreover, the transmission of both UIP and trade rebalancing shocks depends on the degree of risk-bearing capacity captured in the parameter χ, whose role is typically underappreciated in the literature. These findings do not imply that conventional shocks are not important in explaining the various features in the data. Still, without rebalancing and UIP shocks, it is nearly impossible to address all the major exchange rate puzzles.

3 Quantitative Assessment in the Analytical Model

To complement the findings of the previous section, we document how well the simple model with flexible prices and wages captures the key quantitative and qualitative features of exchange rate moments in the data.

[12] Following Engel (2016) and Valchev (2020), in our model the horizon-k expected short-term excess return is

$$E_t \iota_{t+k} = E_t \left(\Delta \hat{q}_{t+k+1} - (r_{1,t+k} - r_{2,t+k}) \right) = E_t \left(\chi \tilde{B}_{1,t+k} + \xi^{UIP}_{t+k} \right).$$

A positive innovation of the UIP shock exerts exogenous upward pressure on the excess return via ξ^{UIP}_{t+k}, but endogenous downward pressure via the decline in the home country's NFA position, $\tilde{B}_{1,t+k}$. In the near term, the exogenous component dominates in determining the excess return, but as the horizon k grows and the direct impact of the shock fades the persistent negative NFA position can dominate. The switch in the sign of the expected excess return is more likely if the UIP shock is not too persistent and χ is not too close to 0.

3.1 Data and Calibration

Using data covering the period 1985Q1-2019:Q2 on the real exchange rate (q), real interest rates (r), real economic GDP (Y), and consumption (C), as well as the trade-balance-to-exports ratio ($\tilde{T}$), we compute key empirical moments against which we assess our theory. The U.S. is treated as the home country (country 1). Data for the foreign country (country 2) are obtained as the trade-weighted average of the respective time series of 34 countries. The countries included represent 85 percent of 2019 world GDP on a PPP basis. Appendix F provides details.

Table 2 lists the empirical moments of interest: the standard deviation of the exchange rate relative to that of output, $\sigma(\Delta\hat{q})/\sigma(\Delta\hat{Y})$, the persistence of the real exchange rate, $\rho(\hat{q})$, the correlation of output across countries, $\rho(\Delta\hat{Y}_1, \Delta\hat{Y}_2)$, the correlation between output and consumption, $\rho(\Delta\hat{Y}, \Delta\hat{C})$, the correlation between the real exchange rate and relative consumption (Backus-Smith puzzle), $\rho(\Delta\hat{q}, \Delta\hat{C}_1 - \Delta\hat{C}_2)$, and the Fama regression coefficient (forward premium puzzle), $\hat{\beta}$, the correlation between the trade balance and the real exchange rate, $\rho(\Delta\tilde{T}, \Delta\hat{q})$, and the standard deviation of the trade balance relative to that of the real exchange rate, $\sigma(\Delta\tilde{T})/\sigma(\Delta\hat{q})$—where "$\Delta$" denotes the change in a variable rather than its level. For each moment, we report GMM standard errors in parentheses.

Before comparing these data moments to our model, we detail our parameter choices in Table 1. Some parameters are set in line with existing estimates while others are chosen so the model loosely matches the empirical features in Table 2. We set the home and foreign parameters at equal values whenever appropriate. The elasticity of substitution between the domestic and foreign good, $\theta \equiv \frac{1+\rho^c}{\rho^c}$, is set at 1.5 which is consistent with long-run estimates for the U.S. at the macro level—see Alessandria and Choi (2021), MacMullen and Woo (2023), and Boehm, Levchenko, and Pandalai-Nayar (2023). The discount factor, β, is set at 0.995 to be consistent with a 2 percent real interest rate. The "home-bias" parameter ω^c of 0.9 matches the cross-country average of domestic sourcing shares of final consumption goods in the World Input-Output Tables.[13]

[13] We measure domestic sourcing shares from 2000-2014 for 44 countries using the World Input-Output Database. In the U.S., a direct reading of the BEA Input-Output tables shows that the domestic sourcing

Table 1: Parameters - Analytical Model

Structural Parameters			
Discount Factor	Trade Elasticity	Home Bias	Extent of Risk Sharing
β	θ	ω^c	χ
0.995	1.5	0.9	0.1
Shocks: Persistence			
ρ_z	$\rho_{\xi^{trade}}$	$\rho_{\xi^{UIP}}$	$\rho(z_1, z_2)$
0.9	0.9	0.9	0.98
Shocks: Standard Deviation			
$100\sigma_z$	$100\sigma_{\xi^{trade}}$	$100\sigma_{\xi^{UIP}}$	
1.05	1.1	1.97	

Notes: Parameters chosen to replicate key exchange rate moments reported in Table 2. Unless otherwise specified, we set the same parameters for the home and foreign economies.

To pin down the key parameter in our exercise χ, we target an auto-correlation of 0.98 for the home country's NFA position, thereby roughly matching the observed auto-correlation of the NFA position of the U.S. with respect to the rest of the world. We explore the role of home bias, ω^c, and the role of limited risk sharing, χ, in Section 4 in greater depth.

Turning to the exogenous shocks, we set the standard deviation of the technology shock relative to the UIP shock at $\frac{\sigma_z}{\sigma_{\xi^{UIP}}} = 0.56$ to match the relative standard deviations of the real exchange rate and output in the data. In addition, the standard deviation of the trade rebalancing shock relative to the UIP shock is $\frac{\sigma_{\xi^{trade}}}{\sigma_{\xi^{UIP}}} = 0.53$ to bring the model close to the relative standard deviations of the exchange rate and the trade balance in the data. We also set the correlation of the innovations to the home and foreign technology shocks equal to $\rho_{\epsilon_z, \epsilon_z^*} = 0.98$ to match the GDP co-movement observed in the data. Finally, the auto-regressive persistence parameters of all shock processes are set at 0.9.

share is slightly higher, averaging 0.94 over the same period. However, since our calibration is symmetric between the U.S. and the foreign block, we prefer the cross-country average estimate, including the U.S. The degree of home bias is tightly linked to the tradability of goods and services. The services sector is almost entirely non-tradable, with a domestic sourcing share of 0.98. By contrast, the manufacturing and agricultural sectors in the economy are substantially more tradable, with a combined sourcing share of about 0.8. For the U.S., the high home bias in the data reflects the large share of the service sector in the U.S. economy.

3.2 Data versus Model

Table 2 compares the selected data and model unconditional moments. We distinguish four groups of moments. First, under the header "Disconnect and PPP," we summarize the excess volatility of the real exchange rate and its near-unit-root behavior. Second, we focus on international business cycle moments under the header "IRBC." The third set corresponds to the Backus-Smith and Forward premium puzzles. In broad terms, the vast majority of the literature studying exchange rate dynamics focuses on these three groups of moments and their nominal counterparts (Corsetti, Dedola, and Leduc 2008; Itskhoki and Mukhin 2021; Itskhoki and Mukhin 2023). We extend this set of statistics by a fourth one, which captures the relationship between the real exchange rate and the trade balance. These moments have received hardly any attention in the context of the exchange rate disconnect analysis.[14] However, as our theoretical results show, these moments provide key identification restrictions to distinguish between UIP and trade rebalancing shocks.

Table 2: Empirical Moments

	Disconnect and PPP		IRBC	
	a.	**b.**	**c.**	**d.**
	$\sigma(\Delta\hat{q})/\sigma(\Delta\hat{Y})$	$\rho(\hat{q})$	$\rho(\Delta\hat{Y}_1, \Delta\hat{Y}_2)$	$\rho(\Delta\hat{Y}, \Delta\hat{C})$
Data	4.23	0.96	0.46	0.63
	(0.43)	(0.01)	(0.15)	(0.11)
Model	4.21	0.9	0.42	0.66
	Backus-Smith and Forward Premium		RER and Trade Balance	
	e.	**f.**	**g.**	**h.**
	$\rho(\Delta\hat{q}, \Delta\hat{C}_1 - \Delta\hat{C}_2)$	Fama $\hat{\beta}$	$\rho(\Delta\tilde{T}, \Delta\hat{q})$	$\sigma(\Delta\tilde{T})/\sigma(\Delta\hat{q})$
Data	0.10	-0.23	0.20	1.24
	(0.16)	(0.31)	(0.14)	(0.08)
Model	-0.99	-3.95	0.21	1.27

Notes: Empirical moments computed using quarterly data from 1985Q1 to 2019Q2. GMM standard errors in parenthesis.

Although we did not target most of the moments in Table 2 as part of our calibration strategy, the model performs remarkably well compared to the data. In particu-

[14] The paper of MacMullen and Woo (2023) explores the exchange rate disconnect puzzle using a dynamic trade model calibrated to U.S. trade data.

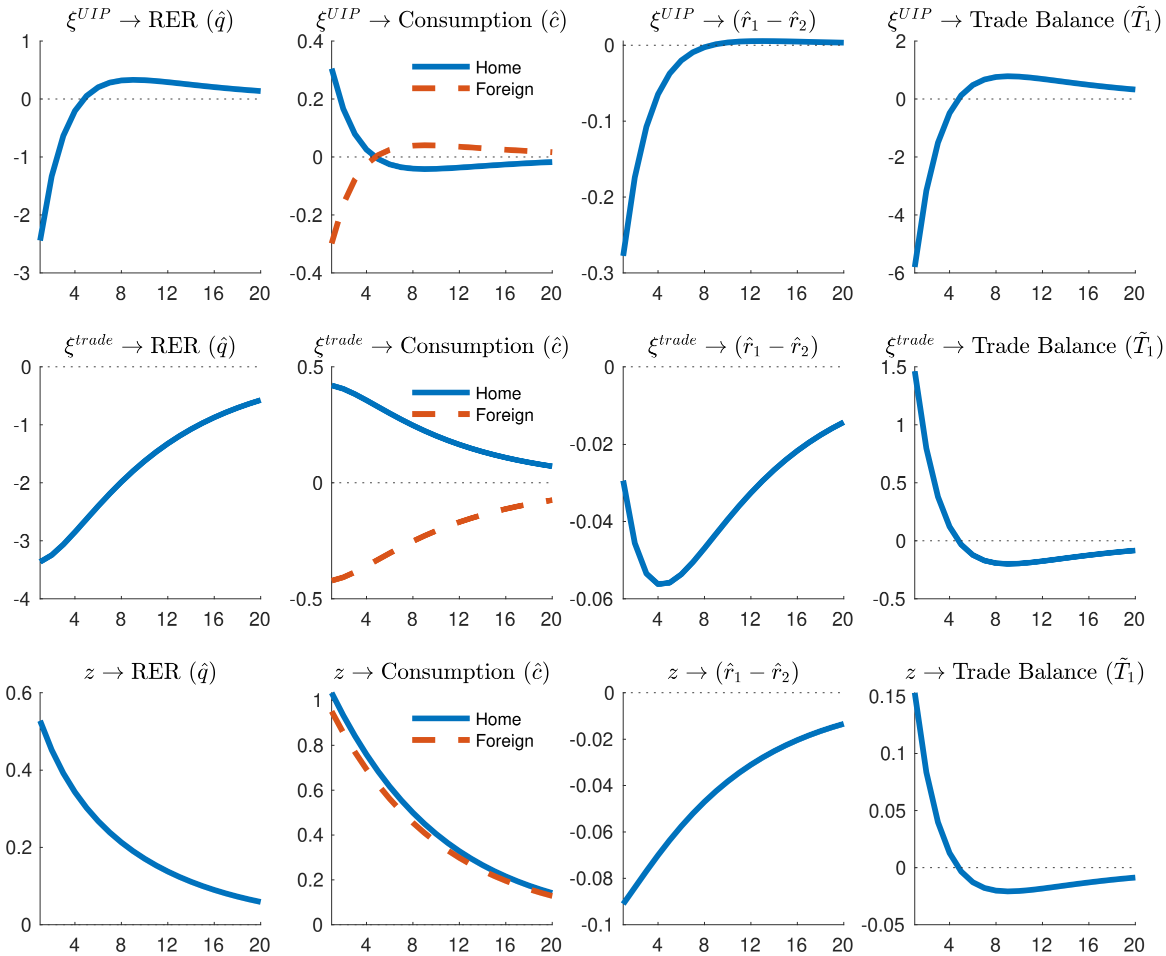
lar, it almost perfectly matches the moments relating to the trade balance. Although our model qualitatively captures the failure of the perfect risk-sharing benchmark, the Backus-Smith correlation is substantially weaker in the data—pointing to the need to introduce additional model features as discussed in Section 5.

3.3 Exchange Rate Puzzles

To understand the exchange rate dynamics associated with each group of shocks, Figure 1 plots the impulse responses to a UIP shock, ξ_1^{UIP}, a trade rebalancing shock towards home goods, ξ_1^{trade}, and a home country technology shock, z_1, under our preferred calibration.

The impulse responses confirm that the UIP shock helps address the major exchange rate puzzles. The UIP shock increases the foreign households' demand for the home country's bond. The resulting drop in the home country's NFA position is reflected in the deterioration of the home country's trade balance. With aggregate foreign consumption, $\hat{C}_2$, being postponed into the future, foreign demand for the foreign good relative to the home good falls at given prices due to home bias in consumption. To equilibrate the goods markets, the terms of trade (measured from the perspective of the home country) improve, and the real exchange rate appreciates on impact so aggregate consumption in the home country, $\hat{C}_1$, can expand via increased demand for the foreign good relative to the home good. As a result, the real exchange rate and relative consumption, $\hat{C}_1 - \hat{C}_2$, are negatively correlated (Backus-Smith puzzle). Although the UIP shock changes the relative demand for the two traded goods in the model, the aggregate consumption response is smaller than the real exchange rate (exchange rate disconnect puzzle). Finally, in addition to the expected depreciation of the real exchange rate following the initial appreciation, the foreign country's increased demand for the home country's bond causes a relative drop in the interest rate paid on the domestic bond so that $r_1 - r_2$ turns negative (forward premium puzzle). Over time, as the direct effects of the shock dissipate, the NFA position determines the dynamics. As the foreign country sells off its accumulated assets, the home country's trade balance turns positive, and the real exchange rate depreciates relative to the steady state.

Figure 1: Impulse Responses



Notes: Impulse responses to one standard deviation shocks. First row: UIP shock. Second row: trade rebalancing shock. Third row: TFP shock. Blue solid lines correspond to the response of variables in the home country. Red dashed lines correspond to the response of variables in the foreign country.

The trade rebalancing shock also addresses the three exchange rate puzzles; however, it moves the trade balance (and the NFA position) in the opposite direction than the UIP shock. The shock raises the home country's appetite for its own goods and shifts home demand away from foreign goods towards home goods at given prices. Thus, the home country's terms of trade must improve for the goods market to clear. Again, the real exchange rate appreciates whereas relative consumption rises (Backus-Smith puzzle), and the increase in the real exchange rate is an order of magnitude larger than the movements in consumption (exchange rate disconnect puzzle). Contrary to the UIP shock, the exogenous boost to the demand for home goods dominates the terms of trade effects and pushes the trade balance into surplus.

As the foreign country increases its borrowing, intermediation costs rise, increasing the overall cost of borrowing in the domestic bond. In equilibrium, the interest rate paid on the domestic bond falls so that $r_1 - r_2$ turns negative, whereas the real exchange rate is expected to depreciate after its initial appreciation (forward premium puzzle).

Technology shocks impact the economy in a fundamentally different manner. While the UIP and trade rebalancing shock primarily reallocates goods between the two countries, the technology shock increases the amount of goods available to both countries. If a positive technology shock increases the production of the home goods, the price of the home goods has to fall, causing the real exchange rate to depreciate. With aggregate consumption in the home country increasing by more than in the foreign country and the real exchange rate depreciating, the technology shock induces a positive comovement between $\hat{q}$ and $\hat{C}_1 - \hat{C}_2$. Similarly, the model delivers the wrong comovement in the interest rate differential. Finally, the technology shock fails to reproduce the exchange rate disconnect as the real exchange rate moves by less than aggregate consumption.

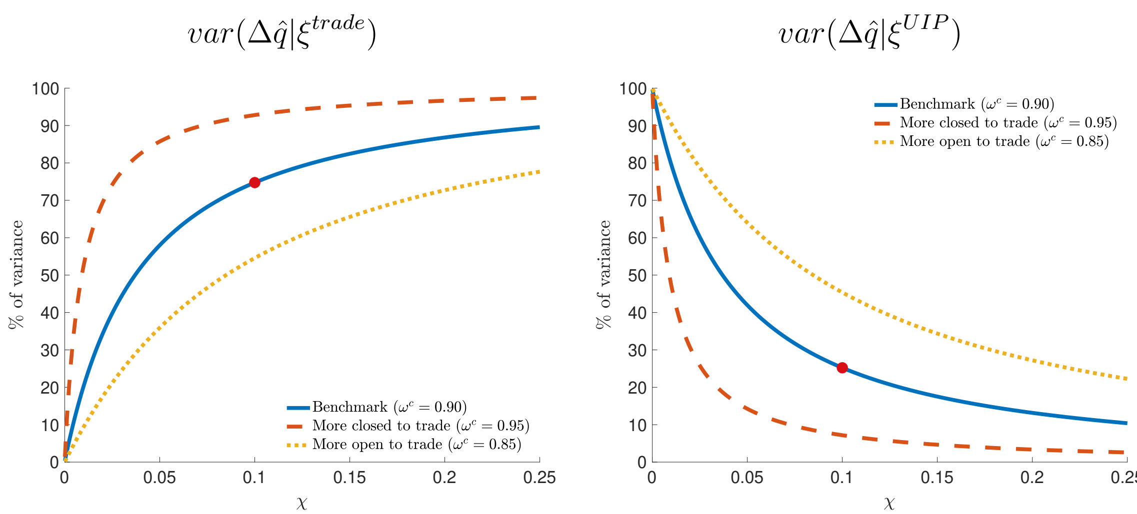
4 Exploring the Disconnect Mechanism

We use our model to explore the interplay of trade and financial integration in determining the drivers of the real exchange rate and the success of our model in matching the data. Our main finding is that trade balancing shocks are important drivers of exchange rate volatility through endogenous UIP deviations. This finding is important because it allows our model to reconnect exchange rate fluctuations to macroeconomic fundamentals while recovering the exchange rate puzzles in the data.

4.1 What Drives the Exchange Rate?

As discussed in Theorem 1, international risk sharing and trade openness modulate endogenous UIP deviations in response to trade rebalancing and UIP shocks. Thus, we explore variance decompositions along these two dimensions. Figure 2 plots the contribution of trade rebalancing and UIP shock to the variance of the real exchange rate. The left panel shows the share of variance of the real exchange rate accounted for by the rebalancing shocks in our calibrated model. The right panel shows the corresponding variance share due to the UIP shock. Solid lines depict the unconditional variance decomposition as a function of costly financial intermediation for a home bias parameter of $\omega^c = 0.9$. Our baseline calibration is marked with a red circle. The red dashed line presents the variance decomposition in an economy more closed to trade, and the yellow dotted line shows the variance decomposition in the case of an economy more open to trade.

Figure 2: Variance Decomposition of the Real Exchange Rate



Notes: Unconditional variance decomposition computed in the theoretical model. The blue line is our baseline model. The red dot marks our preferred calibration. Dashed and dashed-dotted lines correspond to alternative calibrations for different values of the home-bias parameter.

Trade rebalancing shocks explain nearly 75 percent of the variance of the real exchange rate in the benchmark calibration. Shocks to UIP explain about 25 percent of the total variance, and the contribution of technology shocks is negligible. With greater international risk sharing, as measured by a lower value of the parameter χ, the variance contribution of trade rebalancing shocks declines, and the importance of IP shocks increases. Intuitively, when intertemporal substitution is frictionless, as would be the case under $\chi \to 0$, any movement in the trade balance can be accommodated with the corresponding change in the country's NFA position. In other words, if countries can borrow freely, the variation of the trade balance decouples from real shocks. Whether the economy is open or closed to trade is irrelevant when trade in financial assets is frictionless despite the absence of complete markets. In the limit, when $\chi \rightarrow 0$, the NFA asset position inherits a unit-root behavior $\tilde{B}_{1,t} = \tilde{B}_{1,t-1}$.

The level of financial integration (χ) and trade openness (ω^c) significantly affect the contribution of shocks to the total variance of the real exchange rate. When trade integration is low, as illustrated by the calibration with $\omega^c = 0.95$, trade rebalancing shocks dominate the variance of the real exchange rate.

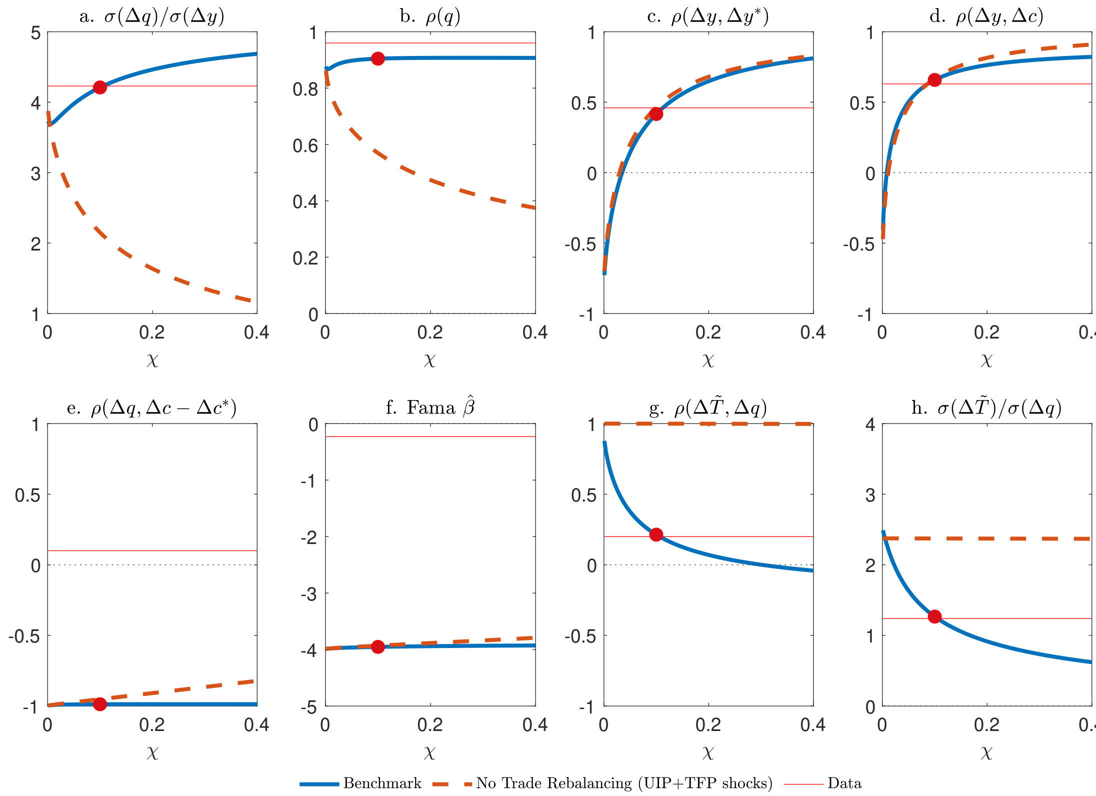
4.2 The Role of International Risk Sharing

The degree of costly financial intermediation, χ, plays a central role in the theoretical predictions of our model. We now illustrate how the exchange rate moments are affected when changing the value of χ. Figure 3 plots theoretical moments from our model. Results for our preferred calibration of $\chi = 0.1$ are depicted with a red circle. Each panel corresponds to the corresponding moment in Table 2, and the red line indicates the associated value computed in our dataset. The solid blue line depicts the model's theoretical moments calculated for different values of χ that range from near-frictionless financial markets ($\chi = 0.001$) to configurations in which costly financial intermediation hampers the flow of financial assets ($\chi = 0.4$).

The top row in Figure 3 shows that reducing risk sharing by costlier financial intermediation amplifies the volatility of the real exchange rate, exacerbating the macroeconomic disconnect (Panel a.), and strengthening the international comovement output and consumption (Panels c. and d.). Costly financial intermediation reduces the flow of the non-state-contingent bond, which reduces the ability of the economy to smooth out shocks that would require a trade balance adjustment. For example, if the preference for domestic goods increases at home, this would require improving the trade balance and the NFA at home. However, because of the higher cost of trading the international bond, the real exchange rate becomes more sensitive to changes in the NFA, triggering a much larger appreciation of the exchange rate.

Turning to the panels in the bottom row of Figure 3, the Backus-Smith correlation (panel e.) and the Fama coefficient (panel f.) are nearly unaffected by the parameter χ. This result is consistent with Theorems 2 and 3 and indicates that adding

Figure 3: Financial Integration and Exchange Rate Moments



Notes: The blue line shows theoretical moments for different values of χ. The red dashed line corresponds to the model without trade rebalancing shocks. The solid red horizontal line indicates the corresponding empirical moment in each panel. The red dot shows our preferred calibration.

technology (TFP) shocks has a limited role in our analysis.[15] One of our central results is that our model can account for the relationship between the exchange rate and trade flows. In particular, our model matches the weak correlation between the trade balance and the real exchange rate (panel g.) and the volatility of trade flows relative to the real exchange rate (panel h.). We return to this point in Sections 4.4 and D.4 in detail.

How do trade rebalancing shocks interact with the degree of risk sharing? The red dashed line in Figure 3 shows the exchange rate moments in a model without rebalancing shocks but keeping the calibration of Table 1 for all other parameters. Two results emerge in the absence of trade rebalancing shocks. First, the model

[15] As discussed in Appendix C.4, this result can be overturned under the assumptions in Corsetti, Dedola, and Leduc (2008).

becomes uninformative about the moments relating to the exchange rate and the trade balance. Second, identifying the parameter χ becomes more difficult, as the model relies only on the moments in panels a. through d. and will ultimately prefer a calibration that implies almost frictionless international bond markets (χ goes to zero) to generate the exchange rate disconnect. In Section 5, we show that this identification issue also emerges in an estimated medium-scale model.

4.3 The Role of Trade Integration

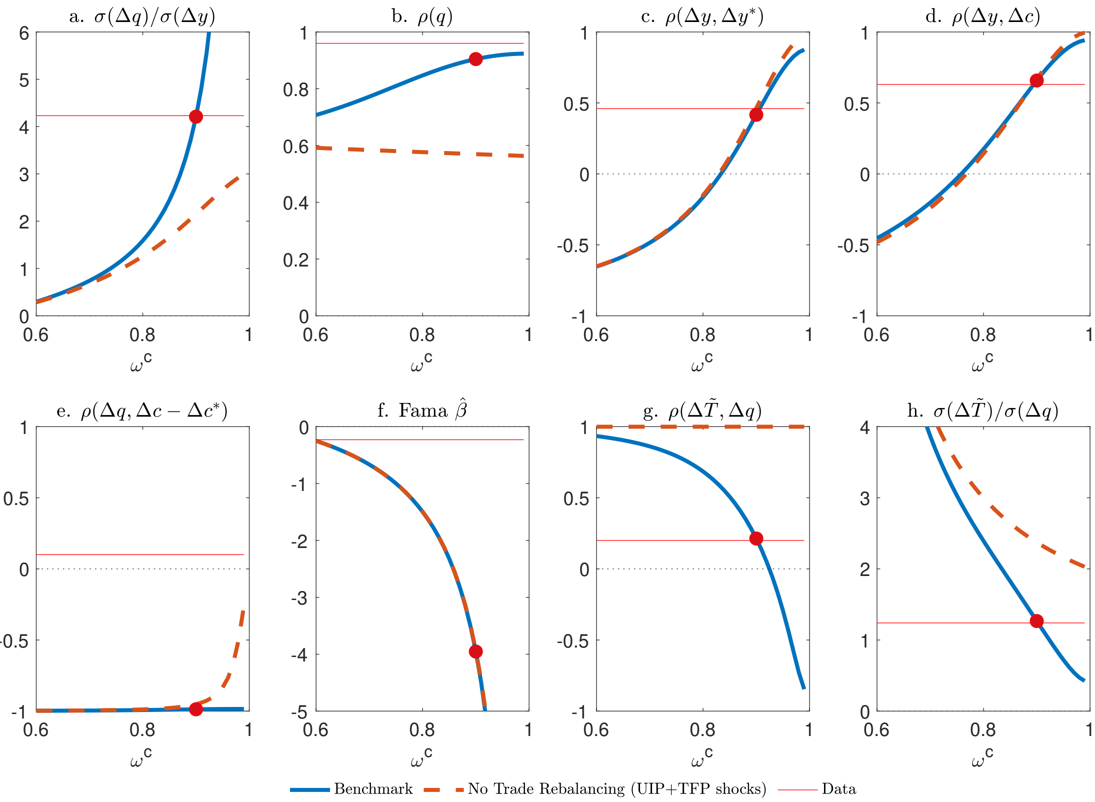
How does trade integration affect the exchange rate puzzles? This question is important to understanding the transmission mechanism of shocks. Figure 4 shows exchange rate moments in our model as a function of the home-bias parameter, ω^c. Lower values of ω^c correspond to a more open economy with greater trade integration. Higher values of ω^c correspond to an economy with less trade integration. The disconnect puzzles (panel a. and panel b.) become starker as the economy moves towards trade autarky. Similarly, the IRBC correlation also increases with the home-bias parameter.

These results follow the intertemporal approach of the current account pioneered in Obstfeld and Rogoff (1995). In our model, the consumption-smoothing mechanism is embedded in Equation 18, which we reproduce below:

$$\tilde{T}_{1,t} = \frac{\omega_1^c}{1-\omega_1^c}\xi_{1,t}^{trade} - \frac{\omega_1^c}{1-\omega_1^c}\xi_{2,t}^{trade} - z_{1,t} + z_{2,t} + \varpi\hat{\delta}_{1,t}.$$

For illustration, consider a trade rebalancing shock that improves the home country's trade balance. Holding the terms of trade unchanged, the trade balance response increases in the degree of home bias. However, as the economy becomes more closed, the aggregate resource constraint implies $\tilde{T}_{1,t} \approx 0$. For the trade balance to remain unchanged, the terms of trade must appreciate. The required real exchange rate appreciation increases with the degree of home bias. Near the trade-autarky limit $\hat{q}_{1,t} = \hat{\delta}_{1,t}$, which implies that the volatility of the real exchange rate increases one-to-one with the volatility of the terms of trade. In other words, the volatility of the exchange rate increases without bounds to induce enough expenditure-switching

Figure 4: Trade Integration and Exchange Rate Moments



Notes: The blue line shows theoretical moments for different values of the home bias parameter, ω^c. The red dashed line corresponds to the model without trade rebalancing shocks. The solid red horizontal line indicates the corresponding empirical moment in each panel. The red dot shows our preferred calibration.

such that the trade balance equals zero.

The intuition behind the response of the IRBC-related moments stems from the resource constraint of a closed economy. As home bias increases, the real exchange neutralizes movements in the trade balance, and productivity shocks become the only shocks affecting domestic output and consumption. In the trade-autarky limit, the correlation between domestic and foreign output will reflect our assumption about the cross-country correlation of technology shocks (panels c. and d).

The Backus-Smith correlation (panel e.) is negative and nearly equal to one, consistent with the predictions of Theorem 2. The degree of trade integration is irrelevant to this correlation. The forward premium puzzle (panel f.), however, is heavily influenced by trade integration. The Fama coefficient becomes increasingly negative as the home-bias parameter approaches the closed-economy limit as stated in Theorem 3.

The correlation of the trade balance and the exchange rate (panel g.) and the volatility of the trade balance relative to the exchange rate (panel h.) decreases with the degree of home bias. As shown in Appendix B.1, the covariance between the trade balance and the exchange rate, absent technology shocks, depends on:

$$cov\left(\Delta\hat{\delta}_{1,t}, \Delta\tilde{T}_{1,t}\right) = \frac{\omega_1^c}{1-\omega_1^c} cov\left(\Delta\hat{\delta}_{1,t}, \Delta\xi_{1,t}^{trade}\right) + \left(1 + 2\frac{\omega_1^c}{\rho^c}\right) var\left(\Delta\hat{\delta}_{1,t}\right) \quad (26)$$

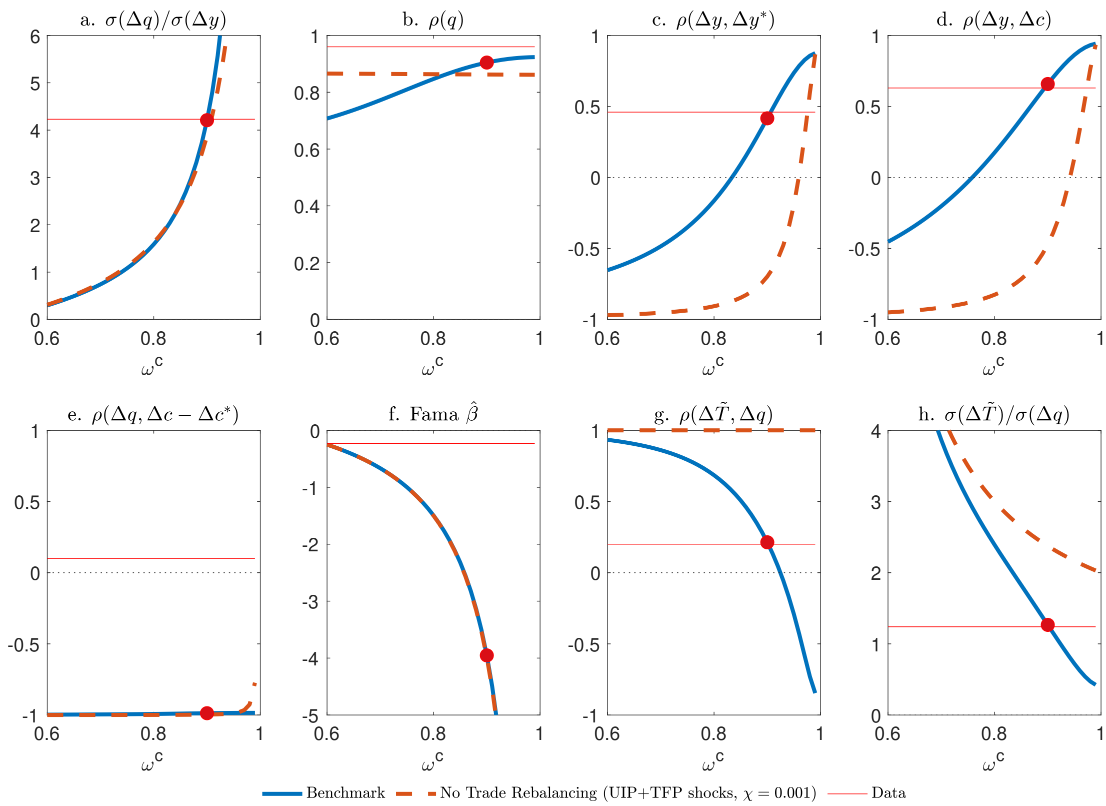
where the covariance between the terms of trade and the trade rebalancing shocks is negative, as previously discussed. With greater home bias, the first term on the right-hand side becomes more negative at a geometric rate, whereas the second term on the right-hand side becomes more positive at a linear rate. This difference implies that even if the variance of the terms of trade grows without bounds as we approach a closed-economy configuration, the correlation between the terms of trade and the trade balance will be more negative with greater home bias. Equation 26 also captures the limitation of models that abstract from trade rebalancing shocks. In that case, $cov\left(\Delta\hat{\delta}_{1,t}, \Delta\xi_{1,t}^{trade}\right) = 0$, implying that the correlation between the trade balance and terms of trade will always be positive and for our parameterization equal to one.

The red dashed line in Figure 4 illustrates the effect of trade openness on the exchange rate moments in a model without trade rebalancing shocks. Models without trade rebalancing shocks are uninformative about the correlation between the real exchange rate and the trade balance (panel g.). Although greater home bias reduces the volatility of the trade balance relative to the real exchange rate, a model without trade rebalancing shocks falls short of matching the empirical value of this moment (panel h.).

4.4 Relation to the Exchange Rate Disconnect Literature

In our setting, trade rebalancing shocks and costly financial intermediation are important to account for the observed exchange rate puzzles. How do we reconcile our results with the literature emphasizing the role of UIP shocks instead? In such models, financial shocks create exogenous UIP deviations consistent with the exchange rate disconnect, and these financial shocks are the main drivers of the real exchange rate—see Itskhoki and Mukhin (2021), Eichenbaum, Johannsen, and Rebelo (2021).

Figure 5: Near Frictionless Risk Sharing and Home Bias



Notes: The blue line shows theoretical moments for different values of ω^c. The red horizontal line indicates the corresponding empirical moment in each panel. The red dot shows our preferred calibration.

Figure 5 shows the different exchange rate moments we analyzed in previous sections. Alongside our benchmark model, shown in the solid blue line, we show a specification that resembles the model in Itskhoki and Mukhin (2021) that abstracts from trade rebalancing shocks and also assumes that the exchange of internationally traded bonds is near frictionless ($\chi = 0.001$). We chart the exchange rate moments as a function of the home-bias parameter (ω^c) to show that this parameter is central for transmitting shocks in the absence of trade rebalancing. The comparison highlights that with greater risk-sharing capacity (lower value of χ) and even as the economy approaches the closed-economy limit with greater home bias, models without trade rebalancing shocks cannot reproduce the moments relating to the real exchange rate and the trade balance (panels g. and h.). Moreover, the Backus-Smith correlation and Fama's regression coefficient do not provide meaningful information to discipline the parameter χ or to distinguish between models.

4.5 Model Extensions

We close this section with a brief discussion of three of our modeling choices: the interpretation of the trade rebalancing shock, the labor supply, and the mechanism that limits households' ability to issue/hold debt. The analytical derivations supporting this discussion can be found in the Appendix C.

4.5.1 Iceberg transportation costs, export subsidies, and import tariffs

The trade rebalancing shock directly alters households' preferences for domestic and foreign goods. We show in Appendix C.2 that the trade rebalancing shock in the original formulation of our model (ξ^{trade}) can be thought of as a linear combination of shocks to iceberg transportation costs (ξ^{ice}), shocks to import tariffs (ξ^{m}), shocks to export subsidies (ξ^{x}), and our original relative demand shock. In detail, it is

$$\begin{aligned} \frac{\bar{\omega}^c}{1-\bar{\omega}^c}\xi^{trade}_{1,t} \quad = \quad & \frac{1}{1-(\tilde{\omega}^c-\omega^c)}\frac{\omega^c}{1-\omega^c}\xi^c_{1,t} \\ & +\left(1+\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\right)\frac{\tau^m}{1-\tau^m}\xi^m_{1,t} \\ & -\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tau^x}{1-\tau^x}\xi^x_{2,t} \\ & +\frac{1}{\rho^c}\frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\frac{\tau^{ice}}{1-\tau^{ice}}\xi^{ice}_{1,t} \end{aligned} \tag{27}$$

where $\bar{\omega}^c$ is the re-scaled home-bias parameter, and the parameter $\tilde{\omega}^c_1$ is a function of the steady state values of transportation costs (τ^{ice}), tariffs (τ^m), and subsidies (τ^x), and is equal to ω^c if those steady-state values are zero, as in our benchmark model.

4.5.2 Labor Supply

Assuming a perfectly elastic labor supply elasticity greatly simplifies the analytical derivations underlying our theorems. When generalizing household preferences to

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln \left(C_{1,t+j} \right) - \frac{\nu_0}{1+\nu} L_{1,t+j}^{1+\nu} \right\} \tag{28}$$

the linearized system of equations that govern the equilibrium dynamics of the terms of trade, the trade balance, and the NFA position are given by

$$-E_t \Delta z_{1,t+1} + E_t \Delta z_{2,t+1} + E_t \Delta \hat{\hat{\delta}}_{1,t+1} + \bar{\nu} E_t \Delta \tilde{T}_{1,t+1} = \chi \tilde{B}_{1,t} + \xi_{1,t}^{UIP} \tag{29}$$

$$\beta \tilde{B}_{1,t} = \tilde{T}_{1,t} + \tilde{B}_{1,t-1} \tag{30}$$

$$\left(1 - \bar{\nu}\right) \tilde{T}_{1,t} = \frac{\omega_1^c}{1 - \omega_1^c} \xi_{1,t}^{trade} - \frac{\omega_1^c}{1 - \omega_1^c} \xi_{2,t}^{trade} - z_{1,t} + z_{2,t} + \varpi \hat{\delta}_{1,t}. \tag{31}$$

If $\nu_0 = 1$ and $\nu = 0$, the composite parameter $\tilde{\nu} = 0$, and our baseline model reemerges. Appendix C.3 repeats the exercise underlying Figure 3 when ν assumes the value of 1. The new model also fits the targeted moments very well.

4.5.3 Endogenous Discounting

Finally, we investigate the consequences of removing the financial intermediation costs mechanism and switching to endogenous discounting as in Uzawa (1968). Endogenous discounting does not change the fundamental feature of the baseline model that the rebalancing shock can be a serious contender to the UIP shock as the driving force of exchange rate fluctuations, see Appendix C.4 for details.

5 Quantitative Assessment

To assess the quantitative relevance of our mechanism when considering a wider range of macroeconomic data we build and estimate an empirical medium-scale DSGE model with trade rebalancing shocks and costly international financial intermediation. Details on the model, the data, calibration, and the Bayesian estimation are given in Appendix D.

The model is a two-country open economy extension of the seminal contribtions of Smets and Wouters (2007) and Christiano, Eichenbaum, and Evans (2005a) along the lines of Erceg, Guerrieri, and Gust (2005). The home country is taken to be the U.S. and the foreign country is the rest of the world. Households consume (with consumption habits), provide labor, and save in bonds. International borrowing and lending is subject to costly financial intermediation as in our analytical model. Nominal wages are sticky as in Erceg, Henderson, and Levin (2000).

The final consumption good consists of a domestically produced and an imported good. Trade rebalancing shocks affect the relative demand for the two goods. Trade adjustment costs imply a difference between the short-and long-term trade elasticity. The production of the investment good follows the same structure.

The domestically produced and the imported good are bundles of a continuum of differentiated varieties. Nominal prices of varieties are rigid as in Calvo (1983). The production of the varieties uses a homogenous input good which itself is produced from labor and capital. The utilization rate of capital is endogenously determined. Capital producers rent out the capital stock to these firms. The installation of new capital by the capital producers is subject investment adjustment costs.

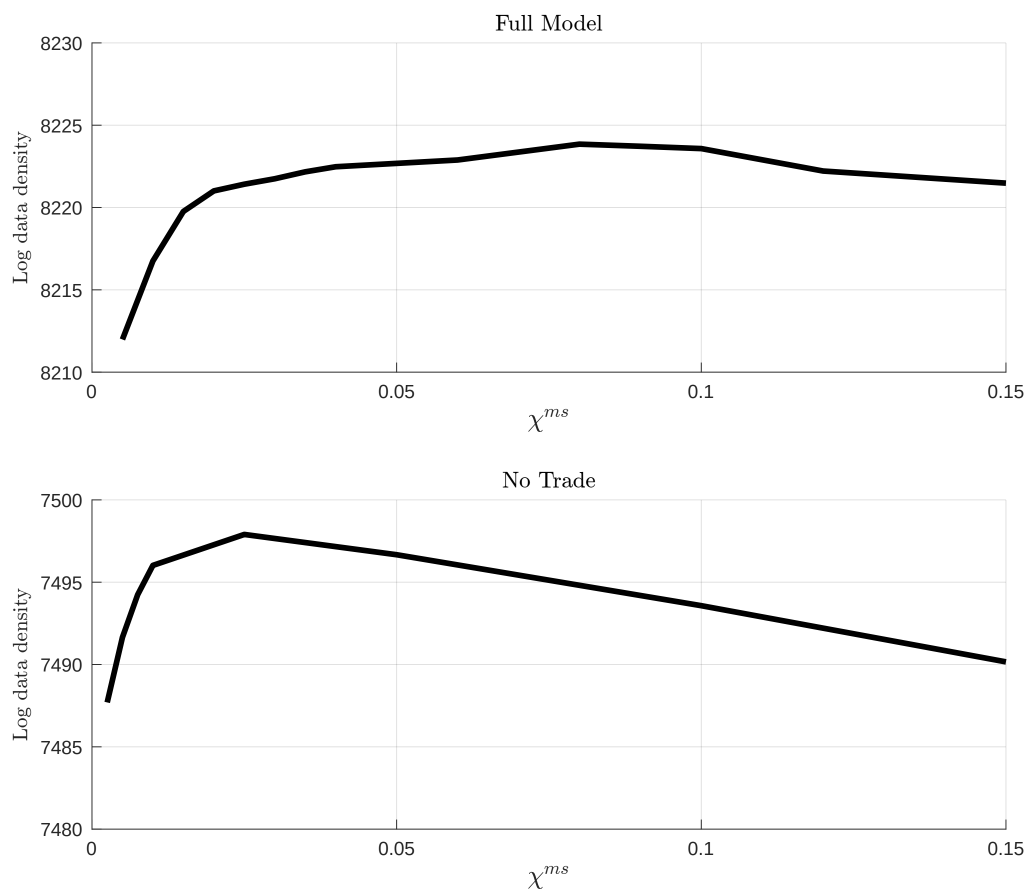
In each country the government issues no debt and runs a balanced budget by financing its expenditures with lump-sum taxes on households. The monetary policy follows a Taylor-like instrument rule which responds to inflation and the output gap. In addition to trade rebalancing, UIP, and total factor productivity shocks, the model features investment-specific shocks, risk premium shocks, price and wage markup shocks, inflation trend shocks, and shocks to monetary and fiscal policy.

We estimate the model with Bayesian methods using using quarterly data for real growth in GDP, consumption, and investment, GDP deflator inflation, and policy rates for the U.S. and the rest of the world. For the U.S., we also use data on the broad real dollar index, real wage growth, labor gap, export and import-to-GDP ratios, and inflation expectations. All series run from 1985Q1 to 2019Q2 and are constructed as in Bodenstein, Cuba-Borda, Gornemann, Presno, Prestipino, Queraltó, and Raffo (2023). We estimate the parameters of all the shock processes and the parameter governing the financial intermediation cost, χ^{ms}. Unless mentioned otherwise, all other parameters are fixed at standard values. One noteworthy feature of our calibration is that investment is more import-intensive than consumption.

5.1 Estimation Results

In order to let the data inform us on the parameter χ^{ms} without prior judgement, we either use a wide uniform prior or re-estimate the model along a grid, fixing the values of χ^{ms} a-priori. In the first case, the posterior distribution peaks at an estimated value of $\chi^{ms} = 0.09$.[16]

Figure 6: Identifying the Risk Sharing Parameter

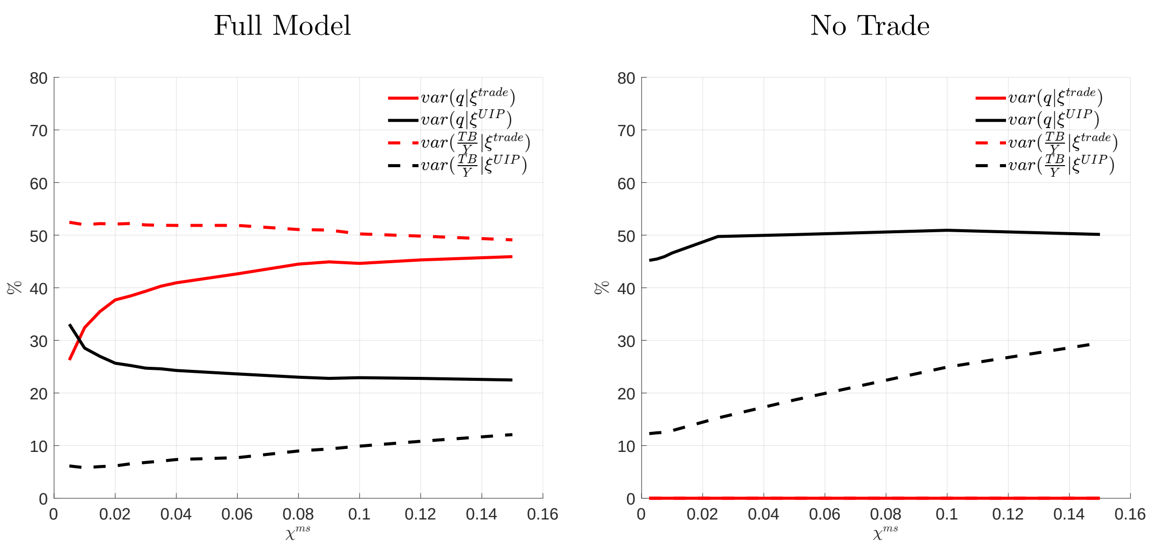


Note: The top panel shows the log-data density of the estimated model for different values of the parameter χ^{ms} when including U.S. exports and imports data in estimation. The bottom panel shows the log-data density in an estimated model without trade data.

[16] Since we scaled the NFA position by GDP in this section, this value is not directly comparable to the values used in the analytical model. To translate χ^{ms} to the setting with a normalization by exports, χ^{ms} needs to be multiplied by the share of exports in GDP.

In the second case, we vary χ^{ms} on a grid between values of 0.001 and 0.15 and re-estimate the rest of the model parameters. Figure 6 shows the resulting log data density for each of these estimations in the top panel. The curve peaks around 0.08.[17] The data favors endogenous UIP deviations that respond to changes in the country's NFA position to a model that features only exogenous UIP deviations. As shown in the bottom panel of Figure 6, using trade data is key for this finding. If we exclude exports and imports from the estimation, the log data density peaks around 0.03, a much lower value than our model with trade data and trade rebalancing shocks.

Figure 7: Variance Share - Business Cycle Frequency



Notes: Solid lines depict the variance decomposition of the real exchange rate. Dashed lines correspond to variance decomposition of the trade balance-to-output ratio. Red lines correspond to the share of variance explained by trade rebalancing shocks. Black lines correspond to the share of variance explained by UIP shocks. All model implied series are bandpass filtered at 6-32 quarters.

Figure 7 shows how different values for χ^{ms} matter for the contribution of the UIP and the trade rebalancing shocks to the variance of the real exchange rate and the trade balance at business cycle frequencies. For this exercise, we also re-estimate the model for different values of χ^{ms} and record the variance shares at the posterior mean. The left panel of Figure 7 shows that the share of the real exchange rate explained by the trade rebalancing shocks grows with the value of the financial intermediation costs, rising from as low as 25 percent to over 45 percent. By contrast, the contribution of the UIP shock falls as the parameter χ^{ms} increases. We also find

[17] This is also close to the value we estimate when including χ^{ms} directly in the estimation.

that the trade rebalancing shocks explain over 50 percent of the variation in trade.

The right panel of Figure 7 shows that the model without trade data and rebalancing shocks assigns a much higher share of the variance of both the real exchange rate and the trade balance to the UIP shock for any value of χ^{ms}. Together with our previous results, we conclude that accounting for trade data and trade rebalancing shocks is important to understand exchange rate determination in this class of open economy models.

5.2 Exchange Rate Moments

This last exercise demonstrates that our estimated model fits key exchange rate disconnect moments. Contrary to the calibration exercise in Section 3, the estimation of the medium-scale model does not explicitly target these specific moments. Instead, the likelihood function trades off model fit along multiple dimensions, including the covariances and cross-correlations with the additional series we include in the estimation beyond those series underlying the exchange rate disconnect moments. Moreover, the medium-scale model imposes additional cross-equation restrictions that may penalize the moments we are interested in. Thus, we are not guaranteed to recover the exchange rate disconnect.

Table 3 shows that our medium-scale model with trade rebalancing shocks leans towards the moments related to exchange rate disconnect despite the additional restrictions in matching a broader set of time series. The data moments lie, for the most part, within the confidence intervals for the moments obtained by simulating the estimated model. In addition to the cost parameter governing financial intermediation, χ^{ms}, import adjustment costs also play an important role in determining model fit. Appendix D.5 compares exchange rate moments for different configurations of these two parameters.[18]

The columns labeled "No Trade" in Table 3 show results when we re-estimate the medium-scale model abstracting from trade rebalancing shocks, trade data and with

[18] Consistent with the results in the simple model, we find that increasing the degree of international risk bearing capacity increases the model implied correlation and the relative volatility of the trade balance and the real exchange rate.

Table 3: Exchange Rate Moments - Medium-Scale Model

		Full Model		No Trade	
	Data	Mean	Std. Error	Mean	Std. Error
Disconnect and PPP puzzles					
$\sigma\Delta q/\sigma\Delta y$	4.23	3.7	(0.4)	3.3	(0.3)
$\rho(q)$	0.96	0.9	(0.04)	0.9	(0.04)
International Co-movement					
$\rho(\Delta y, \Delta y^*)$	0.46	0.1	(0.1)	0.2	(0.1)
$\rho(\Delta y, \Delta c)$	0.63	0.6	(0.1)	0.7	(0.1)
Backus-Smith and Forward Premium					
$\rho(\Delta q, \Delta c - \Delta c^*)$	0.10	0.2	(0.1)	-0.1	(0.1)
Fama (real) $\hat{\beta}$	-0.23	-0.7	(0.4)	-0.9	(0.3)
RER and NX					
$\rho(\Delta\tilde{T}, \Delta q)$	0.20	0.3	(0.1)	0.7	(0.1)
$\sigma(\Delta\tilde{T})/\sigma(\Delta q)$	1.24	1.0	(0.1)	1.5	(0.1)

Notes: Medium-scale models computed using 1000 simulations drawn from the estimated innovations at the posterior mean. Each simulation has a length of 138 quarters to match the observations in our data sample from 1985Q1 to 2019Q2.

nearly frictionless international bond markets. We use the same structural parameters of our benchmark medium-scale model but set the parameter $\chi^{ms} = 0.001$, a common value used in the literature. In the absence of trade data this parameter is not well identified and becomes an additional degree of freedom, rendering the comparison with our benchmark model more difficult. To compensate for the absence of trade shocks, the estimated volatility of the majority of shocks is larger relative to our benchmark estimates. Moreover, the estimated shock processes tend to be more persistent in the model without trade. Despite all these difference in parameter estimates, the model without trade struggles to correctly approximate the correlation and relative volatility between the trade balance and the real exchange rate.[19]

[19] It is possible for the "No Trade" specification to produce a lower correlation between the trade balance and the real exchange rate by lowering the short-run trade elasticity, but the resulting import adjustment cost parameter is significantly larger than existing estimates in the literature. See Eichenbaum, Johannsen, and Rebelo (2021).

5.3 What are Trade Rebalancing Shocks?

To validate the structural interpretation of the trade rebalancing shocks suggested in Equation 27, we review the correlation between our estimates of the trade rebalancing wedge in the model and direct measures of trade costs from the data. We focus on two measures of trade costs, namely iceberg trade costs, τ_t^{ice}, and import tariffs, τ_t^m.

We find a strong correlation between the model's trade rebalancing wedge, ω_t^c, and the two empirical measures of trade costs. For instance, our model implies a strong correlation of 0.88 with the data on effective U.S. import tariffs over the sample period. Iceberg trade costs between the U.S. and its main trading partners, a measure available from 1972 to 2014, are also strongly correlated with estimate trade rebalancing wedge. The correlation with this broader measure reaches 0.92. Our interpretation of trade costs as the driving force of the exchange rate and the trade balance dynamics is strongly supported by this external model evidence.

6 Conclusion

In this paper, we developed an exchange rate determination model consistent with key empirical features of exchange rates. We show, both analytically and numerically, that key exchange rate puzzles, such as the exchange rate volatility, the consumption-real-exchange-rate, and the forward-premium puzzle, do not necessarily imply that the exchange rate is primarily driven by financial shocks and disconnected from the broader macroeconomy. Trade rebalancing shocks and endogenous UIP deviations due to limited risk-bearing capacity are shown to be consistent with these empirical features.

In addition, trade rebalancing shocks paired with imperfect risk sharing due to costly financial intermediation allow our model to also replicate empirical patterns of the real exchange rate relative to the trade balance. In particular, trade rebalancing shocks are necessary to mute the volatility of the trade balance and to deliver the weak correlation between the real exchange rate and the trade balance. Theories of exchange rate determination that rely mainly on exogenous UIP shocks are inconsistent with these empirical patterns. We show that capturing the degree of consumption risk sharing across countries via international financial markets plays a crucial role in reconciling the exchange rate disconnect with the trade balance dynamics.

Our results extend to a New Keynesian open economy model of the U.S. and the Rest of the World that we confront with a broader set of macroeconomic data. In our estimated model, we quantify the main shocks that drive the exchange rate and find that trade rebalancing shocks account for close to 50 percent of the variance of the real exchange rate and the trade balance at business cycle frequencies.

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A Appendix: Model Solution, Decision Rules, and Theoretical Moments of the Simple Model

A.1 First-order and equilibrium conditions

We present the first-order and equilibrium conditions of the model. Optimality of the household's decisions imply

$$C_{1,t} = \frac{P^d_{1,t}}{P^c_{1,t}} \frac{W_{1,t}}{P^d_{1,t}} \tag{A.1}$$

$$P^b_{1,t} = \phi^b_{1,t} \beta E_t \left\{ \frac{C_{1,t}}{C_{1,t+1}} \frac{P^c_{1,t}}{P^c_{1,t+1}} \right\}. \tag{A.2}$$

$$C^d_{1,t} = \omega^c_{1,t} \left(\frac{P^c_{1,t}}{P^d_{1,t}} \right)^{\frac{1+\rho^c}{\rho^c}} C_{1,t} \tag{A.3}$$

$$M_{1,t} = (1 - \omega^c_{1,t}) \left(\frac{P^c_{1,t}}{P^m_{1,t}} \right)^{\frac{1+\rho^c}{\rho^c}} C_{1,t}. \tag{A.4}$$

These conditions and the consumption aggregator imply for the relative prices

$$\frac{P^c_{1,t}}{P^d_{1,t}} = \left[\omega^c_{1,t} + (1 - \omega^c_{1,t}) \delta_{1,t}^{-\frac{1}{\rho^c}} \right]^{-\rho^c} = F_{1,t}^{-\rho^c} \tag{A.5}$$

where the terms of trade $\delta_{1,t}$ is the ratio of import prices expressed in common currency

$$\delta_{1,t} = \frac{e_{1,t} P^d_{2,t}}{P^d_{1,t}}. \tag{A.6}$$

Similar conditions are obtained for country 2, (keeping in mind that the sole internationally traded bond pays in the currency of country 1:

$$C_{2,t} = \frac{P^d_{2,t}}{P^c_{2,t}} \frac{W_{2,t}}{P^d_{2,t}} \tag{A.7}$$

$$P^b_{1,t} = \phi^b_{2,t} \beta E_t \left\{ \frac{C_{2,t}}{C_{2,t+1}} \frac{P^c_{2,t}}{P^c_{2,t+1}} \frac{e_{1,t}}{e_{1,t+1}} \right\} \tag{A.8}$$

$$C^d_{2,t} = \omega^c_{2,t} \left(\frac{P^c_{2,t}}{P^d_{2,t}} \right)^{\frac{1+\rho^c}{\rho^c}} C_{2,t} \tag{A.9}$$

$$M_{2,t} = (1 - \omega^c_{2,t}) \left(\frac{P^c_{2,t}}{P^m_{2,t}} \right)^{\frac{1+\rho^c}{\rho^c}} C_{2,t} \tag{A.10}$$

$$\frac{P^c_{2,t}}{P^d_{2,t}} = \left[\omega^c_{2,t} + (1-\omega^c_{2,t})\delta_{1,t}^{\frac{1}{\rho^c}}\right]^{-\rho^c} = F_{2,t}^{-\rho^c}. \tag{A.11}$$

As we assume that prices and wages are flexible and that the production of each country's good is linear in the use of the country's labor,

$$Y_{1,t} = exp(z_{1,t})L_{1,t} \tag{A.12}$$

the production real wage equals the productivity level

$$\frac{W_{1,t}}{P^d_{1,t}} = exp(z_{1,t}). \tag{A.13}$$

Similarly, for country 2, we obtain

$$Y_{2,t} = exp(z_{2,t})L_{2,t} \tag{A.14}$$

$$\frac{W_{2,t}}{P^d_{2,t}} = exp(z_{2,t}). \tag{A.15}$$

Recall from the main text that market clearance in goods and financial markets requires

$$Y_{1,t} = C^d_{1,t} + M_{2,t} \tag{A.16}$$

$$Y_{2,t} = C^d_{2,t} + M_{1,t} \tag{A.17}$$

$$0 = B_{1,t} + B_{2,t}. \tag{A.18}$$

We also define in the main text that the trade balance (normalized by the value of exports) is

$$T_{1,t} \equiv e_t P^m_{2,t} M_{2,t} - P^m_{1,t} M_{1,t} \tag{A.19}$$

$$\tilde{T}_{1,t} = \frac{T_{1,t}}{e_t P^m_{2,t} M_{2,t}}. \tag{A.20}$$

which implies that the consolidated budget constraint of households in country 1 can be writte as

$$\frac{P^b_{1,t} B_{1,t}}{\phi^b_{1,t}} = T_{1,t} + B_{1,t-1}. \tag{A.21}$$

Finally, the law of one price for the internationally traded bond implies the risk-sharing condition

$$\phi^b_{1,t} E_t \left\{ \frac{C_{1,t}}{C_{1,t+1}} \frac{P^c_{1,t}}{P^c_{1,t+1}} \right\} = \phi^b_{2,t} E_t \left\{ \frac{C_{2,t}}{C_{2,t+1}} \frac{P^c_{2,t}}{P^c_{2,t+1}} \frac{e_{1,t}}{e_{1,t+1}} \right\}. \tag{A.22}$$

A.2 Simplifying the Nonlinear Model

Noticing that consumption in the two countries can be expressed as

$$C_{1,t} = exp(z_{1,t}) F^{\rho^c}_{1,t} \tag{A.23}$$

$$C_{2,t} = exp(z_{2,t}) F^{\rho^c}_{2,t} \tag{A.24}$$

and the consumption real exchange rate is given

$$rer_{1,t} = \frac{e_{1,t} P^c_{2,t}}{P^c_{1,t}} = \left(\frac{F_{1,t}}{F_{2,t}} \right)^{\rho^c} \delta_{1,t} \tag{A.25}$$

we can write the trade balance as

$$\begin{aligned} \tilde{T}_{1,t} &= 1 - \delta_{1,t} \frac{M_{1,t}}{M_{2,t}} \\ &= 1 - \frac{1-\omega^c_{1,t}}{1-\omega^c_{2,t}} \left(\frac{F_{1,t}}{F_{2,t}} \right)^{-1} \frac{exp(z_{1,t})}{exp(z_{2,t})} \delta_{1,t}^{1-2\frac{1+\rho^c}{\rho^c}}. \end{aligned} \tag{A.26}$$

Similarly, we write the risk sharing condition and the evolution of the NFA position as

$$E_t \left\{ \frac{P^c_{1,t}}{F^{\rho^c}_{1,t+1} P^c_{1,t+1}} \left[\phi^b_{1,t} \frac{exp(z_{1,t})}{exp(z_{1,t+1})} - \phi^b_{2,t} \frac{exp(z_{2,t})}{exp(z_{2,t+1})} \frac{\delta_{1,t}}{\delta_{1,t+1}} \right] \right\} = 0 \tag{A.27}$$

$$\frac{P^b_{1,t} \tilde{B}_{1,t}}{\phi^b_{1,t}} = \tilde{T}_{1,t} + \frac{e_{1,t-1} P^m_{2,t-1} M_{2,t-1}}{e_{1,t} P^m_{2,t} M_{2,t}} \tilde{B}_{1,t-1} \tag{A.28}$$

where $\tilde{B}_{1,t} = \frac{B_{1,t}}{e_{1,t} P^m_{2,t} M_{2,t}}$.

We next express all variables entering in equations A.26, A.27, and A.28 and in terms of $\tilde{T}_{1,t}$, $\delta_{1,t}$, $\tilde{B}_{1,t}$. The rebalancing shocks follow $\omega^c_{i,t} = \omega^c_i \exp\left(\xi^{trade}_{i,t}\right)$, for $i = 1, 2$ and we assume that $\phi^b_{1,t} = \exp\left(-\frac{\chi}{2} \frac{B^*_{1,t}}{P^d_{1,t} M^*_{2,t}}\right)$ whereas $\phi^b_{2,t} = \exp\left(-\frac{\chi}{2} \frac{\frac{1}{e_{1,t}} B^*_{2,t}}{P^d_{2,t} M_{1,t}} + \xi^{UIP}_{1,t}\right)$. Then, linearization around the symmetric deterministic steady state with $\omega^c_1 = \omega^c_2$

and balanced trade, i.e. $\tilde{T}_1 = 0$, $\delta_1 = 1$, $B_1 = 0$, yields the linear system

$$\left(z_{1,t} - E_t z_{1,t+1}\right) - \left(z_{2,t} - E_t z_{2,t+1}\right) - \left(\hat{\delta}_{1,t} - E_t \hat{\delta}_{1,t+1}\right) = \chi \tilde{B}_{1,t} + \xi_{1,t}^{UIP} \tag{A.29}$$

$$\beta \tilde{B}_{1,t} = \tilde{T}_{1,t} + \tilde{B}_{1,t-1} \tag{A.30}$$

$$\tilde{T}_{1,t} = \frac{\omega_1^c}{1-\omega_1^c} \xi_{1,t}^{trade} - \frac{\omega_1^c}{1-\omega_1^c} \xi_{2,t}^{trade} - z_{1,t} + z_{2,t} + \varpi \hat{\delta}_{1,t}, \tag{A.31}$$

where $\varpi = 1 + 2\frac{\omega_1^c}{\rho^c}$. The terms trade, $\hat{\delta}_{1,t}$, are measured in log-deviation from the steady state, $\tilde{T}_{1,t}$ and $\tilde{B}_{1,t}$ is in absolute deviation., If the term $\chi \neq 0$, the NFA dynamics are stationary.

For later reference, the remaining model variables can be recovered from the following linear relationships

$$\hat{C}_{1,t} = z_{1,t} - \left(1 - \omega_1^c\right) \hat{\delta}_{1,t} \tag{A.32}$$

$$\hat{C}_{2,t} = z_{2,t} + \left(1 - \omega_1^c\right) \hat{\delta}_{1,t} \tag{A.33}$$

$$\hat{q}_{1,t} = \left(2\omega_1^c - 1\right) \hat{\delta}_{1,t}. \tag{A.34}$$

$$\hat{M}_{1,t} = -\frac{\omega_1^c}{1-\omega_1^c} \xi_{1,t}^{trade} + z_{1,t} - \frac{\varpi + 1}{2} \hat{\delta}_{1,t} \tag{A.35}$$

$$\hat{M}_{2,t} = -\frac{\omega_2^c}{1-\omega_2^c} \xi_{2,t}^{trade} + z_{2,t} + \frac{\varpi + 1}{2} \hat{\delta}_{1,t} \tag{A.36}$$

$$\hat{Y}_{1,t} = \omega_1^c \left(\xi_{1,t}^{trade} - \xi_{2,t}^{trade}\right) + \omega_1^c z_{1,t} + \left(1 - \omega_1^c\right) z_{2,t} + \varpi \left(1 - \omega_1^c\right) \hat{\delta}_{1,t} \tag{A.37}$$

$$\hat{Y}_{2,t} = -\omega_1^c \left(\xi_{1,t}^{trade} - \xi_{2,t}^{trade}\right) + \omega_1^c z_{2,t} + \left(1 - \omega_1^c\right) z_{1,t} - \varpi \left(1 - \omega_1^c\right) \hat{\delta}_{1,t} \tag{A.38}$$

Defining the real interest rate for country i as the real return on a bond that pays one unit of consumption in country i regardless of the state of the world, i.e., $r_{i,t} = \hat{C}_{i,t+1} - \hat{C}_{i,t}$, we can express Equation A.29 in terms of the differential of real interest rates, $r_{1,t} - r_{2,t}$, between countries

$$r_{1,t} - r_{2,t} = E_t \left(\hat{q}_{1,t+1} - \hat{q}_{1,t}\right) - \chi \tilde{B}_{1,t} - \xi_{1,t}^{UIP}. \tag{A.39}$$

A.3 Applying the Method of Undetermined Coefficients

To compute the solution of the dynamic linear system, we employ the method of undetermined coefficients. Starting from the conjecture that in equilibrium the terms of trade evolve according to

$$\hat{\delta}_{1,t} = \gamma_1 \xi^{trade}_{1,t} + \gamma_2 \xi^{trade}_{2,t} + \gamma_3 \xi^{UIP}_{1,t} + \gamma_4 z_{1,t} + \gamma_5 z_{2,t} + \gamma_b \tilde{B}_{1,t-1} \tag{A.40}$$

we compute the values of the unknown coefficients γ_1 through γ_5 and γ_b by substituting the conjectured solution into the dynamic system (A.29)-(A.31). Using Equation A.31, the trade balance follows

$$\begin{aligned} \tilde{T}_{1,t} &= \left(\frac{\omega^c_1}{1-\omega^c_1} + \varpi\gamma_1\right)\xi^{trade}_{1,t} + \left(-\frac{\omega^c_1}{1-\omega^c_1} + \varpi\gamma_2\right)\xi^{trade}_{2,t} \\ &\quad + \varpi\gamma_3 \xi^{UIP}_{1,t} + \left(-1+\varpi\gamma_4\right) z_{1,t} + \left(1+\varpi\gamma_5\right) z_{2,t} + \varpi\gamma_b \tilde{B}_{1,t-1}. \end{aligned} \tag{A.41}$$

Turning to the risk sharing/UIP condition, Equation A.29, we first evaluate the term $\left(\hat{\delta}_{1,t} - E_t\hat{\delta}_{1,t+1}\right)$ using the equation for the evolution of the NFA position, Equation A.30, and Equation A.41 to substitute out for $\tilde{B}_{1,t}$:

$$\begin{aligned} \left(\hat{\delta}_{1,t} - E_t\hat{\delta}_{1,t+1}\right) &= \gamma_1\left(1-\rho^{trade}_1\right)\xi^{trade}_{1,t} + \gamma_2\left(1-\rho^{trade}_2\right)\xi^{trade}_{2,t} + \gamma_3\left(1-\rho^{UIP}_1\right)\xi^{UIP}_{1,t} \\ &\quad + \gamma_4\left(1-\rho^z_1\right) z_{1,t} + \gamma_5\left(1-\rho^z_2\right) z_{2,t} \\ &\quad + \gamma_b\left(\tilde{B}_{1,t-1} - \tilde{B}_{1,t}\right) \\ &= \gamma_1\left(1-\rho^{trade}_1\right)\xi^{trade}_{1,t} + \gamma_2\left(1-\rho^{trade}_2\right)\xi^{trade}_{2,t} + \gamma_3\left(1-\rho^{UIP}_1\right)\xi^{UIP}_{1,t} \\ &\quad + \gamma_4\left(1-\rho^z_1\right) z_{1,t} + \gamma_5\left(1-\rho^z_2\right) z_{2,t} \\ &\quad + \frac{\gamma_b}{\beta}\left(\left(\beta-1\right)\tilde{B}_{1,t-1} - \tilde{T}_{1,t}\right) \\ &= \left(\gamma_1\left(1-\rho^{trade}_1\right) - \frac{\gamma_b}{\beta}\left(\frac{\omega^c_1}{1-\omega^c_1} + \varpi\gamma_1\right)\right)\xi^{trade}_{1,t} \\ &\quad + \left(\gamma_2\left(1-\rho^{trade}_2\right) - \frac{\gamma_b}{\beta}\left(-\frac{\omega^c_1}{1-\omega^c_1} + \varpi\gamma_2\right)\right)\xi^{trade}_{2,t} \\ &\quad + \left(\gamma_3\left(1-\rho^{UIP}_1\right) - \frac{\gamma_b}{\beta}\varpi\gamma_3\right)\xi^{UIP}_{1,t} \\ &\quad + \left(\gamma_4\left(1-\rho^z_1\right) - \frac{\gamma_b}{\beta}\left(-1+\varpi\gamma_4\right)\right) z_{1,t} \\ &\quad + \left(\gamma_5\left(1-\rho^z_2\right) - \frac{\gamma_b}{\beta}\left(1+\varpi\gamma_5\right)\right) z_{2,t} \\ &\quad + \left(\frac{\gamma_b}{\beta}\left(\beta-1\right) - \frac{\gamma_b}{\beta}\varpi\gamma_b\right)\tilde{B}_{1,t-1}. \end{aligned} \tag{A.42}$$

Note that the UIP condition, Equation A.29, can be written as

$$
\begin{aligned}
\left(\hat{\delta}_{1,t} - E_t\hat{\delta}_{1,t+1}\right) \quad = \quad & -\frac{\chi}{\beta}\left(\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_1\right)\xi_{1,t}^{trade} \\
& -\frac{\chi}{\beta}\left(-\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_2\right)\xi_{2,t}^{trade} \\
& -\left(\frac{\chi}{\beta}\varpi\gamma_3 + 1\right)\xi_{1,t}^{UIP} \\
& -\left(\frac{\chi}{\beta}\left(-1 + \varpi\gamma_4\right) - \left(1-\rho_1^z\right)\right) z_{1,t} \\
& -\left(\frac{\chi}{\beta}\left(1 + \varpi\gamma_5\right) + \left(1-\rho_2^z\right)\right) z_{2,t} \\
& -\frac{\chi}{\beta}\left(\varpi\gamma_b + 1\right)\tilde{B}_{1,t-1}.
\end{aligned} \tag{A.43}
$$

Combining the expressions A.42 and A.43 yields the following condition that determines the coefficients of the decision rule for the terms of trade:

$$
\begin{aligned}
\left(\gamma_1\left(1-\rho_1^{trade}\right) - \frac{\gamma_b}{\beta}\left(\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_1\right)\right) &= -\frac{\chi}{\beta}\left(\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_1\right) \\
\left(\gamma_2\left(1-\rho_2^{trade}\right) - \frac{\gamma_b}{\beta}\left(-\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_2\right)\right) &= -\frac{\chi}{\beta}\left(-\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_2\right) \\
\left(\gamma_3\left(1-\rho_1^{UIP}\right) - \frac{\gamma_b}{\beta}\varpi\gamma_3\right) &= -\left(\frac{\chi}{\beta}\varpi\gamma_3 + 1\right) \\
\left(\gamma_4\left(1-\rho_1^z\right) - \frac{\gamma_b}{\beta}\left(-1+\varpi\gamma_4\right)\right) &= -\left(\frac{\chi}{\beta}\left(-1+\varpi\gamma_4\right) - \left(1-\rho_1^z\right)\right) \\
\left(\gamma_5\left(1-\rho_2^z\right) - \frac{\gamma_b}{\beta}\left(1+\varpi\gamma_5\right)\right) &= -\left(\frac{\chi}{\beta}\left(1+\varpi\gamma_5\right) + \left(1-\rho_2^z\right)\right) \\
\left(\frac{\gamma_b}{\beta}\left(\beta-1\right) - \frac{\gamma_b}{\beta}\varpi\gamma_b\right) &= -\frac{\chi}{\beta}\left(\varpi\gamma_b + 1\right).
\end{aligned}
$$

We present the final coefficients next.

A.4 Decision Rules

This subsection collects the decision roles of the main variables in our model, the terms of trade, the trade balance, and the NFA position. These three decision rules are used extensively obtaining the proofs of our theorems. Recall that we assume $\varpi > 0$ throughout the analysis.

A.4.1 Terms of Trade, $\hat{\delta}_{1,t}$

The terms of trade are a linear function of the exogenous shocks and the inherited NFA position, $\tilde{B}_{1,t-1}$

$$\hat{\delta}_{1,t} = \gamma_1 \xi_{1,t}^{trade} + \gamma_2 \xi_{2,t}^{trade} + \gamma_3 \xi_{1,t}^{UIP} + \gamma_4 z_{1,t} + \gamma_5 z_{2,t} + \gamma_b \tilde{B}_{1,t-1} \tag{A.44}$$

with the parameters

$$\gamma_1: \qquad \gamma_1 = \frac{\frac{\gamma_b-\chi}{\beta}\frac{\omega_1^c}{1-\omega_1^c}}{-\frac{\gamma_b-\chi}{\beta}\varpi + \left(1-\rho_1^{trade}\right)} = -\frac{\tilde{\gamma}_b \frac{1}{\varpi}\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_1^{trade}\right)} < 0 \tag{A.45}$$

$$\gamma_2: \qquad \gamma_2 = \frac{-\frac{\gamma_b-\chi}{\beta}\frac{\omega_1^c}{1-\omega_1^c}}{-\frac{\gamma_b-\chi}{\beta}\varpi + \left(1-\rho_2^{trade}\right)} = \frac{\tilde{\gamma}_b \frac{1}{\varpi}\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_2^{trade}\right)} > 0 \tag{A.46}$$

$$\gamma_3: \qquad \gamma_3 = \frac{-1}{-\frac{\gamma_b-\chi}{\beta}\varpi + (1-\rho_1^{UIP})} = -\frac{1}{\tilde{\gamma}_b + (1-\rho_1^{UIP})} < 0 \tag{A.47}$$

$$\gamma_4: \qquad \gamma_4 = \frac{-\frac{\gamma_b-\chi}{\beta} + (1-\rho_1^z)}{-\frac{\gamma_b-\chi}{\beta}\varpi + (1-\rho_1^z)} = \frac{\tilde{\gamma}_b \frac{1}{\varpi} + (1-\rho_1^z)}{\tilde{\gamma}_b + (1-\rho_1^z)} > 0 \tag{A.48}$$

$$\gamma_5: \qquad \gamma_5 = \frac{\frac{\gamma_b-\chi}{\beta} - (1-\rho_2^z)}{-\frac{\gamma_b-\chi}{\beta}\varpi + (1-\rho_2^z)} = -\frac{\tilde{\gamma}_b \frac{1}{\varpi} + (1-\rho_2^z)}{\tilde{\gamma}_b + (1-\rho_2^z)} < 0. \tag{A.49}$$

where the parameter $\tilde{\gamma}_b = -\frac{\gamma_b-\chi}{\beta}\varpi$ and the parameter γ_b is given by

$$\gamma_b \quad = \quad \frac{-(1-\beta-\chi\varpi) - \sqrt{(1-\beta-\chi\varpi)^2 + 4\chi\varpi}}{2\varpi} \tag{A.50}$$

which is the stable root associated with the quadratic equation

$$-\varpi\gamma_b^2 + (\beta - 1 + \chi\varpi)\,\gamma_b + \chi = 0. \tag{A.51}$$

The parameter γ_b is negative and decreasing in χ, i.e., $\frac{\partial \gamma_b}{\partial \chi} < 0$ for $0 < \beta < 1$. For the parameter $\tilde{\gamma}_b$ we therefore obtain $\frac{\partial \tilde{\gamma}_b}{\partial \chi} = \frac{\varpi}{\beta}\left(1 - \frac{\partial \gamma_b}{\partial \chi}\right) > 0$.

A.4.2 Trade Balance, $\tilde{T}_{1,t}$

Using Equation A.41, the trade balance is a linear function of the exogenous shocks and the inherited NFA position, $\tilde{B}_{1,t-1}$

$$\tilde{T}_{1,t} \quad = \quad \alpha_1 \xi_{1,t}^{trade} + \alpha_2 \xi_{2,t}^{trade} + \alpha_3 \xi_{1,t}^{UIP} + \alpha_4 z_{1,t} + \alpha_5 z_{2,t} + \alpha_b \tilde{B}_{1,t-1} \tag{A.52}$$

with the parameters

$$\alpha_1: \qquad \alpha_1 = \frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_1 = \frac{\left(1-\rho_1^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_1^{trade}\right)} > 0 \tag{A.53}$$

$$\alpha_2: \qquad \alpha_2 = -\frac{\omega_1^c}{1-\omega_1^c} + \varpi\gamma_2 = -\frac{\left(1-\rho_2^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_2^{trade}\right)} < 0 \tag{A.54}$$

$$\alpha_3: \qquad \alpha_3 = \varpi\gamma_3 = -\frac{\varpi}{\tilde{\gamma}_b + (1-\rho_1^{UIP})} < 0 \tag{A.55}$$

$$\alpha_4: \qquad \alpha_4 = -1 + \varpi\gamma_4 = \frac{(1-\rho_1^z)(\varpi-1)}{\tilde{\gamma}_b + (1-\rho_1^z)} \tag{A.56}$$

$$\alpha_5: \qquad \alpha_5 = 1 + \varpi\gamma_5 = \frac{-(1-\rho_2^z)(\varpi-1)}{\tilde{\gamma}_b + (1-\rho_2^z)} \tag{A.57}$$

$$\alpha_b: \qquad \alpha_b = \varpi\gamma_b < 0. \tag{A.58}$$

A.4.3 Net Foreign Assets, $\tilde{B}_{1,t}$

The trade balance is a linear function of the exogenous shocks and the inherited NFA position, $\tilde{B}_{1,t-1}$

$$\tilde{B}_{1,t} = \beta_1\xi_{1,t}^{trade} + \beta_2\xi_{2,t}^{trade} + \beta_3\xi_{1,t}^{UIP} + \beta_b\tilde{B}_{1,t-1} + \beta_4 z_{1,t} + \beta_5 z_{2,t} \tag{A.59}$$

with the parameters

$$\beta_1: \qquad \beta_1 = \frac{\alpha_1}{\beta} = \frac{1}{\beta}\frac{\left(1-\rho_1^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_1^{trade}\right)} > 0 \tag{A.60}$$

$$\beta_2: \qquad \beta_2 = \frac{\alpha_2}{\beta} = -\frac{1}{\beta}\frac{\left(1-\rho_2^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1-\rho_2^{trade}\right)} < 0 \tag{A.61}$$

$$\beta_3: \qquad \beta_3 = \frac{\alpha_3}{\beta} = -\frac{1}{\beta}\frac{\varpi}{\tilde{\gamma}_b + (1-\rho_1^{UIP})} < 0 \tag{A.62}$$

$$\beta_4: \qquad \beta_4 = \frac{\alpha_4}{\beta} = \frac{1}{\beta}\frac{(1-\rho_1^z)(\varpi-1)}{\tilde{\gamma}_b + (1-\rho_1^z)} \tag{A.63}$$

$$\beta_5: \qquad \beta_5 = \frac{\alpha_5}{\beta} = -\frac{1}{\beta}\frac{(1-\rho_2^z)(\varpi-1)}{\tilde{\gamma}_b + (1-\rho_2^z)} \tag{A.64}$$

$$\beta_b: \qquad \beta_b = \frac{\alpha_b + 1}{\beta} = \frac{\gamma_b}{\gamma_b - \chi}. \tag{A.65}$$

A.5 Statistical Moments

Using the decision rules of the model, we compute analytical expressions for statistical moments displayed in the main text and the proofs. To ease notation just within this subsection we define

$$\rho_1 = \rho_1^{trade} \tag{A.66}$$

$$\rho_2 = \rho_2^{trade} \tag{A.67}$$

$$\rho_3 = \rho_1^{UIP} \tag{A.68}$$

$$\rho_4 = \rho_1^{z} \tag{A.69}$$

$$\rho_5 = \rho_2^{z} \tag{A.70}$$

and similarly for the variances σ_i. Although we generally assume shock processes to be uncorrelated, we do allow for this possibility in the following. The correlations coefficients between two shocks are denote by ρ_{ij}.

From the decision role for $\tilde{B}_{1,t}$ in Equation A.59, the unconditional variance of $\tilde{B}_{1,t}$ is

$$E_t\left(\tilde{B}_{1,t}^2\right) = \sum_i \sum_j \left\{\frac{\beta_i\beta_j}{1-\beta_b^2}\frac{1+\rho_i\beta_b}{1-\rho_i\beta_b}\frac{\rho_{ij}\sigma_i\sigma_j}{1-\rho_i\rho_j}\right\} \tag{A.71}$$

where i and $j \in \{1,2,3,4,5\}$. The covariance between two exogenous shocks is given by

$$E_t\left(\xi_{i,t}\xi_{j,t}\right) = \frac{\rho_{ij}\sigma_i\sigma_j}{1-\rho_i\rho_j} \tag{A.72}$$

and the covariance between $\tilde{B}_{1,t}$ and shock $\xi_{i,t}$ is given by

$$E_t\left(\xi_{i,t}\tilde{B}_{1,t}\right) = \sum_j \left\{\frac{\beta_j}{1-\rho_i\beta_b}\frac{\rho_{ij}\sigma_i\sigma_j}{1-\rho_i\rho_j}\right\}. \tag{A.73}$$

Let the coefficients in the decision rules for variables x_t and y_t are denoted by γ and α, respectively. The variance/covariance of these two variables (expressed in deviations from the steady state) is

$$E_t\left(x_t y_t\right) = \sum_i \sum_j \left\{ \left(\gamma_i \alpha_j + \Omega_i \beta_j\right) \frac{\rho_{ij}\sigma_i\sigma_j}{1-\rho_i\rho_j} \right\} \tag{A.74}$$

where

$$\Omega_i = \left(\gamma_i \alpha_b + \gamma_b \alpha_i\right) \frac{\rho_i}{1-\rho_i\beta_b} + \gamma_b \alpha_b \frac{\beta_i}{1-\beta_b^2} \frac{1+\rho_i\beta_b}{1-\rho_i\beta_b}. \tag{A.75}$$

Here the coefficients β_i or β_b are those in the decision rules of the NFA position.

Similarly, to compute the moments for $\Delta x_{t,t-1} = x_t - x_{t-1}$ and $\Delta y_{t,t-1} = y_t - y_{t-1}$ (variables in expressed in growth rates), it is

$$E_t\left(\Delta x_{t,t-1} \Delta y_{t,t-1}\right) = \sum_i \sum_j \left\{ \left(d\gamma_i d\alpha_j + \gamma_i \alpha_j \left(1-\rho_i\rho_j\right) + \Gamma_i \beta_j\right) \frac{\rho_{ij}\sigma_i\sigma_j}{1-\rho_i\rho_j} \right\} \tag{A.76}$$

where the coefficients for the decision rules $\Delta x_{t,t-1}$ and $\Delta y_{t,t-1}$ relate to those of the original rules associated with x_t and y_t as follows

$$\Gamma_i = \left(d\gamma_i d\alpha_b + d\gamma_b d\alpha_i\right) \frac{\rho_i}{1-\rho_i\beta_b} + d\gamma_b d\alpha_b \frac{\beta_i}{1-\beta_b^2} \frac{1+\rho_i\beta_b}{1-\rho_i\beta_b} \tag{A.77}$$

$$d\gamma_i = \gamma_i \left(\rho_i - 1\right) + \gamma_b \beta_i \tag{A.78}$$

$$d\gamma_b = \gamma_b \left(\beta_b - 1\right) = \chi \beta_b \tag{A.79}$$

$$d\alpha_i = \alpha_i \left(\rho_i - 1\right) + \alpha_b \beta_i \tag{A.80}$$

$$d\alpha_b = \alpha_b \left(\beta_b - 1\right). \tag{A.81}$$

B Appendix: Proofs of Theorems

B.1 Proof of Theorem 1 and Corollary 1

Theorem 1 *A trade rebalancing shock that improves the home country's terms of trade (appreciates the real exchange rate),* $\xi_{1,t}^{trade} > 0$ *and/or* $\xi_{2,t}^{trade} < 0$*, is associated with an improvement of the trade balance. By contrast, a UIP shock that improves the terms of trade (appreciates the real exchange rate),* $\xi_{1,t}^{UIP} > 0$*, is associated with a deterioration of the trade balance. If financial markets provide less risk sharing due to higher intermediation costs, i.e.,* χ *assumes a higher value, the terms of trade are more (less) sensitive to the trade rebalancing (UIP) shock, and the trade balance is less sensitive to both shocks.*

Proof. We split the proof into two parts.

Claim 1: The home country's terms of trade improve (i.e., the real exchange rate appreciates) after a positive rebalancing shock towards the home country's good, $\xi_{1,t}^{trade} > 0$ and/or $\xi_{2,t}^{trade} < 0$, and a positive UIP shock $\xi_{1,t}^{UIP} > 0$. The magnitude of the terms of trade response to a given-sized shock is increasing in the value of χ for rebalancing shocks, but decreasing for the UIP shock.

Consider the decision rules for the terms of trade computed in Appendix A.4, where

$$\hat{\delta}_{1,t} = \gamma_1 \xi_{1,t}^{trade} + \gamma_2 \xi_{2,t}^{trade} + \gamma_3 \xi_{1,t}^{UIP} + \gamma_4 z_{1,t} + \gamma_5 z_{2,t} + \gamma_b \tilde{B}_{1,t-1}. \tag{B.1}$$

The coefficients of interest rate are γ_1 and γ_3, repeated here for convenience:

$$\gamma_1 = -\frac{\tilde{\gamma}_b \frac{1}{\varpi} \frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b + \left(1 - \rho_1^{trade}\right)} < 0 \tag{B.2}$$

$$\gamma_3 = -\frac{1}{\tilde{\gamma}_b + (1 - \rho_1^{UIP})} < 0 \tag{B.3}$$

where $\tilde{\gamma}_b = -\frac{\gamma_b - \chi}{\beta}\varpi$.

Recall that γ_b is negative and decreasing in χ, i.e., $\frac{\partial \gamma_b}{\partial \chi} < 0$ for $\beta > 0$. For the parameter $\tilde{\gamma}_b$ we therefore obtain $\frac{\partial \tilde{\gamma}_b}{\partial \chi} = \frac{\varpi}{\beta}\left(1 - \frac{\partial \gamma_b}{\partial \chi}\right) > 0$. Hence, the derivatives of the coefficients γ_1 and γ_3 with respect to χ are

$$\frac{\partial \gamma_1}{\partial \chi} = -\frac{\frac{1}{\varpi}\frac{\omega_1^c}{1-\omega_1^c}\left(1-\rho_1^{trade}\right)}{\left(\tilde{\gamma}_b+\left(1-\rho_1^{trade}\right)\right)^2}\frac{\partial \tilde{\gamma}_b}{\partial \chi} < 0 \tag{B.4}$$

$$\frac{\partial \gamma_3}{\partial \chi} = \frac{1}{\left(\tilde{\gamma}_b+(1-\rho_1^{UIP})\right)^2}\frac{\partial \tilde{\gamma}_b}{\partial \chi} > 0. \tag{B.5}$$

Equations B.2 - B.5 establish Claim 1.

Claim 2: The trade and UIP shocks move the terms of trade (the real exchange rate) in the same direction, but move the trade balance in opposite directions. The higher the bond price elasticity to the NFA position, χ, the smaller are the effects on the trade balance of given-size shocks. The effects on the terms of trade is larger (smaller) for the trade (UIP) shock, the larger the value of χ.

Consider the decision rules for the trade balance computed in Appendix A.4, where

$$\tilde{T}_{1,t} = \alpha_1\xi_{1,t}^{trade} + \alpha_2\xi_{2,t}^{trade} + \alpha_3\xi_{1,t}^{UIP} + \alpha_4 z_{1,t} + \alpha_5 z_{2,t} + \alpha_b\tilde{B}_{1,t-1} \tag{B.6}$$

The coefficients of interest rate are α_1 and α_3, repeated here for convenience:

$$\alpha_1 = \frac{\left(1-\rho_1^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\tilde{\gamma}_b+\left(1-\rho_1^{trade}\right)} > 0 \tag{B.7}$$

$$\alpha_3 = -\frac{\varpi}{\tilde{\gamma}_b+(1-\rho_1^{UIP})} < 0. \tag{B.8}$$

The derivatives of the coefficients α_1 and α_3 with respect to χ are

$$\frac{\partial \alpha_1}{\partial \chi} = -\frac{\left(1-\rho_1^{trade}\right)\frac{\omega_1^c}{1-\omega_1^c}}{\left(\tilde{\gamma}_b+\left(1-\rho_1^{trade}\right)\right)^2}\frac{\partial \tilde{\gamma}_b}{\partial \chi} < 0 \tag{B.9}$$

$$\frac{\partial \alpha_3}{\partial \chi} = \frac{\varpi}{\left(\tilde{\gamma}_b+(1-\rho_1^{UIP})\right)^2}\frac{\partial \tilde{\gamma}_b}{\partial \chi} > 0. \tag{B.10}$$

Equations B.7 - B.10 establish Claim 2.

The theorem follows directly from the two claims. ■

These features of the decision rules are also reflected in the unconditional moments of the trade balance and the terms of trade. Abstracting from technology shocks to simplify the exposition, the following corollary applies.

Corollary 1 *The UIP shock induces a positive covariance between the growth rate of the terms of trade and the growth rate of the trade balance. The trade rebalancing shock induces a negative covariance between the two growth rates. When both shocks are present in the model, the overall covariance is determined by the extent of costly financial intermediation as measured by χ.*

Proof. The covariance between the growth rate of the terms of trade and the growth rate of the trade balance is

$$\begin{aligned} cov\left(\Delta\hat{\delta}_{1,t}, \Delta\tilde{T}_{1,t}\right) &= cov\left(\Delta\hat{\delta}_{1,t}, \frac{\omega_1^c}{1-\omega_1^c}\Delta\xi_{1,t}^{trade} + \varpi\Delta\hat{\delta}_{1,t}\right) \\ &= \frac{\omega_1^c}{1-\omega_1^c} cov\left(\Delta\hat{\delta}_{1,t}, \Delta\xi_{1,t}^{trade}\right) + \varpi var\left(\Delta\hat{\delta}_{1,t}\right) \end{aligned} \tag{B.11}$$

where, using Equation A.76,

$$\begin{aligned} cov\left(\Delta\hat{\delta}_{1,t}, \Delta\xi_{1,t}^{trade}\right) &= -\left(1-\rho_1^{trade}\right)\left[d\gamma_1 - \left(1+\rho_1^{trade}\right)\gamma_1 + \gamma_b\frac{\rho_1^{trade}\beta_1}{1-\rho_1^{trade}\beta_b}\right] var\left(\xi_{1,t}^{trade}\right) \\ &= \left[\left(1-\left(\rho_1^{trade}\right)^2\right)\gamma_1 - \frac{\left(1-\rho_1^{trade}\right)\chi\beta_1}{1-\rho_1^{trade}\beta_b}\right] var\left(\xi_{1,t}^{trade}\right) \\ &= -\chi\beta_1\left[\frac{1+\rho_1^{trade}}{1-\beta_b} + \frac{1-\rho_1^{trade}}{1-\rho_1^{trade}\beta_b}\right] var\left(\xi_{1,t}^{trade}\right) < 0 \end{aligned} \tag{B.12}$$

and

$$\begin{aligned} var\left(\Delta\hat{\delta}_{1,t}\right) &= \left[d\gamma_1^2 + 2d\gamma_1 d\gamma_b\frac{\rho_1^{trade}\beta_1}{1-\rho_1^{trade}\beta_b} + d\gamma_b^2\beta_1^2\frac{1}{1-\beta_b^2}\frac{1+\rho_1^{trade}\beta_b}{1-\rho_1^{trade}\beta_b}\right] var\left(\xi_{1,t}^{trade}\right) \\ &\quad + \left(1-\left(\rho_1^{trade}\right)^2\right)\gamma_1^2 var\left(\xi_{1,t}^{trade}\right) \\ &\quad + \left[d\gamma_3^2 + 2d\gamma_3 d\gamma_b\frac{\rho_1^{UIP}\beta_3}{1-\rho_1^{UIP}\beta_b} + d\gamma_b^2\beta_3^2\frac{1}{1-\beta_b^2}\frac{1+\rho_1^{UIP}\beta_b}{1-\rho_1^{UIP}\beta_b}\right] var\left(\xi_{1,t}^{UIP}\right) \\ &\quad + \left(1-\left(\rho_1^{UIP}\right)^2\right)\gamma_3^2 var\left(\xi_{1,t}^{UIP}\right) \\ &= \frac{\chi^2\beta_1^2}{1-\beta_b^2}\frac{1+\rho_1^{trade}\beta_b}{1-\rho_1^{trade}\beta_b} var\left(\xi_{1,t}^{trade}\right) + \gamma_1^2\left(1-\left(\rho_1^{trade}\right)^2\right) var\left(\xi_{1,t}^{trade}\right) \\ &\quad + \frac{\chi^2\beta_3^2}{1-\beta_b^2}\frac{1+\rho_1^{UIP}\beta_b}{1-\rho_1^{UIP}\beta_b} var\left(\xi_{1,t}^{UIP}\right) + \left(1+2\frac{\chi\beta_3}{1-\rho_1^{UIP}\beta_b}\right) var\left(\xi_{1,t}^{UIP}\right) \\ &\quad + \gamma_3^2\left(1-\left(\rho_1^{UIP}\right)^2\right) var\left(\xi_{1,t}^{UIP}\right) \\ &= var\left(\Delta\hat{\delta}_{1,t}|\xi_{1,t}^{trade}\right) + var\left(\Delta\hat{\delta}_{1,t}|\xi_{1,t}^{UIP}\right). \end{aligned} \tag{B.13}$$

The overall variance of the terms of trade is the sum of the terms of trade variance that is due to the rebalancing shock and the terms of trade variance that is due to the UIP shock.

The variance of the change in the trade balance is given by

$$\begin{aligned} var\left(\Delta\tilde{T}_{1,t}\right) &= \left(\frac{\omega_1^c}{1-\omega_1^c}\right)^2 var\left(\Delta\xi_{1,t}^{trade}\right) + \varpi^2 var\left(\Delta\hat{\delta}_{1,t}\right) \\ &\quad +2\left(\frac{\omega_1^c}{1-\omega_1^c}\right)\varpi cov\left(\Delta\hat{\delta}_{1,t},\Delta\xi_{1,t}^{trade}\right). \end{aligned} \tag{B.14}$$

To understand the comovement between the changes in the terms of trade and the trade balance, we consider UIP and rebalancing shocks in turns. If the model admits UIP shocks only, it is $\frac{\omega_1^c}{1-\omega_1^c} cov\left(\Delta\hat{\delta}_{1,t},\Delta\xi_{1,t}^{trade}\right)=0$ and

$$cov\left(\Delta\hat{\delta}_{1,t},\Delta\tilde{T}_{1,t}|\xi_{1,t}^{UIP}\right) = \varpi var\left(\Delta\hat{\delta}_{1,t}|\xi_{1,t}^{UIP}\right) > 0. \tag{B.15}$$

Because $var\left(\Delta\tilde{T}_{1,t}\right)=\varpi^2 var\left(\Delta\hat{\delta}_{1,t}|\xi_{1,t}^{UIP}\right)$, the associated correlation coefficient is equal to 1.

If the model admits rebalancing shocks only,

$$\begin{aligned} cov\left(\Delta\hat{\delta}_{1,t},\Delta\tilde{T}_{1,t}|\xi_{1,t}^{trade}\right) &= \Bigg[-\frac{\omega_1^c}{1-\omega_1^c}\frac{\left(1-\rho_1^{trade}\right)\chi\beta_1}{1-\rho_1^{trade}\beta_b} + \varpi\frac{\chi^2\beta_1^2}{1-\beta_b^2}\frac{1+\rho_1^{trade}\beta_b}{1-\rho_1^{trade}\beta_b} \\ &\quad + \frac{\omega_1^c}{1-\omega_1^c}\left(1-\rho_1^{trade}\right)\gamma_1 + \varpi\gamma_1^2\left(1-\left(\rho_1^{trade}\right)^2\right)\Bigg] var\left(\xi_{1,t}^{trade}\right) \\ &= -\frac{\omega_1^c}{1-\omega_1^c}\chi\beta_1\left[\frac{\left(1-\rho_1^{trade}\right)^2}{1-\rho_1^{trade}\beta_b}\frac{\chi\beta_b+\frac{\beta}{\varpi}\left(1-\beta_b^2\right)}{\chi\frac{1+\beta_b}{1-\rho_1^{trade}}+\frac{\beta}{\varpi}\left(1-\beta_b^2\right)} \right. \\ &\quad \left. +\frac{1-\left(\rho_1^{trade}\right)^2}{1-\beta_b}\right] var\left(\xi_{1,t}^{trade}\right) < 0. \end{aligned} \tag{B.16}$$

The negative covariance implies that the associated correlation coefficient is also negative (but larger than -1).

With the rebalancing shock inducing negative correlation between $\Delta\hat{\delta}_{1,t}$ and $\Delta\tilde{T}_{1,t}$ and the UIP shock inducing positive correlation, the covariance in a model with both shocks being active depends, among other parameters, on the extent of international risk sharing as governed by the value of χ. ■

B.2 Proof of Theorem 2 and Theorem 3

Theorem 2 *Abstracting from technology shocks, the ratio of the standard deviation of the real exchange rate, $\hat{q}_{1,t}$, and consumption, $\hat{C}_{1,t}$, is independent of the relative variances of the trade rebalancing and the UIP shocks,*

$$\frac{std\left(\hat{q}_{1,t}\right)}{std\left(\hat{C}_{1,t}\right)} = \frac{std\left(\Delta\hat{q}_{1,t}\right)}{var\left(\Delta\hat{C}_{1,t}\right)} = \frac{2\omega_1^c - 1}{1-\omega_1^c}. \tag{B.17}$$

The correlation between relative consumption, $\hat{C}_{1,t} - \hat{C}_{2,t}$, and the real exchange rate is equal to minus one regardless of the relative variances of the trade rebalancing and the UIP shocks,

$$corr\left(\hat{C}_{1,t} - \hat{C}_{2,t}, \hat{q}_{1,t}\right) = -1. \tag{B.18}$$

Proof. Absent technology shocks the consumption-real-exchange-rate variance ratio is

$$\frac{std\left(\hat{q}_{1,t}\right)}{std\left(\hat{C}_{1,t}\right)} = \frac{\sqrt{\left(2\omega_1^c - 1\right)^2 var\left(\hat{\delta}_{1,t}\right)}}{\sqrt{\left(1-\omega_1^c\right)^2 var\left(\hat{\delta}_{1,t}\right)}} = \frac{2\omega_1^c - 1}{1-\omega_1^c}. \tag{B.19}$$

The same applies when expressing the variables in growth rates instead of levels.

For the correlation between relative consumption and the real exchange rate it is

$$corr\left(\hat{C}_{1,t} - \hat{C}_{2,t}, \hat{q}_{1,t}\right) = \frac{-2\left(1-\omega_1^c\right)\left(2\omega_1^c - 1\right)E\left(\hat{\delta}_{1,t}^2\right)}{\sqrt{4(1-\omega_1^c)^2 E\left(\hat{\delta}_{1,t}^2\right)}\sqrt{(2\omega_1^c - 1)^2 E\left(\hat{\delta}_{1,t}^2\right)}} = -1. \tag{B.20}$$

Neither the UIP nor the rebalancing shock enter directly into the equations determining the real exchange rate and consumption. Both shocks enter only indirectly through the terms of trade. Hence, the computed moments do not depend on the relative variances of the rebalancing and the UIP shock. ■

Theorem 3 *Suppose the model admits only rebalancing and UIP shocks. In that case, the Fama coefficient is constant and negative independent of the degree of costly* *financial intermediation as measured by χ, as long as $\chi \neq 0$:*

$$\hat{\beta}^{Fama} = \frac{cov\left(E_t \Delta \hat{q}_{1,t+1}, r_{1,t} - r_{2,t}\right)}{var\left(r_{1,t} - r_{2,t}\right)} = -\frac{2\omega_1^c - 1}{2\left(1 - \omega_1^c\right)} = 1 - \frac{1}{2\left(1 - \omega_1^c\right)} < 0 \quad \text{(B.21)}$$

for $\omega_1^c > \frac{1}{2}$.

Proof. We assume that rebalancing shocks and UIP shocks are the only shocks in the model. All shocks are uncorrelated with each other. First, note that using the UIP condition, Equation A.39, it is

$$\begin{aligned} cov\left(\Delta \hat{q}_{1,t+1}, r_{1,t} - r_{2,t}\right) &= var\left(r_{1,t} - r_{2,t}\right) + \chi cov\left(\Delta \hat{q}_{1,t+1}, \tilde{B}_{1,t}\right) \\ &\quad + cov\left(\Delta \hat{q}_{1,t+1}, \xi_{1,t}^{UIP}\right) - var\left(\chi \tilde{B}_{1,t} + \xi_{1,t}^{UIP}\right) \end{aligned}$$

which, after applying the relationship $\hat{q}_{1,t} = \left(2\omega_1^c - 1\right)\hat{\delta}_{1,t}$, implies that the Fama coefficient can be stated as

$$\begin{aligned} \hat{\beta}^{Fama} &= 1 + \frac{\left(2\omega_1^c - 1\right)}{var\left(r_{1,t} - r_{2,t}\right)}\left[\chi cov\left(\Delta \hat{\delta}_{1,t+1}, \tilde{B}_{1,t}\right) + cov\left(\Delta \hat{\delta}_{1,t+1}, \xi_{1,t}^{UIP}\right)\right] \\ &\quad - \frac{1}{var\left(r_{1,t} - r_{2,t}\right)}\left[\chi^2 var\left(\tilde{B}_{1,t}\right) + 2\chi cov\left(\tilde{B}_{1,t}, \xi_{1,t}^{UIP}\right) + var\left(\xi_{1,t}^{UIP}\right)\right]. \end{aligned} \quad \text{(B.22)}$$

The decision rules presented in Appendix A.4, imply that the terms entering Equation B.22 can be expressed solely in terms of the underlying exogenous shocks:

$$\begin{aligned} & cov\left(\Delta \hat{\delta}_{1,t+1}, \tilde{B}_{1,t}\right) = E\left(\left[d\gamma_1 \xi_{1,t}^{trade} + d\gamma_3 \xi_{1,t}^{UIP} + d\gamma_b \tilde{B}_{1,t-1}\right]\tilde{B}_{1,t}\right) \\ & = \chi\beta_1\left(1 + \rho_1^{trade}\beta_b\right)E\left(\xi_{1,t}^{trade}\tilde{B}_{1,t}\right) + \left[1 + \chi\beta_3\left(1 + \rho_1^{UIP}\beta_b\right)\right]E\left(\xi_{1,t}^{UIP}\tilde{B}_{1,t}\right) + \chi\beta_b^2 E\left(\tilde{B}_{1,t}^2\right) \\ & = \chi\frac{\beta_1^2}{1 - \beta_b^2}\frac{1 + \rho_1^{trade}\beta_b}{1 - \rho_1^{trade}\beta_b}var\left(\xi_{1,t}^{trade}\right) + \left[\chi\frac{\beta_3^2}{1 - \beta_b^2}\frac{1 + \rho_1^{UIP}\beta_b}{1 - \rho_1^{UIP}\beta_b} + \frac{\beta_3}{1 - \rho_1^{UIP}\beta_b}\right]var\left(\xi_{1,t}^{UIP}\right) \end{aligned}$$

$$\begin{aligned} & cov\left(\Delta \hat{\delta}_{1,t+1}, \xi_{1,t}^{UIP}\right) = E\left(\left[d\gamma_1 \xi_{1,t}^{trade} + d\gamma_3 \xi_{1,t}^{UIP} + d\gamma_b \tilde{B}_{1,t-1}\right]\xi_{1,t}^{UIP}\right) \\ & = \left(1 + \frac{\chi\beta_3}{1 - \rho_1^{UIP}\beta_b}\right)var\left(\xi_{1,t}^{UIP}\right) \end{aligned}$$

$$\chi^2 var\left(\tilde{B}_{1,t}\right) + 2\chi cov\left(\tilde{B}_{1,t}, \xi_{1,t}^{UIP}\right) + var\left(\xi_{1,t}^{UIP}\right) = \chi^2\frac{\beta_1^2}{1 - \beta_b^2}\frac{1 + \rho_1^{trade}\beta_b}{1 - \rho_1^{trade}\beta_b}var\left(\xi_{1,t}^{trade}\right)$$

$$+\left[\chi^2\frac{\beta_3^2}{1-\beta_b^2}\frac{1+\rho_1^{UIP}\beta_b}{1-\rho_1^{UIP}\beta_b}+\left(1+\frac{2\chi\beta_3}{1-\rho_1^{UIP}\beta_b}\right)\right]var\left(\xi_{1,t}^{UIP}\right)$$

which equals the sum of the two preceding terms, $\Sigma_{\Delta\hat{\delta}_{1,t+1},\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)}=\chi cov\left(\Delta\hat{\delta}_{1,t+1},\tilde{B}_{1,t}\right)+cov\left(\Delta\hat{\delta}_{1,t+1},\xi_{1,t}^{UIP}\right)$.

Before turning to the variance of the interest rate differential, $r_{1,t}-r_{2,t}$, we first establish that the variance of the growth rate of the terms of trade, $\Delta\hat{\delta}_{1,t+1}$, is

$$\begin{aligned}
var\left(\Delta\hat{\delta}_{1,t+1}\right) &= E\left(\left[d\gamma_1\xi_{1,t}^{trade}+d\gamma_3\xi_{1,t}^{UIP}+d\gamma_b\tilde{B}_{1,t-1}\right]^2\right)\\
&= \frac{\chi^2\beta_1^2}{1-\beta_b^2}\frac{1+\rho_1^{trade}\beta_b}{1-\rho_1^{trade}\beta_b}var\left(\xi_{1,t}^{trade}\right)\\
&\quad+\frac{\chi^2\beta_3^2}{1-\beta_b^2}\frac{1+\rho_1^{UIP}\beta_b}{1-\rho_1^{UIP}\beta_b}var\left(\xi_{1,t}^{UIP}\right)\\
&\quad+\left(1+2\frac{\chi\beta_3}{1-\rho_1^{UIP}\beta_b}\right)var\left(\xi_{1,t}^{UIP}\right)\\
&= \Sigma_{\Delta\hat{\delta}_{1,t+1},\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)} \qquad \text{(B.23)}
\end{aligned}$$

Finally, we obtain for the variance of the interest rate differential, $r_{1,t}-r_{2,t}$, that

$$\begin{aligned}
var\left(r_{1,t}-r_{2,t}\right) &= var\left(\Delta\hat{q}_{1,t+1}\right)-2cov\left(\Delta\hat{q}_{1,t+1},\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)\\
&\quad+var\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)\\
&= \left(2\omega_1^c-1\right)^2var\left(\Delta\hat{\delta}_{1,t+1}\right)-\left(4\omega_1^c-3\right)\Sigma_{\Delta\hat{\delta}_{1,t},\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)}\\
&= 4\left(1-\omega_1^c\right)^2\Sigma_{\Delta\hat{\delta}_{1,t},\left(\chi\tilde{B}_{1,t}+\xi_{1,t}^{UIP}\right)}. \qquad \text{(B.24)}
\end{aligned}$$

Applying these findings in Equation B.22, the Fama coefficient reduces to

$$\hat{\beta}^{Fama} = -\frac{2\omega_1^c-1}{2\left(1-\omega_1^c\right)}=1-\frac{1}{2\left(1-\omega_1^c\right)}. \qquad \text{(B.25)}$$

■

C Appendix: Model Extensions and Sensitivity

This section presents details some model extensions:

1. portfolio adjustment costs and borrowing constraints,
2. tariffs and iceberg trade cost,
3. generalized labour supply,
4. endogenous discounting.

C.1 Portfolio Adjustment Costs and Borrowing Constraints

Costly financial intermediation is one of many ways to restrict international borrowing and lending and thus consumption risk sharing across countries in international macro models. In this section we lay out two alternative approaches and show how they relate to our modeling choices in the main text.

Consider the household budget constraint

$$P^c_{1,t+j}C_{1,t+j} + P^b_{1,t+j}B_{1,t+j} + J_{1,t+j} = W_{1,t+j}L_{1,t+j} + B_{1,t-1+j} \tag{C.1}$$

where the term $J_{1,t+j}$ stands for mechanisms that limit the ability of households to issue/hold debt. In the case of costly financial intermediation used in the main text it is

$$J_{1,t+j} = \left(\frac{1}{\phi^b_{1,t+j}} - 1\right) P^b_{1,t+j}B_{1,t+j} \tag{C.2}$$

where, as shown in more detail below, $\phi^b_{1,t+j}$ is a function of aggregate aggregate amount of bonds issued.

Here we present two alternative approaches:

1. a quadratic portfolio adjustment cost, as in Fukui, Nakamura, and Steinsson (2023) or Guo, Ottonello, and Perez (2023), with

$$J_{1,t+j} = \frac{1}{2}\tau\left(\frac{B_{1,t+j}}{P^d_{1,t+j}M^*_{2,t+j}}\right)^2 P^d_{1,t+j}M^*_{2,t+j} \tag{C.3}$$

where here the costs are measured by household bond holdings relative to the aggregate value of exports (not the individual household's choice), $P^d_{1,t+j}M^*_{2,t+j}$;

2. a borrowing constraint on debt, as in de Groot, Durdu, and Mendoza (2023), with

$$J_{1,t+j} = -\mu_{1,t+j}\left(B_{1,t+j} + \bar{B}\right) \tag{C.4}$$

where household assets cannot fall below the adhoc level $-\bar{B}$, i.e., $B_{1,t+j} > -\bar{B}$. Denoting the Lagrange multiplier on the time-t budget constraint by $\beta^t\lambda_{1,t+j}$, the borrowing constraint can be absorbed into the budget constraint since $\beta^t\lambda_{1,t+j}\mu_{1,t+j}\left(B_{1,t+j} + \bar{B}\right) = 0$.

As for the case of costly financial intermediation, the portfolio adjustment cost affects the household's ability to smooth consumption because the level of asset holdings directly influences the price of debt. When asset holdings are high, higher borrowing costs discourage further accumulation of debt and limits risk sharing (or consumption smoothing). Borrowing constraints introduce a hard limit on the quantity of bonds in circulation. Once the constraint binds, no more bonds can be issued. The closer the quantity of outstanding debt is to the limit $\bar{B}$, the higher the cost of borrowing.[20]

C.2 Tariffs and Iceberg Trade Costs

In this section, we show that the rebalancing shock is closely related to shocks to tariffs and trade costs. In detail, we distinguish between import tariffs, export subsidies, and iceberg trade costs.

We denote the import price of country 1 by $P^m_{1,t}$ and the producer price in country 2 by $P^d_{2,t}$. The different trading frictions affect international prices as follows:

[20] Costly financial intermediation and portfolio adjustment costs are widely used in the international macro literature to render the dynamics of the NFA position stationary under popular local perturbation methods. See Schmitt-Grohé and Uribe (2003), Bodenstein (2011) and references therein. As shown in de Groot, Durdu, and Mendoza (2023), under suitable parameter choices, these two approaches imply similar dynamics for the endogenous variables as the use of borrowing constraints, for which obtaining the equilibrium dynamics requires global solution methods and numerical algorithms.

- under iceberg trade costs a share $\tau^{ice}_{1,t}$ of the shipped good is lost in the shipping process, implying an import price $P^m_{1,t}$ to be

$$P^m_{1,t} = \frac{1}{1-\tau^{ice}_{1,t}} e_{1,t} P^d_{2,t} \tag{C.5}$$

- under an import tariff $\tau^m_{1,t}$ the import price $P^m_{1,t}$ increases over the producer price according to

$$P^m_{1,t} = (1+\tau^m_{1,t}) e_{1,t} P^d_{2,t} \tag{C.6}$$

- under an export subsidy $\tau^x_{2,t}$ the import price $P^m_{1,t}$ falls below the producer price according to

$$P^m_{1,t} = \left(1-\tau^x_{2,t}\right) e_{1,t} P^d_{2,t} = e_{1,t} \tilde{P}^d_{2,t} \tag{C.7}$$

If all three elements are present, the following relationship applies between the import price of country 1 and the foreign production price of country 2

$$P^m_{1,t} = (1+\tau^m_{1,t}) \frac{1-\tau^x_{2,t}}{1-\tau^{ice}_{1,t}} e_{1,t} P^d_{2,t} = \frac{1+\tau^m_{1,t}}{1-\tau^{ice}_{1,t}} e_{1,t} \tilde{P}^d_{2,t}. \tag{C.8}$$

Similarly, import prices of country 2 and the foreign production price of country 1 are related via

$$P^m_{2,t} = (1+\tau^m_{2,t}) \frac{1-\tau^x_{1,t}}{1-\tau^{ice}_{2,t}} \frac{1}{e_{1,t}} P^d_{1,t} = \frac{1+\tau^m_{2,t}}{1-\tau^{ice}_{2,t}} \frac{1}{e_{1,t}} \tilde{P}^d_{1,t}. \tag{C.9}$$

Using Equations A.3 and A.4, the relative prices $\frac{P^c_{1,t}}{P^d_{1,t}}$ and $\frac{P^c_{2,t}}{P^d_{2,t}}$ are shown to be

$$\frac{P^c_{1,t}}{P^d_{1,t}} = \left[\omega^c_{1,t} + (1-\tilde{\omega}^c_{1,t}) \delta_{1,t}^{-\frac{1}{\rho^c}}\right]^{-\rho^c} = F_{1,t}^{-\rho^c} \tag{C.10}$$

$$\frac{P^c_{2,t}}{P^d_{2,t}} = \left[\omega^c_{2,t} + (1-\tilde{\omega}^c_{2,t}) \delta_{1,t}^{\frac{1}{\rho^c}}\right]^{-\rho^c} = F_{2,t}^{-\rho^c} \tag{C.11}$$

where

$$1-\tilde{\omega}^c_{1,t} = (1-\omega^c_{1,t}) \left((1+\tau^m_{1,t}) \frac{1-\tau^x_{2,t}}{1-\tau^{ice}_{1,t}} \right)^{-\frac{1}{\rho^c}} \tag{C.12}$$

$$1-\tilde{\omega}^c_{2,t} = \left(1-\omega^c_{2,t}\right) \left((1+\tau^m_{2,t}) \frac{1-\tau^x_{1,t}}{1-\tau^{ice}_{2,t}} \frac{1}{\delta_{1,t}} \right)^{-\frac{1}{\rho^c}}. \tag{C.13}$$

In the presence of iceberg trade costs the market clearing condition is given by

$$Y_{1,t} = C^d_{1,t} + \frac{1}{1-\tau^{ice}_{2,t}} M_{2,t} \tag{C.14}$$

$$Y_{2,t} = C^d_{2,t} + \frac{1}{1-\tau^{ice}_{1,t}} M_{1,t} \tag{C.15}$$

where $M_{1,t}$ and $M_{2,t}$ denote the final consumption of imports (net of iceberg costs).

The government's net receipts from import tariffs and export subsidies amount to

$$U_{t,1} = \tau^m_{1,t} e_{1,t} \tilde{P}^d_{2,t} \frac{M_{1,t}}{1-\tau^{ice}_{1,t}} - \tau^x_{1,t} P^d_{1,t} \frac{M_{2,t}}{1-\tau^{ice}_{2,t}}. \tag{C.16}$$

From the consolidated budget constraint we obtain

$$P^d_{1,t} C^d_{1,t} + P^m_{1,t} M_{1,t} + \frac{P^b_{1,t}}{\phi^b_{1,t}} B_{1,t} \;=\; P^d_{1,t} C^d_{1,t} + P^d_{1,t} \frac{M_{2,t}}{1-\tau^{ice}_{2,t}} + B_{1,t-1} + U_{t,1} \tag{C.17}$$

or

$$\frac{P^b_{1,t} B_{1,t}}{\phi^b_{1,t}} = T_{1,t} + B_{1,t-1} \tag{C.18}$$

with

$$T_{1,t} \;=\; P^d_{1,t} \frac{M_{2,t}}{1-\tau^{ice}_{2,t}} - P^m_{1,t} M_{1,t} + Z_{t,1} \tag{C.19}$$

$$\;=\; \frac{1-\tau^x_{1,t}}{1-\tau^{ice}_{2,t}} P^d_{1,t} M_{2,t} - \frac{1-\tau^x_{2,t}}{1-\tau^{ice}_{1,t}} e_{1,t} P^d_{2,t} M_{1,t} \tag{C.20}$$

We define

$$\tilde{T}_{1,t} \;=\; \frac{T_{1,t}}{\frac{1-\tau^x_{1,t}}{1-\tau^{ice}_{2,t}} P^d_{1,t} M_{2,t}} = 1 - \frac{1-\tau^x_{2,t}}{1-\tau^x_{1,t}} \frac{1-\tau^{ice}_{2,t}}{1-\tau^{ice}_{1,t}} \delta_{1,t} \frac{M_{1,t}}{M_{2,t}} \tag{C.21}$$

As the first order conditions for consumption are unchanged, the model dynamics can be summarized by the same three equations as before:

$$E_t \left\{ \frac{P^c_{1,t}}{F^{\rho^c}_{1,t+1} P^c_{1,t+1}} \left[\phi^b_{1,t} \frac{exp(z_{1,t})}{exp(z_{1,t+1})} - \phi^b_{2,t} \frac{exp(z_{2,t})}{exp(z_{2,t+1})} \frac{\delta_{1,t}}{\delta_{1,t+1}} \right] \right\} = 0 \tag{C.22}$$

$$\frac{P^b_{1,t} \tilde{B}_{1,t}}{\phi^b_{1,t}} = \tilde{T}_{1,t} + \frac{\frac{1-\tau^x_{1,t-1}}{1-\tau^{ice}_{2,t-1}} P^d_{1,t-1} M_{2,t-1}}{\frac{1-\tau^x_{1,t}}{1-\tau^{ice}_{2,t}} P^d_{1,t} M_{2,t}} \tilde{B}_{1,t-1} \tag{C.23}$$

with

$$\tilde{T}_{1,t} = 1 - \frac{1-\tilde{\omega}^c_{1,t}}{1-\tilde{\omega}^c_{2,t}} \frac{1+\tau^m_{2,t}}{1+\tau^m_{1,t}} \left(\frac{F_{1,t}}{F_{2,t}}\right)^{-1} \frac{exp(z_{1,t})}{exp(z_{2,t})} \delta_{1,t}^{1-2\frac{1+\rho^c}{\rho^c}}$$
$$= 1 - \frac{1+\tau^m_{2,t}}{1+\tau^m_{1,t}} \left(\frac{\frac{\omega^c_{1,t}}{1-\tilde{\omega}^c_{1,t}} + \delta_{1,t}^{-\frac{1}{\rho^c}}}{\frac{\omega^c_{2,t}}{1-\tilde{\omega}^c_{2,t}} + \delta_{1,t}^{\frac{1}{\rho^c}}} \right)^{-1} \frac{exp(z_{1,t})}{exp(z_{2,t})} \delta_{1,t}^{1-2\frac{1+\rho^c}{\rho^c}}. \quad (C.24)$$

C.2.1 Linearized Model

We assume a symmetric steady state with $\omega^c_1 = \omega^c_2 = \omega^c$, $\tau^i_1 = \tau^i_2 = \tau^i$ with $i \in \{m, x, ice\}$ and $\delta_1 = 1$, $\tilde{B}_1 = 0$, the dynamics around the steady state are approximated by the equations

$$(z_{1,t} - E_t z_{1,t+1}) - (z_{2,t} - E_t z_{2,t+1}) - \left(\hat{\delta}_{1,t} - E_t \hat{\delta}_{1,t+1}\right) = \chi \tilde{B}_{1,t} + \xi^{UIP}_{1,t} \quad (C.25)$$
$$\beta \tilde{B}_{1,t} = \tilde{T}_{1,t} + \tilde{B}_{1,t-1} \quad (C.26)$$
$$\tilde{T}_{1,t} = \frac{\bar{\omega}^c}{1-\bar{\omega}^c}\left(\xi^{trade}_{1,t} - \xi^{trade}_{2,t}\right) - (z_{1,t} - z_{2,t}) + \bar{\varpi} \hat{\delta}_{1,t} \quad (C.27)$$

where we now define $\bar{\varpi} = 1 + 2\frac{\omega^c}{\rho^c}\frac{1}{1-(\tilde{\omega}^c-\omega^c)}$. The trade shock reflects movements in the four underlying shocks to preferences, import tariffs, export subsidies, and transportation costs:

$$\frac{\bar{\omega}^c}{1-\bar{\omega}^c}\xi^{trade}_{1,t} = \frac{1}{1-(\tilde{\omega}^c-\omega^c)} \frac{\omega^c}{1-\omega^c} \xi^c_{1,t}$$
$$+ \left(1 + \frac{1}{\rho^c} \frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)}\right) \frac{\tau^m}{1-\tau^m} \xi^m_{1,t}$$
$$- \frac{1}{\rho^c} \frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)} \frac{\tau^x}{1-\tau^x} \xi^x_{2,t}$$
$$+ \frac{1}{\rho^c} \frac{\omega^c}{1-(\tilde{\omega}^c-\omega^c)} \frac{\tau^{ice}}{1-\tau^{ice}} \xi^{ice}_{1,t}. \quad (C.28)$$

The solution for the terms of trade, the trade balance, and the NFA position in the model with differentiated trade shocks is isomorphic with the solution to the model with a single trade rebalancing shock.

C.3 Labor Supply

Our baseline preference specification assumes perfectly elastic labor supply. While convenient to derive theoretical predictions, this assumption could limit the generality of our results. We explore this issue in this section. Let the intertemporal preferences of the household be

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \ln\left(C_{1,t+j}\right) - \frac{\nu_0}{1+\nu} L_{1,t+j}^{1+\nu} \right\}. \tag{C.29}$$

Focusing on the first-order conditions of the home country, this generalization in preferences only affects the first-order condition with respect to labor. Consequently, the consumption-labor trade-off is

$$\nu_0 L_{1,t+j}^{\nu} C_{1,t} = \frac{P_{1,t}^d}{P_{1,t}^c} \frac{W_{1,t}}{P_{1,t}^d} = exp\left(z_{1,t}\right) F_{1,t}^{\rho_c}. \tag{C.30}$$

where

$$F_{1,t} = \omega_{1,t}^c + (1-\omega_{1,t}^c)\delta_{1,t}^{-\frac{1}{\rho^c}} \tag{C.31}$$

as before. When $\nu = 0$ as under our baseline specification, Equation C.30 yields consumption as a function of the terms of trade and technology. If $\nu \neq 0$, it is of advantage to substitute out for the labor supply.

Using the demand functions for goods 1 and 2, the respective market clearing conditions imply

$$Y_{1,t}^d = C_{1,t}^d + M_{2,t} = A_{1,t} C_{1,t} \tag{C.32}$$

$$Y_{2,t}^d = C_{2,t}^d + M_{1,t} = A_{2,t} C_{2,t} \tag{C.33}$$

where

$$A_{1,t} = \left[\omega_1^c + (1-\omega_1^c)\delta_{1,t}^{-\frac{1}{\rho^c}} \frac{1}{1-\tilde{T}_{1,t}}\right] F_{1,t}^{-(1+\rho^c)} \tag{C.34}$$

$$A_{2,t} = \left[\omega_2^c + (1-\omega_2^c)\delta_{1,t}^{\frac{1}{\rho^c}} \frac{1-\tilde{T}_{1,t}}{\delta_{1,t}}\right] F_{2,t}^{-(1+\rho^c)} \tag{C.35}$$

$$1-\tilde{T}_{1,t} = \frac{1-\omega_{1,t}^c}{1-\omega_{2,t}^c} \left(\frac{F_{1,t}}{F_{2,t}}\right)^{-(1+\rho^c)} \delta_{1,t}^{1-2\frac{1+\rho^c}{\rho^c}} \frac{C_{1,t}}{C_{2,t}}. \tag{C.36}$$

Using Equation C.30 and the fact that $L_{1,t} = \frac{1}{\exp(z_{1,t})} Y^d_{1,t}$, we can express consumption as

$$C_{1,t} \;=\; \exp\left(z_{1,t}\right)\left(\frac{1}{\nu_0} F^{\rho^c}_{1,t}\left(A_{1,t}\right)^{-\nu}\right)^{\frac{1}{1+\nu}}. \tag{C.37}$$

C.3.1 Linearized Model

Approximating Equation C.37 around the deterministic steady state yields

$$\hat{C}_{1,t} \;=\; z_{1,t} - \left(1-\omega^c_1\right)\hat{\delta}_{1,t} - \frac{1}{2}\bar{\nu}\tilde{T}_{1,t}. \tag{C.38}$$

where $\bar{\nu} = 2\frac{\nu}{1+\nu}\left(1-\omega^c_1\right)$. If $\nu \neq 0$, the trade balance enters the consumption equation. Without additional algebra, it is easy to see how this feature impacts the correlation between relative consumption,

$$\hat{C}_{1,t} - \hat{C}_{2,t} \;=\; \left(z_{1,t} - z_{2,t}\right) - 2\left(1-\omega^c_1\right)\hat{\delta}_{1,t} - \bar{\nu}\tilde{T}_{1,t} \tag{C.39}$$

and the real exchange rate (or terms of trade). For shocks like the rebalancing shock that drive the terms of trade and the trade balance in opposite directions—i.e., negative covariance between the two variables—the (shock-specific) correlation between relative consumption and the real exchange rate (or terms of trade) is not perfectly negative.

Applying this insight when linearizing the definition of the trade balance in Equation C.36 and the risk sharing condition, the equilibrium dynamics of $\hat{\delta}_{1,t}$, $\tilde{T}_{1,t}$, and $\tilde{B}_{1,t}$ are governed by

$$-E_t\Delta z_{1,t+1} + E_t\Delta z_{2,t+1} + E_t\Delta\hat{\delta}_{1,t+1} + \bar{\nu}E_t\Delta\tilde{T}_{1,t+1} = \chi\tilde{B}_{1,t} + \xi^{UIP}_{1,t} \tag{C.40}$$

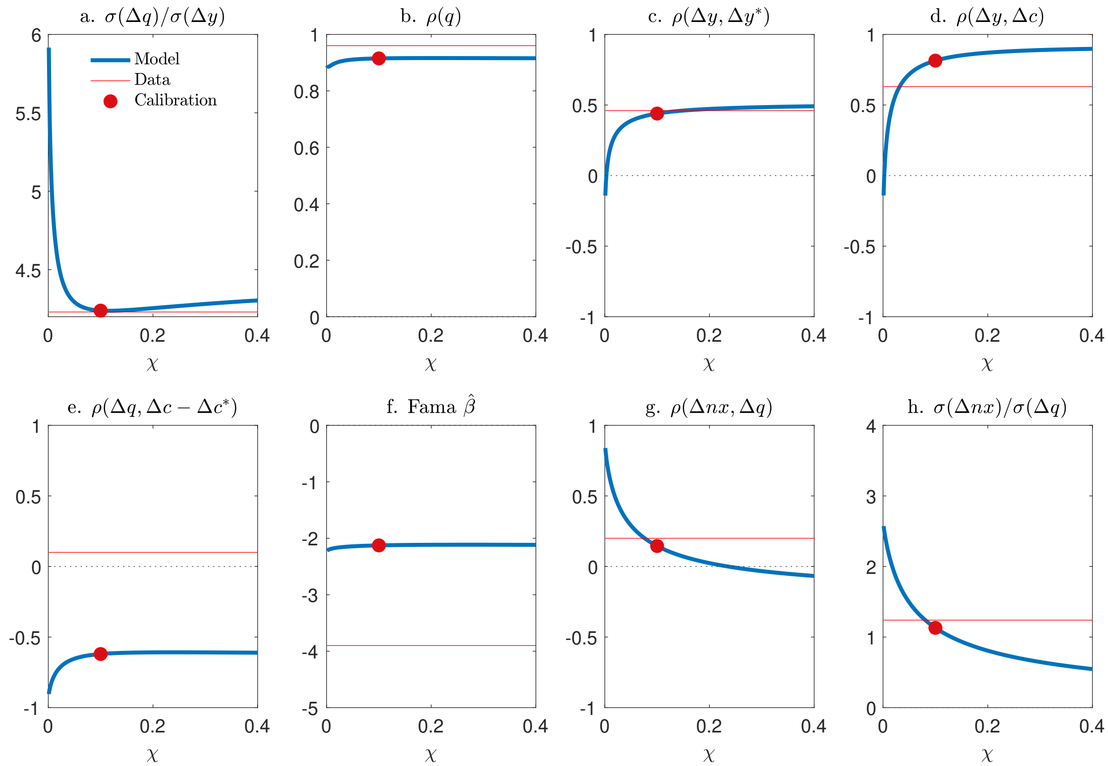
$$\beta\tilde{B}_{1,t} = \tilde{T}_{1,t} + \tilde{B}_{1,t-1} \tag{C.41}$$

$$\left(1-\bar{\nu}\right)\tilde{T}_{1,t} = \frac{\omega^c_1}{1-\omega^c_1}\xi^{trade}_{1,t} - \frac{\omega^c_1}{1-\omega^c_1}\xi^{trade}_{2,t} - z_{1,t} + z_{2,t} + \varpi\hat{\delta}_{1,t} \tag{C.42}$$

using the notation $\Delta x_t \equiv x_t - x_{t-1}$ to express variable x in terms of its growth rate.

Figure A.1 repeats the exercise from Figure 3 for $\nu = 1$.

Figure A.1: Standard Labor Supply and Exchange Rate Moments



Notes: The blue line shows theoretical moments for different values of χ. The red horizontal line indicates the corresponding empirical moment in each panel. The red dot shows our preferred calibration. The parameters used for this exercise are: $\omega^c = 0.915$, $\rho_{\xi^{trade}} = \rho_{\xi^{UIP}} = \rho_z = 0.91$, $100\sigma_{\xi^{trade}} = 0.75$,$100\sigma_{\xi^{UIP}} = 1.58$, $100\sigma_z = 1.1$, $\rho(z_1, z_2) = 0.51$.

C.4 Endogenous Discounting

Using a model with endogenous discounting as in Uzawa (1968) instead, Corsetti, Dedola, and Leduc (2008) show that a positive technology shock in the home country can induce an appreciation of the real exchange rate under specific assumptions on the value of the trade elasticity of substitution. They argue this to be the case if the trade elasticity is either very low or quite high (if paired with fairly persistent technology shocks). The assumption of Uzawa-type preferences is key to their work. As shown in Bodenstein (2011), the dynamics around the symmetric deterministic steady state are explosive for very low trade elasticities under the assumptions of Section 2, but stable under endogenous discounting.

This section serves two purposes. First, we establish that the model with endogenous discounting displays similar equilibrium dynamics as our baseline model for standard calibrations. Second, we demonstrate analytically the results of Corsetti, Dedola, and Leduc (2008).

Let the intertemporal preferences of the household in country 1 be

$$E_t \sum_{j=0}^{\infty} \Theta_{1,t+j+1} \left\{ \ln\left(C_{1,t+j}\right) - \frac{\nu_0}{1+\nu} L_{1,t+j}^{1+\nu} \right\}. \tag{C.43}$$

where

$$\Theta_{1,t+j+1} = \Psi_0 \left(1 + exp\left(\ln\left(C_{1,t+j}\right) - \frac{\nu_0}{1+\nu} L_{1,t+j}^{1+\nu}\right)\right)^{-\Psi} \Theta_{1,t+j}. \tag{C.44}$$

With this change in preferences and an analogous change for country 2, the risk sharing condition changes from A.27 to

$$E_t \left\{ \frac{P_{1,t}^c}{F_{1,t+1}^{\rho^c} P_{1,t+1}^c} \left[\frac{\Theta_{1,t+1}}{\Theta_{1,t}} \frac{exp(z_{1,t})}{exp(z_{1,t+1})} - \frac{\Theta_{2,t+1}}{\Theta_{2,t}} \frac{exp(z_{2,t})}{exp(z_{2,t+1})} \frac{\delta_{1,t}}{\delta_{1,t+1}} \right] \right\} = 0. \tag{C.45}$$

With the equations for the trade balance and the evolution of the NFA position unchanged, we linear system that governs the equilibrium dynamics of $\hat{\delta}_{1,t}$, $\tilde{T}_{1,t}$, and $\tilde{B}_{1,t}$ is given by

$$+\widehat{\Delta\Theta}_t - E_t \Delta z_{1,t+1} + E_t \Delta z_{2,t+1} + E_t \Delta \hat{\delta}_{1,t+1} + \bar{\nu} E_t \Delta \tilde{T}_{1,t+1} = \xi_{1,t}^{UIP} \tag{C.46}$$

$$\beta \tilde{B}_{1,t} = \tilde{T}_{1,t} + \tilde{B}_{1,t-1} \tag{C.47}$$

$$(1-\bar{\nu})\,\tilde{T}_{1,t} = \frac{\omega_1^c}{1-\omega_1^c} \xi_{1,t}^{trade} - \frac{\omega_1^c}{1-\omega_1^c} \xi_{2,t}^{trade} - z_{1,t} + z_{2,t} + \varpi \hat{\delta}_{1,t} \tag{C.48}$$

where differences in endogenous discounting across countries result in the wedge

$$\begin{aligned} \widehat{\Delta\Theta}_t &= -\tilde{\Psi}\left(\hat{C}_{1,t} - \hat{L}_{1,t}\right) + \tilde{\Psi}\left(\hat{C}_{2,t} - \hat{L}_{2,t}\right) \\ &= -\tilde{\Psi}\left(z_{1,t} - z_{2,t} - 2\left(1-\omega_1^c\right)\left(\hat{\delta}_{1,t} + \tilde{T}_{1,t}\right)\right), \end{aligned} \tag{C.49}$$

with $\tilde{\Psi} = \frac{exp\left(\ln(C) - \frac{\nu_0}{1+\nu} L^{1+\nu}\right)}{1+exp\left(\ln(C) - \frac{\nu_0}{1+\nu} L^{1+\nu}\right)} \Psi$.

We solve for the coefficients in the policy rule for the terms of trade using the method of undetermined coefficients

$$\hat{\delta}_{1,t} = \gamma_1^{end}\xi_{1,t}^{trade} + \gamma_2^{end}\xi_{2,t}^{trade} + \gamma_3^{end}\xi_{1,t}^{UIP} + \gamma_4^{end}z_{1,t} + \gamma_5^{end}z_{2,t} + \gamma_b^{end}\tilde{B}_{1,t-1} \quad \text{(C.50)}$$

with the parameters

$$\gamma_1^{end}: \quad \gamma_1^{end} = -\frac{\frac{1-\beta}{\beta} + 2\tilde{\Psi}\left(1-\omega_1^c\right)}{\varpi\left(\frac{1}{\beta} - \rho_1^{trade}\right)}\frac{\omega_1^c}{1-\omega_1^c} < 0 \quad \text{(C.51)}$$

$$\gamma_2^{end}: \quad \gamma_2^{end} = \frac{\frac{1-\beta}{\beta} + 2\tilde{\Psi}\left(1-\omega_1^c\right)}{\varpi\left(\frac{1}{\beta} - \rho_2^{trade}\right)}\frac{\omega_1^c}{1-\omega_1^c} > 0 \quad \text{(C.52)}$$

$$\gamma_3^{end}: \quad \gamma_3^{end} = -\frac{1}{\frac{1}{\beta} - \rho_1^{UIP}} < 0 \quad \text{(C.53)}$$

$$\gamma_4^{end}: \quad \gamma_4^{end} = \frac{\left(1-\rho_1^z\right) - \tilde{\Psi}}{\left(\frac{1}{\beta} - \rho_1^z\right)} + \frac{\frac{1-\beta}{\beta} + 2\tilde{\Psi}\left(1-\omega_1^c\right)}{\varpi\left(\frac{1}{\beta} - \rho_1^z\right)} \quad \text{(C.54)}$$

$$\gamma_5^{end}: \quad \gamma_5^{end} = -\frac{\left(1-\rho_2^z\right) - \tilde{\Psi}}{\left(\frac{1}{\beta} - \rho_2^z\right)} - \frac{\frac{1-\beta}{\beta} + 2\tilde{\Psi}\left(1-\omega_1^c\right)}{\varpi\left(\frac{1}{\beta} - \rho_2^z\right)} \quad \text{(C.55)}$$

$$\gamma_b^{end}: \quad \gamma_b^{end} = -\frac{\beta}{\varpi}\left(\frac{1-\beta}{\beta} + 2\tilde{\Psi}\left(1-\omega_1^c\right)\left(1+\varpi\right)\right) < 0. \quad \text{(C.56)}$$

With this information in hand, we can explore the conditions under which a positive technology shock causes an appreciation on impact. As the terms of trade and the real exchange rate are proportional to each other, a positive home (foreign) technology shock is followed by an appreciation if $\gamma_4^{end} < 0$ (or $\gamma_5^{end} > 0$). Assuming that the home technology shock follows (close to) a unit root process, γ_4^{end} reduces to

$$\gamma_4^{end} = \frac{-\tilde{\Psi}}{\frac{1}{\beta} - \rho_1^z} + \frac{\frac{1-\beta}{\beta} + 2\tilde{\Psi}\left(1-\omega_1^c\right)}{\varpi\left(\frac{1}{\beta} - \rho_1^z\right)}. \quad \text{(C.57)}$$

Recalling that the trade elasticity is given by $\frac{1+\rho^c}{\rho^c}$ and $\varpi = 1 + \frac{\omega^c}{\rho^c}$, the second term in Equation C.57 decreases as the trade elasticity rises and vanishes as the trade elasticity approaches infinity. By contrast, the first term is independent of the trade elasticity. Thus, there is a threshold value of the trade elasticity above which $\gamma_4^{end} < 0$. As for the low-elasticity case discussed in Corsetti, Dedola, and Leduc (2008), notice that ϖ is negative for low values of the trade elasticity, as is γ_4^{end}.

For completeness, we report the coefficients in the decision rule for the trade balance

$$\tilde{T}_{1,t} = \alpha_1^{end}\xi_{1,t}^{trade} + \alpha_2^{end}\xi_{2,t}^{trade} + \alpha_3^{end}\xi_{1,t}^{UIP} + \alpha_4^{end}z_{1,t} + \alpha_5^{end}z_{2,t} + \alpha_b^{end}\tilde{B}_{1,t-1} \tag{C.58}$$

which are given by

$$\alpha_1^{end}: \qquad \alpha_1^{end} = \left[1 - \frac{\frac{1-\beta}{\beta} + 2\tilde{\Psi}\left(1-\omega_1^c\right)}{\left(\frac{1}{\beta} - \rho_1^{trade}\right)}\right]\frac{\omega_1^c}{1-\omega_1^c} \tag{C.59}$$

$$\alpha_2^{end}: \qquad \alpha_2^{end} = -\left[1 - \frac{\frac{1-\beta}{\beta} + 2\tilde{\Psi}\left(1-\omega_1^c\right)}{\left(\frac{1}{\beta} - \rho_2^{trade}\right)}\right]\frac{\omega_1^c}{1-\omega_1^c} \tag{C.60}$$

$$\alpha_3^{end}: \qquad \alpha_3^{end} = -\frac{\varpi}{\frac{1}{\beta} - \rho_1^{UIP}} \tag{C.61}$$

$$\alpha_4^{end}: \qquad \alpha_4^{end} = \frac{(1-\rho_1^z) - \tilde{\Psi}}{\left(\frac{1}{\beta} - \rho_1^z\right)}\varpi - \left[1 - \frac{\frac{1-\beta}{\beta} + 2\tilde{\Psi}\left(1-\omega_1^c\right)}{\left(\frac{1}{\beta} - \rho_1^z\right)}\right] \tag{C.62}$$

$$\alpha_5^{end}: \qquad \alpha_5^{end} = -\frac{(1-\rho_2^z) - \tilde{\Psi}}{\left(\frac{1}{\beta} - \rho_2^z\right)}\varpi + \left[1 - \frac{\frac{1-\beta}{\beta} + 2\tilde{\Psi}\left(1-\omega_1^c\right)}{\left(\frac{1}{\beta} - \rho_2^z\right)}\right] \tag{C.63}$$

$$\alpha_b^{end}: \qquad \alpha_b^{end} = -\beta\left(\frac{1-\beta}{\beta} + 2\tilde{\Psi}\left(1-\omega_1^c\right)\left(1+\varpi\right)\right). \tag{C.64}$$

Finally, the coefficients in the decision rules for the NFA position

$$\tilde{B}_{1,t} = \beta_1^{end}\xi_{1,t}^{trade} + \beta_2^{end}\xi_{2,t}^{trade} + \beta_3^{end}\xi_{1,t}^{UIP} + \beta_4^{end}z_{1,t} + \beta_5^{end}z_{2,t} + \beta_b^{end}\tilde{B}_{1,t-1} \tag{C.65}$$

satisfy $\beta_i = \frac{\alpha_i}{\beta}$ for $i = \{1,2,3,4,5\}$ and $\beta_b^{end} = 1 - 2\tilde{\Psi}\left(1-\omega_1^c\right)\left(1+\varpi\right)$.

Endogenous discounting does not change the fundamental feature of the baseline model that the trade rebalancing shock can be a serious contender to the UIP shock as the driving force of exchange rate fluctuations.

D Appendix: Medium-scale Model

D.1 Model Description

The model consists of two country blocs, which we index with the subscript $j = \{1, 2\}$. The home country, which we associate with the U.S., is indexed as $j = 1$, and a foreign block, which we associate with the rest of the world, is indexed as $j = 2$. Country sizes are given by the parameter $n_j \in (0, 1)$, with the restriction that $n_1 + n_2 = 1$. Each country is populated by households, wholesale retailers, and intermediate goods producers.

Households. Each country has a continuum of risk-averse households of measure one. Each household consists of a continuum of workers who supply differentiated labor to firms through an employment agency. We assume perfect risk-sharing within the household. Households derive utility from consumption and bond holdings and disutility from labor.

Let $C_{j,t}$ denote households' consumption of the final good in country j, $B^k_{j,t}$ their holdings of the bonds denominated in the currency of country k, with price $P^{b,k}_{j,t}$, for $k = \{1, 2\}$. Also, $\Pi_{j,t}$ are the profits received from firms, $T_{j,t}$ are lump-sum taxes collected by the government, $n_{j,t}(i)$ the labor supply of differentiated labor variety $i \in (0, 1)$, and $w_{j,t}(i)$ its the associated nominal wage. Then households in country j choose $C_{j,t}, B^k_{j,t}$, and $\{n_{j,t}(i), w_{j,t}(i)\}$ to maximize their expected lifetime utility given by[21]

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \log\left(C_{j,t} - b\tilde{C}_{j,t-1}\right) - \frac{\psi_N}{1+\eta} \int n_{j,t}(i)^{1+\eta} di + \exp(\zeta^{RP}_{j,t}) \sum_{k=1}^{2} U(B^k_{j,t}) \right\}. \tag{D.1}$$

where $\eta > 0$ is the Frisch elasticity of labor supply, ψ_N controls the disutility from labor, $\xi^{RP}_{j,t}$ is a risk-premium shock that shifts the demand for bonds, and $\tilde{C}_{j,t}$ is aggregate consumption in country j, which implies that households exhibit external

[21] Later, when we log-linearize the equilibrium conditions, we will assume that in the non-stochastic steady state $\frac{U}{[C(1-b)]^{-1}} = 1$.

habits on their consumption decisions.[22] The households' budget constraint is

$$P_{j,t}C_{j,t} + \sum_{k=1}^{2} \frac{P_{j,t}^{b,k} B_{j,t}^{k}}{e_{j,t}^{k} \phi_{j,t}^{k}} = \int w_{2,t}(i) n_{2,t}(i) di + \sum_{k=1}^{2} \frac{B_{j,t-1}^{k}}{e_{j,t}^{k}} + \Pi_{j,t} + T_{j,t},$$

where $e_{j,t}^{k}$ is the price of currency from country k in units of currency of country j, the nominal exchange rate, and obviously $e_{j,t}^{k} = 1$ for $k = j$. As in our analytical model, $\phi_{j,t}^{k}$ captures the financial intermediation cost of trading bonds from country k in country j. The budget constraint states that purchasing final consumption and new bonds must equal total labor income, proceeds from existing bond holdings, and firms' profits, net of lump-sum taxes.

Asset Markets. International asset markets are incomplete, and the only asset traded internationally is the non-state-contingent bond denominated in the currency of country 1. Thus, international borrowing and lending occur only in U.S. dollars, which implies that $B_{1,t}^{2} = 0$. We assume that financial intermediation costs affect bond trade across borders but not within borders, thus $\phi_{j,t}^{k} = 1$ for $k = j$. With these restrictions the only non-trivial intermediation cost is $\phi_{2,t}^{1}$, which we assume the functional form $\phi_{2,t}^{1} = \exp\left(-\chi^{ms} \frac{\tilde{B}_{2,t}^{1}}{e_{2,t}^{1} P_{2,t} Y_{2,t}} + \xi_{1,t}^{UIP}\right)$, with $\chi^{ms} > 0$. Notice that we normalize the NFA position by aggregate output in this section and not by exports as we did in the analytical model for convenience. The term $\xi_{1,t}^{UIP}$ is the UIP shock that scales the return of the dollar bonds for foreign households.

Labor markets. Workers supply differentiated labor through employment agencies, which bundle the differentiated varieties into a homogeneous labor input $N_{j,t}$ and sell it to intermediate producers at a wage $W_{j,t}$. Therefore, the demand for the labor varieties is given by $n_{1,t}(i) = \left(\frac{w_{1,t}(i)}{W_{1,t}}\right)^{-(1+\mu_{1,t}^{w})/\mu_{1,t}^{w}} N_{1,t}$, where $\mu_{1,t}^{w} = \mu^{w} \exp(\xi_{1,t}^{w})$ and $\xi_{1,t}^{w}$ is a wage-markup shock. As in Erceg, Henderson, and Levin (2000), the employment agency sets the wages for each labor variety subject to Calvo-type frictions. The opportunity to reset the wage occurs with probability $1-\theta_{w}$ every period. There is no indexation when wages are not reset.

Final Consumption and Investment Goods. The production of the final

[22] For notation, we use X to denote the allocation by an individual agent whereas $\tilde{X}$ denotes the allocation by the representative agent.

consumption and investment goods is conducted by perfectly competitive firms and is symmetric between the two countries. We describe a generic country $j \in \{1,2\}$. The final consumption good $\mathcal{C}_{j,t}$ is produced by combining the final intermediate good $C^d_{j,t}$ and imports $M^c_{j,t}$ of the final intermediate according to

$$\mathcal{C}_{j,t} = \left[\omega_t^{1/\theta} C^d_{j,t}{}^{\frac{\theta-1}{\theta}} + (1-\omega_t)^{1/\theta} \left((1-\psi^C_{j,t}) M^c_{j,t} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

where $\theta > 1$ is the elasticity of substitution between home and foreign intermediate goods, with $\omega_t \equiv \omega \exp(\xi^\omega_{j,t})$, where $\omega \in (0.5, 1]$ is the home-bias parameter, and $\xi^\omega_{j,t}$ is a shock to the home bias in domestic consumption.

Similarly, the final investment good $I_{j,t}$ is produced as a composite of the final intermediate good $I^d_{j,t}$ and imports $M^i_{j,t}$ of the final intermediate from the other economy according to

$$I_{j,t} = \left[\omega^i_t{}^{1/\theta} I^d_{j,t}{}^{\frac{\theta-1}{\theta}} + \left(1-\omega^i_t\right)^{1/\theta} \left((1-\psi^i_{j,t}) M^i_{j,t} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

with $\omega^i_t \equiv \omega^i \exp(\xi^{\omega^i}_{j,t})$ where $\omega^i \in (0.5, 1]$ is the home-bias parameter for investment goods and $\xi^{\omega^i}_{j,t}$ is a shock to the home bias in domestic investment.

The import adjustment costs for consumption, $\psi^c_{j,t}$, and for investment, $\psi^i_{j,t}$, attenuate the response of imports to changes in relative prices in the short run, allowing for differences between the short-term and the long-term trade elasticities. We assume these adjustment costs take the quadratic form proposed in Erceg, Guerrieri, and Gust (2005).

Domestic and imported intermediate goods are bundles of a continuum of intermediate varieties aggregated according to the following technologies

$$Y^d_{j,t} = \left(\int_0^1 (Y^d_{j,t}(h))^{\frac{1}{1+\mu^d_{j,t}}} dh \right)^{1+\mu^d_{j,t}} \qquad M_{j,t} = \left(\int_0^1 (M_{j,t}(h))^{\frac{1}{1+\mu^M_{j,t}}} dh \right)^{1+\mu^M_{j,t}}$$

where $\mu^k_{j,t}$ are time-varying markups, defined as $\mu^k_{j,t} = \mu^k \exp(\xi^{\mu^k_j}_{j,t})$, for $k \in \{d, M\}$, with shocks $\xi^{\mu^k_j}_{j,t}$ following ARMA(1,1) processes.

In equilibrium, the supply of final consumption goods has to equal the consumption demand by households and the government, $\mathcal{C}_{j,t} = C_{j,t} + G_{j,t}$. The market clearing conditions for the demand for domestic and imported goods are $Y^d_{j,t} = C^d_{j,t} + I^d_{j,t}$, and $M_{j,t} = M^c_{j,t} + M^i_{j,t}$.

Intermediate Goods Producers. A continuum of perfectly competitive firms produces a homogeneous intermediate good sold to intermediate retailers. Intermediate producers rent labor from the employment agency and rent capital from capital goods producers to operate a Cobb-Douglas production technology $Y_{j,t} = \exp(\xi^A_{j,t})\bar{K}^{\alpha}_{j,t}N^{1-\alpha}_{j,t}$, where $\alpha \in (0,1)$, $\xi^A_{j,t}$ is an aggregate technology shock, and $\bar{K}_{j,t}$ is effective units of capital. We allow variable capital utilization, with $u_{j,t}$ the utilization rate. Therefore, effective capital is related to installed capital as follows: $\bar{K}_{j,t} = K_{j,t-1}u_{j,t}$. We assume that adjusting the utilization rate is costly and proportional to the level of capital, $\mathcal{A}(u_{j,t})K_{j,t-1}$, where $\mathcal{A}(u_{j,t})$ has the following functional form $\mathcal{A}(u_{j,t}) = r^K \frac{\exp(\xi(u_{j,t}-1)-1)}{\xi}$, where $\xi > 0$ and r^K is the steady-state rental rate of capital.

Capital Goods Producers. Every period, perfectly competitive firms, investment goods to augment the undepreciated capital stock $K_{j,t} = (1-\delta)K_{j,t-1} + \exp(\xi^I_{j,t})F(I_{j,t}, I_{j,t-1})$, where $\xi^I_{j,t}$ is a shock to the marginal efficiency of investment as in Justiniano, Primiceri, and Tambalotti (2010), and $F(I_{j,t}, I_{j,t-1}) = I_{j,t}\left[1 - S\left(\frac{I_{j,t}}{I_{j,t-1}}\right)\right]$ represents flow adjustments costs, and $S(.)$ is a convex adjustment cost function, as in Christiano, Eichenbaum, and Evans (2005b).

Intermediate Retailers. Intermediate retailers purchase the homogenous intermediate goods and produce a differentiated variety at no cost. Retailers operate in monopolistically competitive markets and sell the intermediate varieties to the wholesale retailers described earlier. We assume that intermediate retailers set prices as in Calvo (1983).

Each country has two types of retailers: domestic retailers who sell in local markets and exporters. With probability $1-\theta_{p,j}$ the domestic retailer chooses optimal reset prices $P^o_{j,t}(h)$ to maximize their profits given by

$$\mathbb{E}_t \sum_{i=0}^{\infty} (\theta_{p,j})^i \Lambda_{j,t,t+i} \left(\frac{P^{o,d}_{j,t}(h)}{P^c_{j,t+i}} - MC_{j,t+i} \right) Y^d_{j,t+i}(h)$$

where $MC_{j,t}$ is the price of the homogeneous intermediate input sold to retailers, and $\Lambda_{j,t,t+1} \equiv \beta \frac{\tilde{C}_{j,t} - b\tilde{C}_{j,t-1}}{\tilde{C}_{j,t+1} - b\tilde{C}_{j,t}}$ is the households' stochastic discount factor in country j. Also, let $P^d_{j,t}$, and $P^M_{j,t}$ be the prices of the domestic and non-domestic consumption goods in the country j. There is no indexation when prices are not reset.

Exporters set their prices in the currency where the good is sold, therefore featuring "local-currency pricing" (LCP). They are also subject to price-setting frictions and reoptimize their price with probability $1 - \theta^x_{p,j}$, with $\theta^x_{p,j}$ potentially different than $\theta_{p,j}$. Their optimal export price $P^{o,x}_{j,t}(h)$ is chosen to maximize profits given by

$$\mathbb{E}_t \sum_{i=0}^{\infty} (\theta^x_{p,j})^i \Lambda_{j,t,t+i} \left(\frac{P^{o,x}_{j,t}(h)}{P^c_{j,t+i}} \frac{1}{e_{j,t+i}} - MC_{j,t+i} \right) X_{j,t+i}(h),$$

where $X_{j,t}(h) \equiv M_{-j,t}(h)$ is the quantity of variety h exported from country j into the country different than j. From the LCP assumption, $P^x_{j,t}(h) \equiv P^M_{-j,t}(h)$, is the price of imported goods from country j into the country different than j.

Monetary and Fiscal Policy. The government issues no debt and runs a balanced budget by financing its expenditures with lump-sum taxes levied on the households, that is, $T_{j,t} = G_{j,t}$. Government expenditures are given by $G_{j,t} = \exp(\xi^G_{j,t})G$, where G is the steady-state level and $\xi^G_{j,t}$ is a government expenditure shock.

The monetary authority in each country sets nominal interest rates following a Taylor-like rule which reacts to inflation and the output gap:

$$\frac{R_{j,t}}{R} = \left(\frac{R_{j,t-1}}{R} \right)^{\varphi_R} \left[\left(\frac{\bar{\pi}_{j,t}}{\exp(\xi^\pi_{j,t})\bar{\pi}_j} \right)^{\varphi_\pi} \left(\frac{Y_{j,t}}{Y^{flex}_{j,t}} \right)^{\varphi_Y} \right]^{1-\varphi_R} \exp(\xi^R_{j,t}) \tag{D.2}$$

where, $\bar{\pi}_{j,t} = \left(\prod_{s=0}^{3} \pi_{j,t-s} \right)^{1/4}$ is the 4-quarter average inflation with $\pi_{j,t} \equiv \frac{P^c_{j,t}}{P^c_{j,t-1}}$, and $Y^{flex}_{j,t}$ is aggregate output in the flexible-price version of the economy, $\xi^\pi_{j,t}$ is a shock to the inflation target $\bar{\pi}_j$, and $\xi^R_{j,t}$ is a monetary policy shock. The inflation target follows an AR(1) process common across blocks.

D.2 Calibrated Parameters

We calibrate most model parameters to standard values in the literature. Table A.1 reports the calibrated parameters. On the household side, we set the (inverse) Frisch elasticity to 1 and target total hours of 0.33 in steady state. External habit is set at 0.8, and we target an interest rate of 4 percent annually.[23] The wage stickiness parameter, θ_w, is set to 0.85, and we target an average wage markup of ten percent. Regarding production-related parameters, we target a labor share of 65 percent and a depreciation rate of 2.5 percent per quarter and set the curvature parameters of the cost of capital utilization and investment adjustment to 1 and 5, respectively. The "Calvo" parameter controlling the frequency of domestic price changes is set to 0.85, which implies a frequency of price adjustment of about seven quarters. The Calvo parameter for internationally traded goods is set to 0.85 to balance short- and medium-term pass-through estimates.

We set $\theta = 1.5$ and $\psi^C = \psi^i = 10$ for the parameters affecting trade flows, implying a long-run and a short-run trade elasticity of 1.5 and around 0.5, respectively. Regarding country size, we assume that the U.S. is 25 percent of the world economy. We set the share of imported goods in U.S. consumption to 5 percent and 50 percent for U.S. investment. For the rest of the world, we re-scale these two moments proportionally to obtain balanced trade in a steady state.

Concerning policy parameters, we set the government expenditure-to-GDP ratio to 22 percent and assume that the Taylor rule responds to the lagged interest rate with a weight of 0.8 to 4-quarter average inflation with a weight of $1.5 \times (1 - 0.8)$, and to the output gap with a weight of $0.125 \times (1 - 0.8)$.

D.3 Data and Inference

We estimate the model using quarterly data for real growth in GDP, consumption, and investment, GDP deflator inflation, and policy rates for the U.S. and the rest of the world. For the U.S., we also use data on the broad real dollar index, real wage growth, labor gap, export and import-to-GDP ratios, and inflation expectations.

[23] Our measurement equation for the interest rate adjusts the intercept to be consistent with average short-term interest rates.

Table A.1: Calibrated Parameters

Parameter	Description	Value
η	Inverse Frisch Elasticity	1
b	Consumption Habit	0.8
$\bar{R}$	Nominal Interest Rate Steady State	1.01
θ	Elasticity of Substitution Domestic Foreign Good	1.5
ψ^C, ψ^i	Trade Adjustment Costs	10
θ_w	Calvo Wage	0.85
θ_p	Calvo Domestic	0.85
θ_p^x	Calvo Export	0.85
α	Capital Elasticity Production	0.29
δ_k	Depreciation Capital	0.025
S''	Investment Adjustment Costs	5
ξ	Slope Utilization	1
$\bar{g}$	Government Expenditures Share GDP	0.22
φ_R	Inertia Taylor Rule	0.8
φ_π	Taylor Rule Inflation Response	1.5
φ_Y	Taylor Rule Output Gap Response	0.125

Notes: This table lists the parameters that are calibrated to values shown here. See the text for the details on the calibration targets. We omit scale parameters like $\bar{\psi}_1$, as they depend on estimated parameters and vary across different exercises.

All series run from 1985Q1 to 2019Q2 and are constructed as in Bodenstein, Cuba-Borda, Gornemann, Presno, Prestipino, Queraltó, and Raffo (2023). In matching the data to the model, we allow for intercept terms in the measurement equations and measurement error in all the observable time series of the rest of the world. Details on the data construction are provided in Appendix F.[24]

We use Bayesian methods to estimate the parameters governing the shock processes and the parameter χ^{ms} that governs the costly international financial intermediation process. Table A.2 gives details of the prior specification for the persistence and standard deviations of the exogenous processes. When estimating the parameter χ^{ms}, we either use a wide uniform prior or re-estimate the model along a grid, fixing the values of χ^{ms} a-priori. We discuss the results of both estimation strategies in the next section.

[24] Measurement error is set to equal five percent of the in-sample variance of the underlying series. We use DYNARE to implement a standard RWMH algorithm for our estimation. See Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto, and Villemot (2011) for details.

Table A.2: Estimated Parameters - Medium-Scale Model

Parameter	Description	Prior Distribution		Posterior Distribution	
		Family	[P(1), P(2)]	U.S.	Rest of World
Financial Integration					
χ^{ms}	Fin. Intermediation Cost	$\mathcal{U}$	$[0, 0.2]$	–	0.09 [0.05,0.14]
Standard Deviations					
$100 \times \sigma_R$	Monetary Policy Shock	$\mathcal{IG}$	$[0.1, 0.1]$	0.09 [0.08,0.10]	0.12 [0.09,0.16]
$100 \times \sigma_G$	Government Policy Shock	$\mathcal{IG}$	$[1, 5]$	2.04 [1.83,2.24]	0.63 [0.34,0.89]
$100 \times \sigma_I$	MEI Shock	$\mathcal{IG}$	$[1, 5]$	2.75 [2.37,3.11]	3.44 [2.68,4.19]
$100 \times \sigma_{RP}$	Risk Premium Shock	$\mathcal{IG}$	$[1, 1]$	0.88 [0.68,1.07]	0.24 [0.17,0.31]
$100 \times \sigma_A$	TFP Shock	$\mathcal{IG}$	$[1, 5]$	0.48 [0.43,0.53]	1.39 [1.05,1.72]
$100 \times \sigma_\omega$	Home Bias Shock	$\mathcal{IG}$	$[1, 5]$	1.93 [1.69,2.18]	0.91 [0.78,1.03]
$100 \times \sigma_\mu$	Markup Shock	$\mathcal{IG}$	$[1, 5]$	2.43 [2.16,2.66]	1.42 [1.14,1.71]
$100 \times \sigma_W$	Wage Markup Shock	$\mathcal{IG}$	$[1, 5]$	1.43 [1.29,1.59]	–
$100 \times \sigma_\pi$	Inflation Trend Shock	$\mathcal{IG}$	$[0.01, 0.1]$	0.05 [0.04,0.05]	0.05 [0.04,0.05]
$100 \times \sigma_{UIP}$	UIP Shock	$\mathcal{IG}$	$[1, 5]$	–	0.22 [0.16,0.27]
Persistence Parameters					
ρ_R	Monetary Policy Shock	$\mathcal{B}$	$[0.4, 0.125]$	0.68 [0.63,0.74]	0.50 [0.35,0.66]
ρ_G	Government Policy Shock	$\mathcal{B}$	$[0.6, 0.125]$	0.94 [0.92,0.96]	0.77 [0.59,0.93]
ρ_I	MEI Shock	$\mathcal{B}$	$[0.6, 0.125]$	0.87 [0.83,0.91]	0.47 [0.32,0.63]
ρ_{RP}	Risk Premium Shock	$\mathcal{B}$	$[0.6, 0.125]$	0.74 [0.69,0.80]	0.95 [0.92,0.97]
ρ_A	TFP Shock	$\mathcal{B}$	$[0.6, 0.125]$	0.96 [0.94,0.98]	0.90 [0.82,0.98]
ρ_ω	Home Bias Shock	$\mathcal{B}$	$[0.6, 0.125]$	0.90 [0.88,0.92]	0.80 [0.74,0.86]
ρ_μ	Markup Shock	$\mathcal{B}$	$[0.6, 0.125]$	0.93 [0.91,0.95]	0.62 [0.47,0.78]
θ_μ	MA Price Markup Shock	$\mathcal{B}$	$[0.5, 0.125]$	0.52 [0.41,0.63]	0.41 [0.22,0.59]
ρ_W	Wage Markup Shock	$\mathcal{B}$	$[0.6, 0.125]$	0.55 [0.38,0.71]	–
θ_W	MA Wage Markup Shock	$\mathcal{B}$	$[0.5, 0.125]$	0.53 [0.39,0.69]	–
ρ_π	Inflation Trend Shock	$\mathcal{B}$	$[0.995, 0.002]$	0.62 [0.47,0.78]	0.62 [0.47,0.78]
ρ_{UIP}	UIP Shock	$\mathcal{B}$	$[0.6, 0.125]$	–	0.99 [0.99,1.00]

Notes: $\mathcal{U}$ is Uniform distribution; $\mathcal{IG}$ is Inverse Gamma distribution; $\mathcal{B}$ is Beta distribution. P(1) and P(2) are the mean and standard deviations for Beta and Inverse Gamma distributions. For Uniform distributions, P(1) and P(2) represent the hyper-parameters determining the lower and upper bound of the support of the distribution. The table reports the posterior mean and the 90% credible set in square brackets. We omit the level shifters in the measurement equations. We rounded to the second decimal. As a result, ρ_{UIP} lies between 0.99 and 1.00. Using greater decimal precision, the interval is $[0.9876, 0.9951]$.

D.4 Estimation Results

Table A.2 shows the posterior distribution of parameter estimates. The estimated persistence of UIP shocks is close to unity, while the home bias and productivity shocks have significantly lower persistence. Notably, UIP shocks are not as volatile as in our calibrated example, and the trade rebalancing shocks, both at home and abroad, are over five times more volatile than UIP shocks and about two times more volatile than total factor productivity (TFP) shocks. Nonetheless, as we show later in this section, UIP shocks still significantly contribute to the exchange rate. This is consistent with our theoretical prediction that puts weight on both exogenous and endogenous UIP deviations to match key moments in the data.

As previously discussed, the transmission of endogenous UIP deviations hinges on the degree of financial integration. The top row in Table A.2 shows the prior and posterior distribution of the parameter χ^{ms}. Although there is substantial uncertainty in the estimation, the posterior distribution peaks at an estimated value of $\chi^{ms} = 0.09$. We note that this estimated value of χ^{ms} is crucial to recovering the endogenous deviations of UIP from the data. Using χ^{ms} to induce first-order stationary dynamics results in a substantial deterioration of model fit.

D.5 Additional Moment Simulations in Medium Scale Model

Table A.3: Exchange Rate Moments - Additional Specifications

		Full Model			
		$\chi^{ms} = 0.09$	$\chi^{ms} = 0.001$	$\chi^{ms} = 0.09$	$\chi^{ms} = 0.001$
		$\psi_i = 10$	$\psi_i = 10$	$\psi_i = 0$	$\psi_i = 0$
	Data	(Benchmark)			
Disconnect and PPP puzzles					
$\sigma\Delta q/\sigma\Delta y$	4.23	3.7	7.0	1.6	2.1
$\rho(q)$	0.96	0.9	0.9	0.9	0.9
International Co-movement					
$\rho(\Delta y, \Delta y^*)$	0.46	0.1	0.1	-0.8	-0.8
$\rho(\Delta y, \Delta c)$	0.63	0.6	0.5	0.1	0.0
Backus-Smith and Forward Premium					
$\rho(\Delta q, \Delta c - \Delta c^*)$	0.10	0.2	-0.2	0.2	-0.2
Fama (real) $\hat{\beta}$	-0.23	-0.7	-0.9	-1.0	-1.0
RER and NX					
$\rho(\Delta\tilde{T}, \Delta q)$	0.20	0.3	0.7	-0.7	0.1
$\sigma(\Delta\tilde{T})/\sigma(\Delta q)$	1.24	1.0	1.2	2.9	2.8

Notes: Medium-scale models computed using 1000 simulations drawn from the estimated innovations at the posterior mean. Each simulation has a length of 138 quarters to match the observations in our data sample from 1985Q1 to 2019Q2.

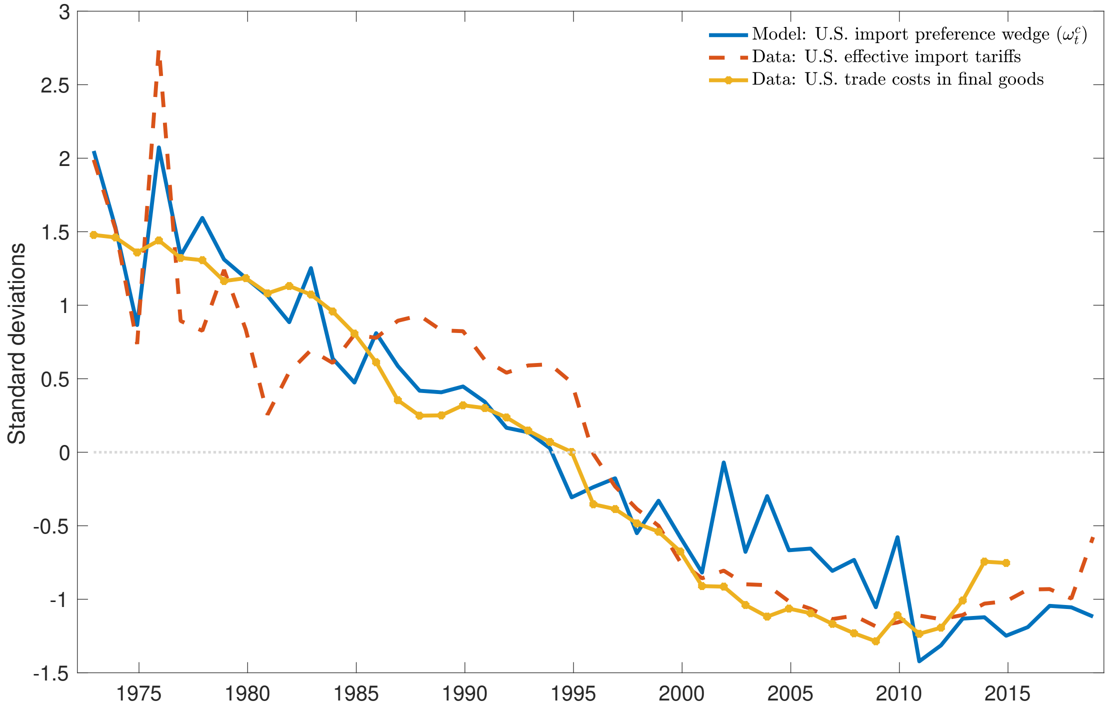
E Appendix: External Validation

To provide external validity of the rebalancing shocks we compare the model implied import wedge ω_t^c extracted from our estimation, with two measures measuring the relative cost of imports in the data. For comparability, we report the resulting time series at the annual frequency for comparability and in standard deviation units.

First, we measure the effective import tariff rate using data from the U.S. Federal Government's Current Receipts and Expenditures from the National Income and Products Accounts (Table 3.2). We collect total custom duties, CD_t, and construct import tariffs as $\tau_t = \frac{CD_t}{CD_t + M_t}$, where M_t is the dollar value of total imports, net of tariffs. Second, we construct a broad measure of trade costs using data from the World Input-Output Database. We follow the methodology from (Cuba-Borda, Reyes-Heroles, Queralto, and Scaramucci 2024) to recover bilateral trade costs in final consumption goods. This measure of trade costs encompasses tariffs, non-tariff barriers such as trade quotas, and any other barrier to trade, such as shipping costs or trade uncertainty. We compute these bilateral trade costs for the U.S. and 25 trading partners from 1972 to 2014.

Figure A.2 shows that the model implied trade wage, ω_t^c, shown in the blue solid line, tracks the evolution of the two empirical counterparts in the data. Our model's trade wedge picks up the secular decline in trade costs and tariffs and some important trade shocks, such as President Ford's tariff on oil imports in 1975 or the increase in trade costs in 2009 associated with the Great Trade Collapse.

Figure A.2: U.S. Tariffs and Model Implied Home Bias



F Appendix: Data

We estimate the exchange rate moments presented in Section 3 and the medium-scale model in Section 5 using quarterly macroeconomic data of a trade-weighted sum of foreign economies and the United States.

We collect quarterly time series for the following 35 country/blocs: Argentina, Australia, Brazil, Bulgaria, Canada, Colombia, Chile, China, Croatia, Czech Republic, Denmark, Euro Area, Hong Kong, Hungary, India, Indonesia, Israel, Japan, Malaysia, Mexico, New Zealand, Philippines, Poland, Romania, Russian Federation, Saudi Arabia, Singapore, South Africa, South Korea, Sweden, Taiwan, Thailand, Turkey, United Kingdom and the United States. Our sample of countries represents about 85 % of PPP-adjusted world GDP in 2019. Unless otherwise note, all data is seasonally adjusted. Our data covers the period 1985Q1-2019Q2.

Below we list the variables used in the analysis together with the required transformations. For a detailed list of data sources, see (Bodenstein, Cuba-Borda, Gornemann, Presno, Prestipino, Queraltó, and Raffo 2023).

F.1 Interest rates and Inflation

For consistency across countries, we measure the policy rate using the money market interest rate where available, otherwise we use the deposit rate. Our measure of inflation is the quarterly log change in the GDP implicit price deflator expressed in annual rate terms. The real interest rate is the difference between the nominal interest rate and ex-post one-period ahead GDP deflator inflation.

F.2 Real GDP, consumption and investment

We source nominal GDP, nominal personal consumption expenditures, nominal gross private investment, from quarterly national accounts data obtained through Haver Analytics. We convert GDP and its components to per capita terms using the "Resident Working Age Population: 15-64 years" or an equivalent concept from the United Nations World Population Prospects database. We linearly interpolate annual population estimates to quarterly frequency. We use the implicit GDP price deflator to express all variables in real terms.

F.3 Real exchange rate

Our measure of the real exchange rate is the ”Real Broad Effective Exchange Rate” for United States obtained from FRED and extended back in time using trade-weighted averages of real effective exchange rate of major trading partners.

F.4 Trade flows

We collect data of U.S. nominal imports of goods and services and U.S. nominal exports of goods and services from the Bureau of Economic Analysis. For the exchange rate moments reported in Section 3 we construct exports minus imports relative to total exports. For the estimation of the medium-scale model in Section 5 we deflate nominal trade flows using the implict GDP price deflator.